

MathWorks

11

Workbook Solutions

MathWorks 11 Workbook

SOLUTIONS

Pacific Educational Press
Vancouver, Canada

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ISBN 978-1-926966-05-2

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Chapter 1

Slope and Rate of Change

1.1

Rise Over Run

REVIEW: WORKING WITH RATIO AND SLOPE

BUILD YOUR SKILLS, p. 11

1. Calculate the ratio of concentrate to water.

$$\text{concentrate:water} = 250:750$$

$$\text{concentrate:water} = (250 \div 250):(750 \div 250)$$

$$\text{concentrate:water} = 1:3$$

2. Calculate the ratio of blue paint to yellow paint.

$$\frac{\text{blue paint}}{\text{yellow paint}} = \frac{2.3}{1.7}$$

$$\frac{\text{blue paint}}{\text{yellow paint}} = \frac{2.3 \times 10}{1.7 \times 10}$$

$$\frac{\text{blue paint}}{\text{yellow paint}} = \frac{23}{17}$$

3. a) Calculate the ratio of oats to raisins.

$$\text{oats:raisins} = 6:1$$

- b) Calculate the ratio of almonds to coconut.

$$\text{almonds:coconut} = 2:\frac{3}{4}$$

$$\text{almonds:coconut} = (2 \times 4):(\frac{3}{4} \times 4)$$

$$\text{almonds:coconut} = 8:3$$

- c) Calculate the total amount of ingredients.

$$\text{total} = \text{oats} + \text{almonds} + \text{raisins} + \text{coconut}$$

$$\text{total} = 6 + 2 + 1 + \frac{3}{4}$$

$$\text{total} = 9\frac{3}{4} \text{ cups}$$

Calculate the ratio of oats to total ingredients.

$$\text{oats:total} = 6:9\frac{3}{4}$$

$$\text{oats:total} = 6:(\frac{36}{4} + \frac{3}{4})$$

$$\text{oats:total} = 6:(\frac{39}{4})$$

$$\text{oats:total} = (6 \times 4):(\frac{39}{4} \times 4)$$

$$\text{oats:total} = 24:39$$

$$\text{oats:total} = (24 \div 3):(39 \div 3)$$

$$\text{oats:total} = 8:13$$

4. Set up a proportion to solve for x , the number of cans of paint needed.

$$\frac{4}{15} = \frac{x}{20}$$

$$20 \times \frac{4}{15} = \frac{x}{20} \times 20$$

$$\frac{20 \times 4}{15} = x$$

$$5.3 \text{ L} \approx x$$

About 5.3 L of oil will be needed.

5. Set up a proportion to solve for x , the number of centimetres used to represent 45 km of actual ground.

$$\frac{10}{25} = \frac{x}{45}$$

$$45 \times \frac{10}{25} = \frac{x}{45} \times 45$$

$$\frac{45 \times 10}{25} = x$$

$$18 \text{ cm} = x$$

On the map, 45 km will be represented as 18 cm.

6. Set up a proportion to solve for x , the amount of corn needed.

$$\frac{3}{1.5} = \frac{x}{4.5}$$

$$4.5 \times \frac{3}{1.5} = \frac{x}{4.5} \times 4.5$$

$$\frac{4.5 \times 3}{1.5} = x$$

$$9 \text{ cups} = x$$

NEW SKILLS: WORKING WITH SLOPE

BUILD YOUR SKILLS, P. 14

7. a) $m = \frac{\text{rise}}{\text{run}}$

$$m = \frac{3}{18}$$

$$m = \frac{1}{6} \text{ or approx. } 0.17$$

b) $m = \frac{\text{rise}}{\text{run}}$

$$m = \frac{1.25}{5}$$

$$m = \frac{1}{4} \text{ or } 0.25$$

c) $m = \frac{\text{rise}}{\text{run}}$

$$m = \frac{10}{8}$$

$$m = \frac{10 \div 2}{8 \div 2}$$

$$m = \frac{5}{4} \text{ or } 1.25$$

d) $m = \frac{\text{rise}}{\text{run}}$

$$m = \frac{8}{18}$$

$$m = \frac{8 \div 2}{18 \div 2}$$

$$m = \frac{4}{9} \text{ or approx. } 0.44$$

e) $m = \frac{\text{rise}}{\text{run}}$

$$m = \frac{2.4}{3.5}$$

$$m = \frac{2.4 \times 10}{3.5 \times 10}$$

$$m = \frac{24}{35} \text{ or approx. } 0.69$$

8. Set up a proportion to solve for x , the run.

$$m = \frac{\text{rise}}{\text{run}}$$

$$\frac{3}{190} = \frac{400}{x}$$

$$x \times \frac{3}{190} = \frac{400}{x} \times x$$

$$x \times \frac{3}{190} = 400$$

$$x = 400 \div \frac{3}{190}$$

$$x \approx 25\,333 \text{ m}$$

The hill covers a horizontal distance of about 25 333 m.

9. Solve for x , the run of the staircase.

$$m = \frac{\text{rise}}{\text{run}}$$

$$0.95 = \frac{210}{x}$$

$$x \times 0.95 = \frac{210}{x} \times x$$

$$x = \frac{210}{0.95}$$

$$x \approx 221 \text{ cm}$$

The run of the staircase is about 221 cm.

10. Solve for x , the rise of the street.

$$m = \frac{\text{rise}}{\text{run}}$$

$$0.54 = \frac{x}{28}$$

$$28 \times 0.54 = x$$

$$15.1 \text{ m} \approx x$$

The rise of the street is about 15.1 m.

11. $m = \frac{\text{rise}}{\text{run}}$

$$m = \frac{3.5}{6.0}$$

$$m = \frac{35}{60}$$

$$m = \frac{35 \div 5}{60 \div 5}$$

$$m = \frac{7}{12} \text{ or approx. } 0.58$$

12. Solve for x , the run of the stairway.

$$m = \frac{\text{rise}}{\text{run}}$$

$$0.89 = \frac{203}{x}$$

$$x \times 0.89 = \frac{203}{x} \times x$$

$$x \times 0.89 = 203$$

$$x = \frac{203}{0.89}$$

$$x \approx 228 \text{ cm}$$

The run of the stairway is about 228 cm.

13. Solve for x , the rise of the slide.

$$m = \frac{\text{rise}}{\text{run}}$$

$$\frac{17}{10} = \frac{x}{1.5}$$

$$\frac{1.5 \times 17}{10} = x$$

$$2.55 \text{ m} = x$$

PRACTICE YOUR NEW SKILLS, p. 17

1.

Rise	Run	Slope	
		As a fraction ($m = \frac{\text{rise}}{\text{run}}$)	As a decimal
18 m	63 m	$m = \frac{18}{63}$ $m = \frac{18 \div 9}{63 \div 9}$ $m = \frac{2}{7}$	$m = \frac{2}{7}$ $m \approx 0.29$
21 m	49 m	$m = \frac{21}{49}$ $m = \frac{21 \div 7}{49 \div 7}$ $m = \frac{3}{7}$	$m = \frac{3}{7}$ $m \approx 0.43$
1.2 cm	0.6 cm	$m = \frac{1.2}{0.6}$ $m = 2$	2
12.4 mm	4.6 mm	$m = \frac{12.4}{4.6}$ $m = \frac{124}{46}$ $m = \frac{124 \div 2}{46 \div 2}$ $m = \frac{62}{23}$	$m = \frac{62}{23}$ $m \approx 2.70$
300 ft	900 ft	$m = \frac{300}{900}$ $m = \frac{1}{3}$	$m = \frac{1}{3}$ $m \approx 0.33$

2.

Rise	Run	Slope
15 ft	60 ft	$m = \frac{\text{rise}}{\text{run}}$ $m = \frac{15}{60}$ $m = \frac{1}{4}$
12 cm	$m = \frac{\text{rise}}{\text{run}}$ $0.375 = \frac{12}{\text{run}}$ $\text{run} = \frac{12}{0.375}$ $\text{run} = 32 \text{ cm}$	0.375
$m = \frac{\text{rise}}{\text{run}}$ $\frac{9}{5} = \frac{\text{rise}}{16}$ $16 \times \frac{9}{5} = \text{rise}$ $28.8 \text{ m} = \text{rise}$	16 m	$\frac{9}{5}$
$m = \frac{\text{rise}}{\text{run}}$ $\frac{32}{7} = \frac{\text{rise}}{42}$ $42 \times \frac{32}{7} = \text{rise}$ $192 \text{ in} = \text{rise}$	42 in	$\frac{32}{7}$
63 m	$m = \frac{\text{rise}}{\text{run}}$ $3.0 = \frac{63}{\text{run}}$ $\text{run} = \frac{63}{3.0}$ $\text{run} = 21 \text{ m}$	3.0
19.5 ft	$m = \frac{\text{rise}}{\text{run}}$ $0.25 = \frac{19.5}{\text{run}}$ $\text{run} = \frac{19.5}{0.25}$ $\text{run} = 78 \text{ ft}$	0.25

3. $m = \frac{\text{rise}}{\text{run}}$

$$m = \frac{56}{120}$$

$$m = \frac{56 \div 8}{120 \div 8}$$

$$m = \frac{7}{15} \text{ or approx. } 0.47$$

4. Set up a proportion to solve for
- x
- , the horizontal distance from the base of the ladder to the house.

$$\frac{4}{1} = \frac{22}{x}$$

$$x \times 4 = \frac{22}{x} \times x$$

$$4x = 22$$

$$x = \frac{22}{4}$$

$$x = 5.5 \text{ ft}$$

The base of the ladder should be 5.5 ft from the house.

5. Set up a proportion to solve for
- x
- , the rise of the ditch.

$$\frac{3}{1.5} = \frac{x}{25}$$

$$\frac{25 \times 3}{1.5} = x$$

$$50 \text{ cm} = x$$

The drainage ditch will drop 50 cm over a horizontal distance of 25 m.

6. $m = \frac{\text{rise}}{\text{run}}$

$$0.64 = \frac{x}{32}$$

$$32 \times 0.64 = x$$

$$20.5 \text{ m} \approx x$$

The hill will rise 20.5 m.

7. Calculate the difference in elevation.

$$895 - 752 = 143 \text{ m}$$

Convert to kilometres. $143 \text{ m} = 0.143 \text{ km}$

Calculate the slope.

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{0.143}{1.2}$$

$$m = \frac{143}{1200} \text{ or approx. } 0.12$$

8. Roof:

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{3.2}{8}$$

$$m = \frac{32}{80}$$

$$m = \frac{32 \div 16}{80 \div 16}$$

$$m = \frac{2}{5} \text{ or } 0.4$$

Trusses:

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{1.6}{4}$$

$$m = \frac{16}{40}$$

$$m = \frac{16 \div 8}{40 \div 8}$$

$$m = \frac{2}{5} \text{ or } 0.4$$

The roof and the trusses have the same slope, 0.4.

9. The run of the actual house will be half the total width.

$$10.8 \text{ m} \div 2 = 5.4 \text{ m}$$

The run of the doll house will be half the total width.

$$1.6 \text{ m} \div 2 = 0.8 \text{ m}$$

Set up a proportion to solve for x , the rise of the roof.

$$\frac{2.4}{5.4} = \frac{x}{0.8}$$

$$\frac{0.8 \times 2.4}{5.4} = x$$

$$0.36 \text{ m} \approx x$$

The rise of the doll house roof will be about 0.36 m.

1.2

Grade, Angle of Elevation, and Distance

REVIEW: THE PYTHAGOREAN THEOREM AND THE TANGENT RATIO

BUILD YOUR SKILLS, p. 22

1. a) $h^2 = a^2 + b^2$

$$h^2 = 1.9^2 + 4.3^2$$

$$h = \sqrt{1.9^2 + 4.3^2}$$

$$h \approx 4.7 \text{ cm}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan G = \frac{1.9}{4.3}$$

$$G = \tan^{-1}\left(\frac{1.9}{4.3}\right)$$

$$G \approx 23.8^\circ$$

b) $r^2 = a^2 + b^2$

$$r^2 = 7.8^2 + 8.2^2$$

$$r = \sqrt{7.8^2 + 8.2^2}$$

$$r \approx 11.3 \text{ m}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan S = \frac{8.2}{7.8}$$

$$S = \tan^{-1}\left(\frac{8.2}{7.8}\right)$$

$$S \approx 46.4^\circ$$

c) $f^2 = a^2 + b^2$

$$f^2 = 6.5^2 + 9.4^2$$

$$f = \sqrt{6.5^2 + 9.4^2}$$

$$f \approx 11.4 \text{ in}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan E = \frac{9.4}{6.5}$$

$$E = \tan^{-1}\left(\frac{9.4}{6.5}\right)$$

$$E \approx 55.3^\circ$$

2. a) $\tan \theta = \frac{\text{opp}}{\text{adj}}$

$$\tan \theta = \frac{11}{25}$$

$$\theta = \tan^{-1}\left(\frac{11}{25}\right)$$

$$\theta \approx 23.7^\circ$$

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{11}{25}$$

$$m = 0.44$$

b) $\tan \theta = \frac{\text{opp}}{\text{adj}}$

$$\tan \theta = \frac{29}{14}$$

$$\theta = \tan^{-1}\left(\frac{29}{14}\right)$$

$$\theta \approx 64.2^\circ$$

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{29}{14}$$

$$m \approx 2.07$$

3. a) $\ell^2 = a^2 + b^2$

$$\ell^2 = 3^2 + 7^2$$

$$\ell = \sqrt{3^2 + 7^2}$$

$$\ell = \sqrt{9 + 49}$$

$$\ell \approx 7.6 \text{ ft}$$

b) $\tan \theta = \frac{\text{rise}}{\text{run}}$

$$\tan \theta = \frac{3}{7}$$

$$\theta = \tan^{-1}\left(\frac{3}{7}\right)$$

$$\theta \approx 23.2^\circ$$

4. a) $\ell^2 = a^2 + b^2$

$$\ell^2 = 2.2^2 + 5.8^2$$

$$\ell = \sqrt{2.2^2 + 5.8^2}$$

$$\ell = \sqrt{4.84 + 33.64}$$

$$\ell \approx 6.2 \text{ m}$$

b) $\tan \theta = \frac{\text{rise}}{\text{run}}$

$$\tan \theta = \frac{2.2}{5.8}$$

$$\theta = \tan^{-1}\left(\frac{2.2}{5.8}\right)$$

$$\theta \approx 20.8^\circ$$

5. a) Set up a proportion to solve for
- x
- , the run of the wheelchair ramp.

$$\frac{2.5}{30} = \frac{2.4}{x}$$

$$x \times \frac{2.5}{30} = \frac{2.4}{x} \times x$$

$$x \times \frac{2.5}{30} = 2.4$$

$$x = 2.4 \div \left(\frac{2.5}{30}\right)$$

$$x \approx 28.8 \text{ m}$$

b) $\ell^2 = a^2 + b^2$

$$\ell^2 = 28.8^2 + 2.4^2$$

$$\ell = \sqrt{28.8^2 + 2.4^2}$$

$$\ell = \sqrt{4.84 + 33.64}$$

$$\ell \approx 28.9 \text{ m}$$

NEW SKILLS: WORKING WITH GRADE

6. percent grade = $\frac{\text{rise}}{\text{run}} \times 100$

percent grade = $\frac{0.9}{5} \times 100$

percent grade = 18%

7. a) Convert the run to inches.

$$10 \text{ ft} \times 12 \text{ in/ft} = 120 \text{ in}$$

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = -\frac{2.5}{120}$$

$$m \approx -0.021$$

b) percent grade = $\frac{\text{rise}}{\text{run}} \times 100$

percent grade = 0.021×100

percent grade = 2.1%

8. a) percent grade = $\frac{\text{rise}}{\text{run}} \times 100$

percent grade = $m \times 100$

$$13.5 = m \times 100$$

$$\frac{13.5}{100} = m$$

$$0.135 = m$$

b) $\tan \theta = \frac{\text{rise}}{\text{run}}$

$$\tan \theta = m$$

$$\tan \theta = 0.135$$

$$\theta = \tan^{-1}(0.135)$$

$$\theta \approx 7.7^\circ$$

$$c) \quad m = \frac{\text{rise}}{\text{run}}$$

$$0.135 = \frac{\text{rise}}{15}$$

$$15 \times 0.135 = \text{rise}$$

$$2.03 \text{ m} \approx \text{rise}$$

NEW SKILLS: WORKING WITH PITCH

9. A pitch of 2:5 means a slope of $\frac{2}{5}$.

$$m = \frac{\text{rise}}{\text{run}}$$

$$\frac{2}{5} = \frac{h}{4.8}$$

$$4.8 \times \frac{2}{5} = h$$

$$1.92 \text{ m} \approx h$$

The lean-to reaches 1.92 m up the building.

10. $m = \frac{\text{rise}}{\text{run}}$

$$m = \frac{3.6}{(5.4 \div 2)}$$

$$m = \frac{3.6}{2.7}$$

$$m = \frac{36}{27}$$

$$m = \frac{36 \div 9}{27 \div 9}$$

$$m = \frac{4}{3}$$

The pitch of the roof is 4:3.

11. $m = \frac{\text{rise}}{\text{run}}$

$$\frac{2}{5} = \frac{x}{(7 \div 2)}$$

$$\frac{2}{5} = \frac{x}{3.5}$$

$$\frac{3.5 \times 2}{5} = x$$

$$1.4 \text{ ft} = x$$

The rise of the roof is 1.4 ft.

PRACTICE YOUR NEW SKILLS, p. 32

1. a) $\tan \theta = \frac{\text{opp}}{\text{adj}}$

$$\tan A = \frac{35}{29}$$

$$A = \tan^{-1}\left(\frac{35}{29}\right)$$

$$A \approx 50.4^\circ$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan B = \frac{29}{35}$$

$$B = \tan^{-1}\left(\frac{29}{35}\right)$$

$$B \approx 39.6^\circ$$

ALTERNATIVE SOLUTION

$$90^\circ - A = B$$

$$90^\circ - 50.4^\circ = 39.6^\circ$$

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{29}{35} \text{ or approx. } 0.83$$

b) $\tan \theta = \frac{\text{opp}}{\text{adj}}$

$$\tan A = \frac{68}{102}$$

$$A = \tan^{-1}\left(\frac{68}{102}\right)$$

$$A \approx 33.7^\circ$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan B = \frac{102}{68}$$

$$B = \tan^{-1}\left(\frac{102}{68}\right)$$

$$B \approx 56.3^\circ$$

ALTERNATIVE SOLUTION

$$90^\circ - A = B$$

$$90^\circ - 33.7^\circ = 56.3^\circ$$

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{102}{68}$$

$$c) \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan A = \frac{26}{19}$$

$$A = \tan^{-1}\left(\frac{26}{19}\right)$$

$$A \approx 53.8^\circ$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan B = \frac{19}{26}$$

$$B = \tan^{-1}\left(\frac{19}{26}\right)$$

$$B \approx 36.2^\circ$$

ALTERNATIVE SOLUTION

$$90^\circ - A = B$$

$$90^\circ - 53.8^\circ = 36.2^\circ$$

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{19}{29} \text{ or approx}$$

$$d) \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan A = \frac{22}{16}$$

$$A = \tan^{-1}\left(\frac{22}{16}\right)$$

$$A = 54^\circ$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan B = \frac{16}{22}$$

$$B = \tan^{-1}\left(\frac{16}{22}\right)$$

$$B = 36^\circ$$

ALTERNATIVE SOLUTION:

$$90^\circ - A = B$$

$$90^\circ - 54^\circ = 36^\circ$$

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{16}{22}$$

$$m = \frac{8}{11} \text{ or approx. } 0.73$$

$$2. \quad a) \quad \ell^2 = a^2 + b^2$$

$$\ell^2 = 1.2^2 + 7.2^2$$

$$\ell = \sqrt{1.2^2 + 7.2^2}$$

$$\ell = \sqrt{1.44 + 51.84}$$

$$\ell \approx 7.3 \text{ m}$$

$$b) \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{\text{rise}}{\text{run}}$$

$$\tan \theta = \frac{1.2}{7.2}$$

$$\theta = \tan^{-1}\left(\frac{1.2}{7.2}\right)$$

$$\theta \approx 9.5^\circ$$

$$3. \quad a) \quad \tan \theta_1 = \frac{\text{rise}}{\text{run}}$$

$$\tan \theta_1 = m_1$$

$$\tan 35^\circ = m_1$$

$$0.7 \approx m_1$$

$$\tan \theta_2 = \frac{\text{rise}}{\text{run}}$$

$$\tan \theta = m_2$$

$$\tan 45^\circ = m_2$$

$$1 = m_2$$

$$\text{b) percent grade}_1 = \frac{\text{rise}}{\text{run}} \times 100$$

$$\text{percent grade}_1 = m_1 \times 100$$

$$\text{percent grade}_1 = 0.7 \times 100$$

$$\text{percent grade}_1 = 70\%$$

$$\text{percent grade}_2 = \frac{\text{rise}}{\text{run}} \times 100$$

$$\text{percent grade}_2 = m_2 \times 100$$

$$\text{percent grade}_2 = 1 \times 100$$

$$\text{percent grade}_2 = 100\%$$

$$4. \text{ a) } m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{0.015}{2.5}$$

$$m = \frac{3}{500} \text{ or } 0.006$$

$$\text{b) } m = \frac{\text{rise}}{\text{run}}$$

$$\frac{3}{500} = \frac{x}{12}$$

$$12 \times \frac{3}{500} = x$$

$$0.072 \text{ m} = x$$

The pipe will have a drop of 0.072 m or 7.2 cm.

$$\text{c) } \tan \theta = \frac{\text{rise}}{\text{run}}$$

$$\tan \theta = m$$

$$\tan \theta = 0.006$$

$$\theta = \tan^{-1}(0.006)$$

$$\theta \approx 0.34^\circ$$

5. a) Calculate the change in altitude.

$$1257 - 982 = 275 \text{ m}$$

$$\ell^2 = a^2 + b^2$$

$$3000^2 = a^2 + 275^2$$

$$3000^2 - 275^2 = a^2$$

$$\sqrt{3000^2 - 275^2} = a$$

$$2987 \text{ m} \approx a$$

$$\text{b) percent grade} = \frac{\text{rise}}{\text{run}} \times 100$$

$$\text{percent grade} = \frac{275}{3000} \times 100$$

$$\text{percent grade} = 9.2\%$$

6. a) The roof has a rise of 2 ft and a run of 5 ft.
This means a pitch of 2:5.

$$\text{b) } m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{2}{5}$$

$$m = 0.4$$

$$\text{c) percent grade} = \frac{\text{rise}}{\text{run}} \times 100$$

$$\text{percent grade} = \frac{2}{5} \times 100$$

$$\text{percent grade} = 40\%$$

7. Calculate the slope of each roof.

$$m_1 = \frac{\text{rise}}{\text{run}}$$

$$m_1 = \frac{4.2}{12}$$

$$m_1 = 0.35$$

$$m_2 = \frac{\text{rise}}{\text{run}}$$

$$m_2 = \frac{7.8}{20}$$

$$m_2 = 0.39$$

The roof of the second house is steeper.

8. Calculate the change in elevation.

$$1132 - 1070 = 62 \text{ m}$$

$$\text{percent grade} = \frac{\text{rise}}{\text{run}} \times 100$$

$$\text{percent grade} = \frac{62}{1300} \times 100$$

$$\text{percent grade} \approx 4.8\%$$

1.3

Rate of Change

NEW SKILLS: USING COORDINATES TO CALCULATE SLOPE

BUILD YOUR SKILLS, p. 39

1. For line 1, use points (5,3) and (10,6).

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{6 - 3}{10 - 5}$$

$$m_1 = \frac{3}{5} \text{ or } 0.6$$

For line 2, use points (2,3) and (6,9).

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_2 = \frac{9 - 3}{6 - 2}$$

$$m_2 = \frac{6}{4}$$

$$m_2 = \frac{3}{2} \text{ or } 1.5$$

Line 2 has a greater slope, so it is steeper.

2. For line 1, use points (5,6) and (9,6).

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{6 - 6}{9 - 5}$$

$$m_1 = \frac{0}{4}$$

$$m_1 = 0$$

For line 2, use points (4,2) and (4,9).

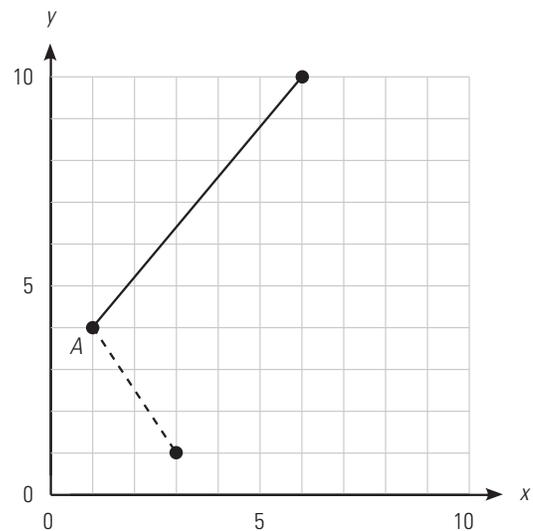
$$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_2 = \frac{9 - 2}{4 - 4}$$

$$m_2 = \frac{7}{0}$$

$$m_2 = \text{undefined}$$

3. a) If the slope is $\frac{6}{5}$, this means that for a rise of 6 units, there is a run of 5 units. Starting at A, go up 6 units then to the right 5 units. Join A to B.
- b) If the slope is $\frac{3}{2}$, the rise is -3 and the run is 2. Start at A and go down 3 units, then to the right 2 units. Join A to C.



NEW SKILLS: WORKING WITH SLOPE AS A RATE OF CHANGE

4. a) Divide the distance walked by the time in hours.

$$3 \text{ km} \div 0.5 \text{ h} = 6 \text{ km/h}$$

Reggie's average walking rate is 6 km/h.

- b) The independent variable is time. If you were to graph the data, time would be on the x -axis.

- c) Divide the total distance by his walking rate.

$$36 \text{ km} \div 6 \text{ km/h} = 6 \text{ h}$$

It would take Reggie 6 hours to walk the marathon.

5. a) Let p be the amount owed and w be the number of weeks she has paid.

$$p = 600 - 40w$$

This equation shows that Jenita started off owing \$600.00, but pays back \$40.00 per week.

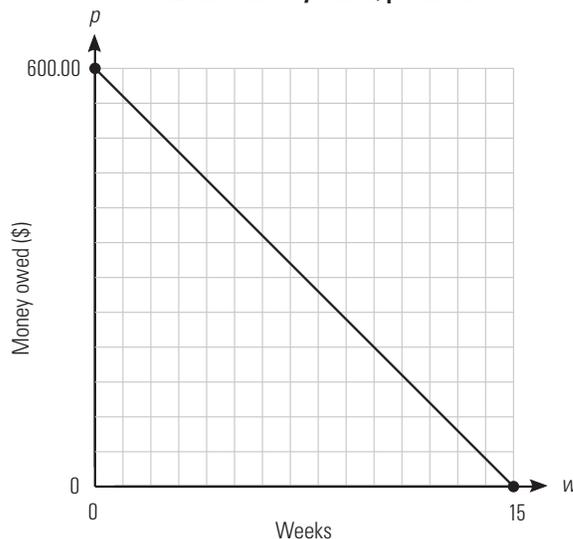
- b) Construct a graph that has time in weeks on the x -axis, and money owed on the y -axis. Calculate 2 or 3 data points by determining how much money Jenita owes after different numbers of weeks.

At 0 weeks, Jenita owed \$600.00, so one point will be (0,600).

After 3 weeks, Jenita will have paid back \$120.00 and will still owe \$480.00. The data point for this will be (3,480).

After 5 weeks, Jenita will owe \$400.00, so the data point will be (5,400).

Amount Owed by Jenita, per Week



Calculate the slope using two of your data points, for example (600,0) and (5,400).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{400 - 600}{5 - 0}$$

$$m = \frac{-200}{5}$$

$$m = -40$$

The slope of the line is -40 . The slope represents the amount of money Jenita pays per week.

6. a) Calculate George's earnings by multiplying by the rate of commission, 12% (expressed as a decimal, 0.12).

$$\$500.00 \times 0.12 = \$60.00$$

$$\$1000.00 \times 0.12 = \$120.00$$

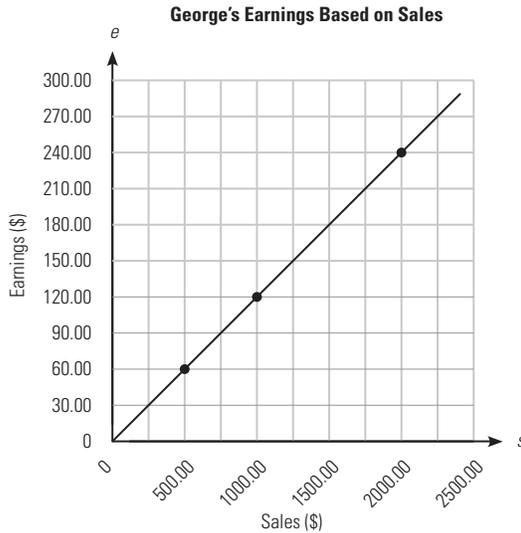
$$\$2000 \times 0.12 = \$240.00$$

- b) George's earnings are 0.12 times his sales. Let e represent his earnings and s represent total sales.

$$e = 0.12s$$

The independent variable is the total sales.
It will be on the x -axis of the graph.

c)



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{120 - 60}{1000 - 500}$$

$$m = \frac{60}{500}$$

$$m = 0.12$$

The slope is 0.12 or $\frac{3}{25}$. It represents his rate of commission.

d) $e = 0.12s$

$$\$210.00 = 0.12s$$

$$\frac{\$210.00}{0.12} = s$$

$$\$1750.00 = s$$

If George earned a commission of \$210.00, he sold \$1750.00 worth of furniture.

7. a) Divide the amount of water in the pool by the amount of time.

$$2250 \text{ gal} \div 45 \text{ min} = 50 \text{ gal/min}$$

ALTERNATIVE SOLUTION

$$2250 \text{ gal} \div 0.75 \text{ h} = 3000 \text{ gal/h}$$

The pool was being filled at a rate of 50 gal/min or 3000 gal/h.

- b) Divide the total capacity by the rate of filling.

$$13\,500 \div 3000 = 4.5 \text{ h}$$

ALTERNATIVE SOLUTION

$$13\,500 \div 50 = 270 \text{ min}$$

It will take 4.5 h or 270 min to fill the pool.

8. a) Divide the total distance travelled by the total time.

$$380 \text{ km} \div 5 \text{ h} = 76 \text{ km/h}$$

- b) Calculate the distance he travelled between the end of the 5th hour and the end of the 9th hour.

$$764 - 380 = 384 \text{ km}$$

Divide by the time travelled at the increased speed (4 hours).

$$384 \text{ km} \div 4 \text{ h} = 96 \text{ km/h}$$

9. a) Divide the total number of pages by the rate.

$$321 \text{ pages} \div 0.6 \text{ pages/min} = 535 \text{ min}$$

Convert the time in minutes to hours and minutes.

$$535 \text{ min} \div 60 \text{ min/h} \approx 8.92 \text{ h}$$

$$0.92 \text{ h} \times 60 \text{ min/h} \approx 55 \text{ min}$$

It will take Maryam 535 minute or 8 hours 55 minutes to read the book.

- b) Multiply the rate by the number of minutes spent reading.

$$2 \text{ h} = 120 \text{ min}$$

$$0.6 \text{ pages/min} \times 120 \text{ min} = 72 \text{ pages}$$

After 2 hours, she will have read 72 pages.

PRACTISE YOUR NEW SKILLS, p. 50

1. a) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{204 - 0}{4 - 0}$$

$$m = \frac{204}{4}$$

$$m = 51$$

b) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{4.5 - 3}{16 - 12}$$

$$m = \frac{1.5}{4}$$

$$m = \frac{15}{40}$$

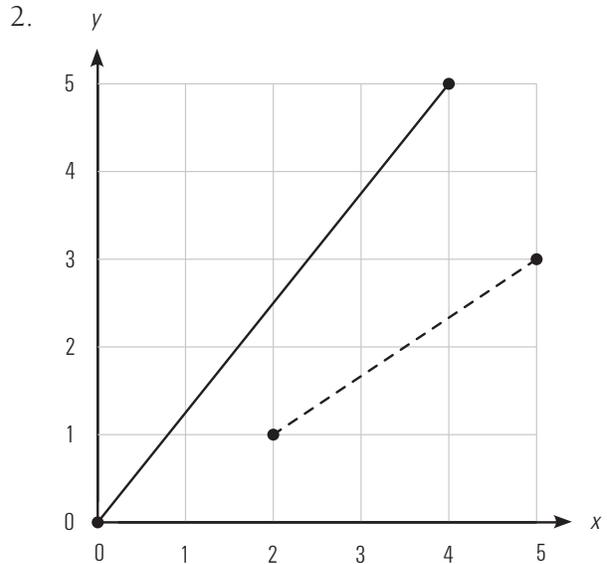
$$m = \frac{15 \div 5}{40 \div 5}$$

$$m = \frac{3}{8} \text{ or } 0.375$$

c) $m = \frac{y_2 - y_1}{x_2 - x_1}$

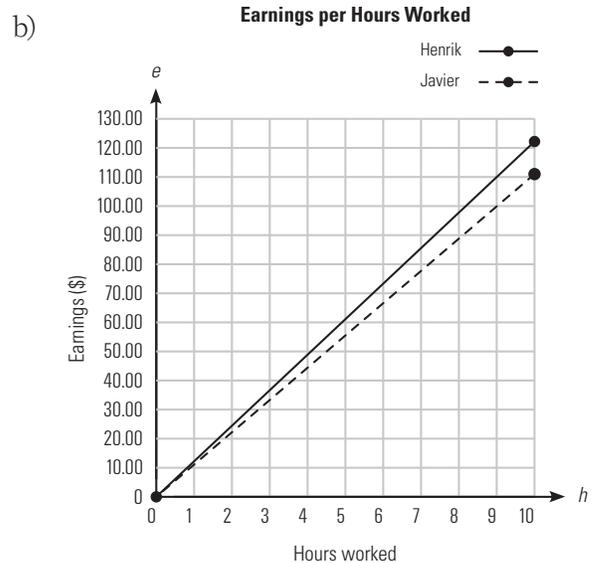
$$m = \frac{6 - 3}{25 - 150}$$

$$m = -\frac{3}{125} \text{ or } -0.024$$



3. a) Answers will vary, depending on the number of hours chosen. Answers for times of 1 h to 5 h are provided.

Hours	Henrik's earnings	Javier's earnings
1	\$12.20	\$11.10
2	\$24.40	\$22.20
3	\$36.60	\$33.30
4	\$48.80	\$44.40
5	\$61.00	\$55.50



- c) Calculate how much money each will make after 8 hours.

$$\text{Henrik} = \$12.20/\text{h} \times 8 \text{ h} = \$97.60$$

$$\text{Javier} = \$11.10 \times 8 \text{ h} = \$88.80$$

Henrik will earn more money after an 8-hour shift.

- d) The rate of earning is the slope of the line on the graph.

Henrik:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{61 - 0}{5 - 0}$$

$$m = \frac{61}{5}$$

$$m = 12.20$$

Henrik earns \$12.20/h.

Javier:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{55.50 - 0}{5 - 0}$$

$$m = \frac{55.50}{5}$$

$$m = 11.10$$

Javier earns \$11.10/h.

4. Write an equation showing Janice's earnings compared to sales.

$$\text{earnings} = 0.04 \times \text{sales}$$

Substitute in her earnings and solve for x , her sales.

$$\text{earnings} = 0.04 \times \text{sales}$$

$$\$7560.00 = 0.04x$$

$$\frac{\$7560.00}{0.04} = x$$

$$\$189\,000.00 = x$$

The selling price was \$189 000.00.

5. a) Consider the data as if it were plotted on a graph of Time (in hours; on the x -axis) vs. Distance travelled (in kilometres; on the y -axis). The data points would be: (2.25,40) and (4,75). Solve for the slope, which represents the rate of travel.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{75 - 40}{4 - 2.25}$$

$$m = \frac{35}{1.75}$$

$$m = 20$$

Rita's average rate of travel was 20 km/h.

- b) Calculate how far Rita will travel in 2 hours (from the 4-hour point to the 6-hour point).

$$2 \text{ h} \times 20 \text{ km/h} = 40 \text{ km}$$

Add this to the distance she had already travelled in the first 4 hours.

$$75 \text{ km} + 40 \text{ km} = 115 \text{ km}$$

After 6 hours, Rita will have travelled 115 km.

6. a) Divide their total distances by their walking times.

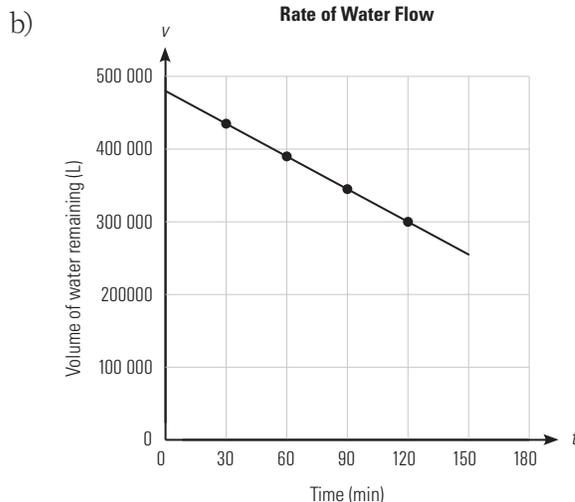
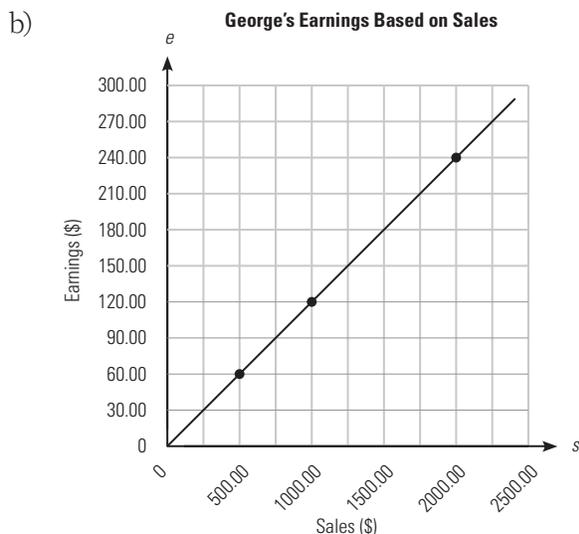
Sheila:

$$28.8 \text{ km} \div 6 \text{ h} = 4.8 \text{ km/h}$$

Brandy:

$$32.2 \text{ km} \div 7 \text{ h} = 4.6 \text{ km/h}$$

Sheila is the faster walker.



The slopes of the lines on the graph are equal to their walking rates (Sheila: 4.8 km/h; Brandy: 4.6 km/h).

c) Divide the total distance by the walking rate.

Sheila:

$$10 \text{ km} \div 4.8 \text{ km/h} = 2.1 \text{ h}$$

Brandy:

$$10 \text{ km} \div 4.6 \text{ km/h} = 2.2 \text{ h}$$

It will take Sheila 2.1 h and Brandy 2.2 h to walk 10 km.

7. a)

Time elapsed (minutes)	Volume of water remaining (L)
0	480 000
30	435 000
60	390 000
90	345 000
120	300 000
150	255 000

c) Calculate the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{435\,000 - 480\,000}{30 - 0}$$

$$m = \frac{-45\,000}{30}$$

$$m = -1500$$

The slope is the rate at which the water volume decreases, -1500 L/min .

d) Divide the total volume by the rate.

$$480\,000 \text{ L} \div 1500 \text{ L/min} = 320 \text{ min}$$

It will take 320 min for the tank to empty.

CHAPTER TEST, p. 55

$$1. \quad m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{1.8}{4.5}$$

$$m = \frac{18}{45}$$

$$m = \frac{18 \div 9}{45 \div 9}$$

$$m = \frac{2}{5} \text{ or } 0.4$$

$$2. \quad m = \frac{\text{rise}}{\text{run}}$$

$$\frac{2}{3} = \frac{x}{250}$$

$$250 \times \frac{2}{3} = x$$

$$167 \text{ cm} \approx x$$

The railing will be about 167 cm higher.

$$3. \quad \tan \theta = \frac{\text{rise}}{\text{run}}$$

$$\tan 20^\circ = \frac{x}{2.6}$$

$$2.6 \tan 20^\circ = x$$

$$0.95 \text{ m} \approx x$$

The rise of the waterslide is about 0.95 m.

$$4. \quad \text{a) } m = \frac{\text{rise}}{\text{run}}$$

$$0.375 = \frac{3.75}{x}$$

$$0.375x = 3.75$$

$$x = \frac{3.75}{0.375}$$

$$x = 10 \text{ ft}$$

The shortest horizontal distance from the base of the patio to the base of the ramp is 10 ft.

$$\text{b) } \ell^2 = a^2 + b^2$$

$$\ell^2 = 10^2 + 3.75^2$$

$$\ell = \sqrt{10^2 + 3.75^2}$$

$$\ell = \sqrt{100 + 14.0625}$$

$$\ell \approx 10.7 \text{ ft}$$

The surface of the ramp must be at least 10.7 ft long.

$$\text{c) } \tan \theta = \frac{\text{rise}}{\text{run}}$$

$$\tan \theta = m$$

$$\tan \theta = 0.375$$

$$\theta = \tan^{-1}(0.375)$$

$$\theta \approx 20.6^\circ$$

The angle of elevation of the ramp will be 20.6° .

5. a) A pitch of 4:12 means a slope of $\frac{4}{12}$. This can be simplified.

$$m = \frac{4}{12}$$

$$m = \frac{4 \div 4}{12 \div 4}$$

$$m = \frac{1}{3} \text{ or about } 0.33$$

$$\text{b) } \tan \theta = \frac{\text{rise}}{\text{run}}$$

$$\tan \theta = m$$

$$\tan \theta = \frac{1}{3}$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right)$$

$$\theta \approx 18.4^\circ$$

The angle of elevation of the roof is about 18.4° .

6. percent grade = $\frac{\text{rise}}{\text{run}} \times 100$

$$6 = \frac{x}{15} \times 100$$

$$\frac{6}{100} = \frac{x}{15}$$

$$15 \times \frac{6}{100} = x$$

$$0.9 \text{ km} = x$$

The vertical change is 0.9 km or 900 m.

7. a) $\ell^2 = a^2 + b^2$

$$\ell^2 = 6.8^2 + 125.9^2$$

$$\ell = \sqrt{6.8^2 + 125.9^2}$$

$$\ell \approx 126.1 \text{ m}$$

The railroad track will be about 126.1 m long.

b) percent grade = $\frac{\text{rise}}{\text{run}} \times 100$

$$\text{percent grade} = \frac{6.8}{125.9} \times 100$$

$$\text{percent grade} \approx 5.4\%$$

8. A pitch of 2:5 means a slope of $\frac{2}{5}$.

$$m = \frac{\text{rise}}{\text{run}}$$

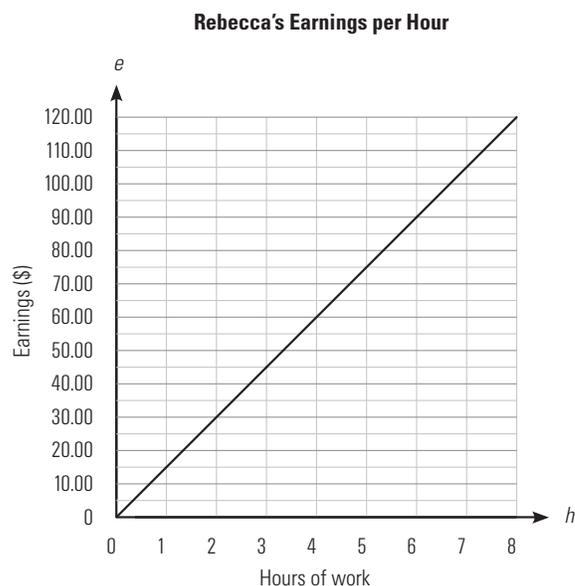
$$\frac{2}{5} = \frac{x}{3.5}$$

$$3.5 \times \frac{2}{5} = x$$

$$1.4 \text{ m} = x$$

The roof will rise 1.4 m.

9. a)



b) Choose two points on the graph, such as (1, 15) and (2, 30).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{30 - 15}{2 - 1}$$

$$m = \frac{15}{1}$$

$$m = 15$$

The slope of the graph is 15, which represents Rebecca's earnings per hour.

c) Calculate Rebecca's earnings after 8 hours.

$$8 \text{ h} \times \$15.00/\text{h} = \$120.00$$

After 8 hours, she will have earned \$120.00.

Chapter 2

Graphical Representations

2.1

Broken Line Graphs

BUILD YOUR SKILLS, p. 60

- He spent about \$3.00 on Wednesday and nothing on Friday.
 - He spent slightly less than \$19.00 for lunch on Sunday. He may have gone out with friends for lunch.
- The graph shows Katie's heart rate each hour in beats per minute from 8:00 am to 9:00 pm.
 - Her heart rate was lowest at 8:00 am, 2:00 pm, 5:00 pm and 9:00 pm. It was 68 beats per minute. Katie was probably relaxed at these times.
 - Katie's heart rate was highest at 1:00 pm. She may have just been involved in some physical activity.
- The Week 1 dot is the value Vince bought the stock at. Students can count that there are five equal segments between 0 and 3.00.

Calculate the value of each segment.

$$\frac{3.00}{5} = 0.60$$

The dot is just below the third segment.

$$0.60 \times 3 = 1.80$$

The dot is just below \$1.80 so you can estimate Vince bought the stock at a value near \$1.75.

- You can estimate that the stock was worth \$12.15 at Week 4.

$$12.15 - 1.80 = 10.40$$

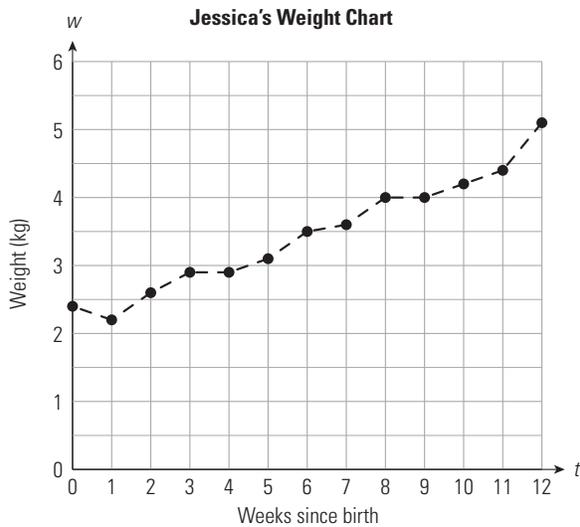
He would have made \$10.40 per stock.

4.



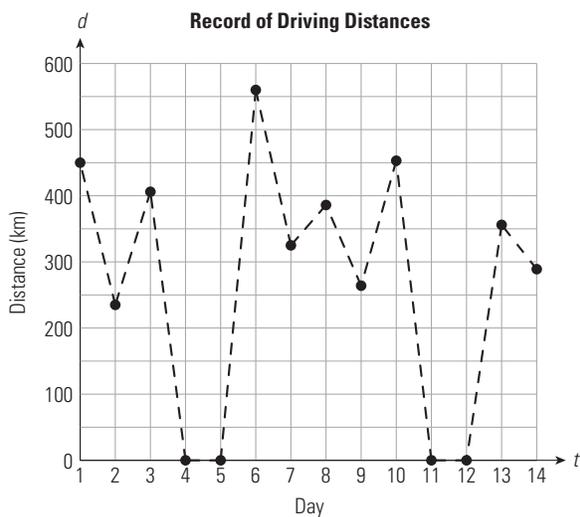
Sales of the magazine seem to be slowly increasing.

5.



Jessica lost weight during the first week, and stayed the same between the 8th and 9th, but other than that, there was a weight gain each week.

6. a)



b) Answers will vary but should include things such as:

The graph indicates the distance he drove each day and that these vary considerably.

The anomalies of zero are probably his days off.

c) The graph is an okay representation. However, since it is not looking at a trend over time, but is discrete data, it doesn't give much useful information.

7. Answers will vary slightly but should suggest:

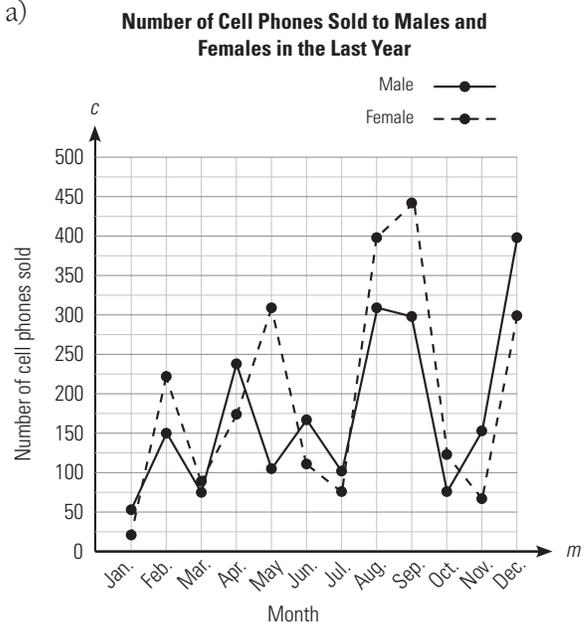
Thérèse was driving her car from 8:30 am until 10:30. It was parked from 10:30 to 12:30 at which time she put gas in the tank. She then drove for an hour (until 1:30) and stopped again for an hour until 2:30. Between 2:30 and 4:30, she drove again, but not so far, and she stopped for the last hour. drove a bit

8. a) The cost seems to be declining slightly.

b) Draw a straight line between the point in July and September and estimate where it would meet for August. Answers will vary but should be approximately \$325.00.

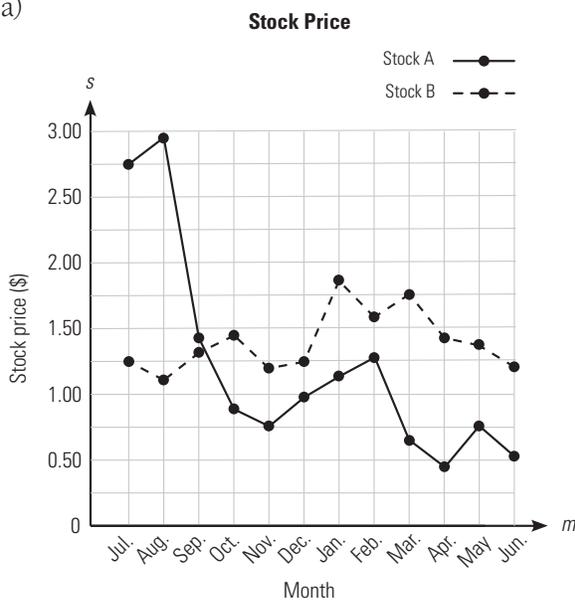
c) Answers will vary, but should be between \$300.00 and \$325.00.

9. a)



- b) The number of cell phones bought fluctuated greatly from month to month, but over all there tended to be an increase.
- c) No, there is no real relationship as some months females bought more and other months it was males.
- d) It is not very useful. A stacked bar graph or even a bar graph would better indicate the numbers sold.

10. a)



- b) Stock A started off high and made a sudden drop in August and September, increasing again a bit over the next few months, then dropping more in March. Stock B prices fluctuated slightly over the year but remained relatively even compared to Stock A.
- c) Answers will vary but may include:

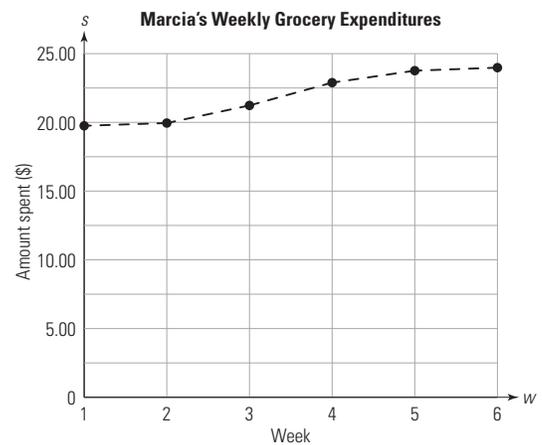
- She should sell stock A because it has lost a lot and doesn't look like it is going to increase.

- She should sell stock B because it has not made her any money and likely won't.
- She should sell stock B because she needs to make up the money she lost on A, so if she buys more of them she can maybe do that.

11. a) The graph represents the amount of money Marcia spent weekly on groceries.

- b) It appears that her expenditures increased greatly over the 6 weeks.
- c) The vertical axis starts at \$19.50 rather than \$0.

d)



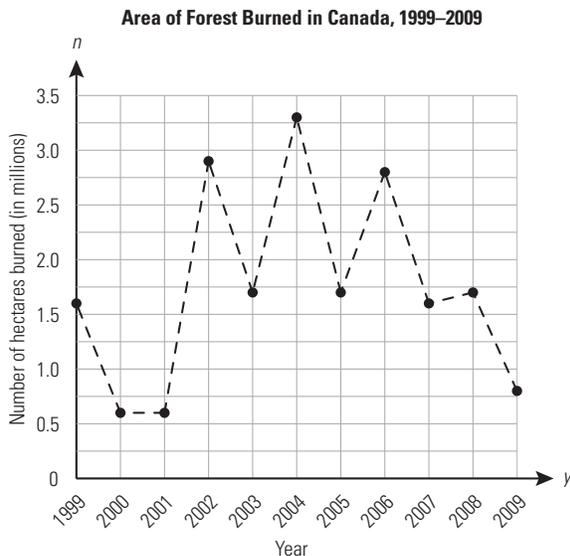
- 12. a) The first graph makes the decrease look more rapid because the same width (years) represents twice as many years.
- b) The second graph is better because more years are represented.
- c) Estimating the halfway point between 1911 and 1921 should get students a number near 61%.
- d) Estimating the 50% point between students should get approximately 1952.

- e) The graph indicates a decline so students should estimate about 17% of the population will be rural in 2011.

PRACTISE YOUR NEW SKILLS, p. 76

1. a) She walked approximately 56 000 steps the first week and 81 000 the last week.
- b) She had the fewest steps during weeks 5 and 9, each with about 54 000 steps.
- c) With the exception of the two low weeks, weeks 5 and 9, there was a general trend of gradual increase in number of steps per week.

2.



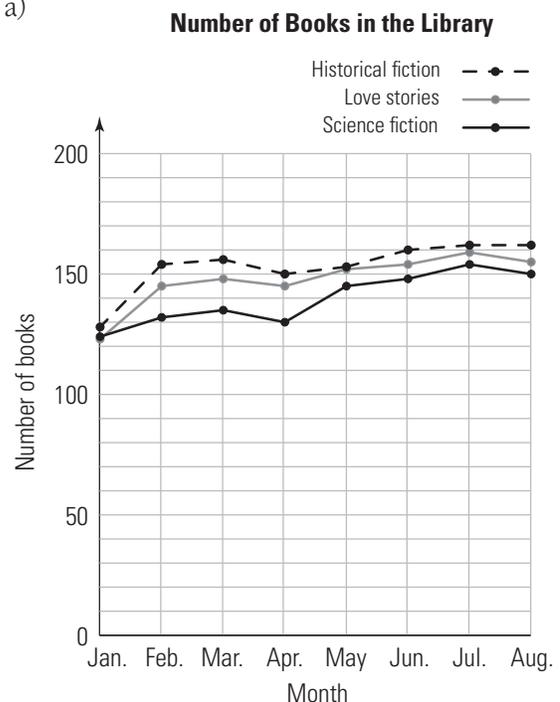
There is no general trend of increase or decrease although if one year was high, the next year tended to be lower for a few years. There seemed to be a lot of fires from 2002 to 2006.

3. Answers will vary but should include the following ideas: At the beginning of the year there was a steady rise in prices, but from June

to August they dropped. Prices remained about the same from August to September, then rose again until the end of the year. The highest prices were in December.

4. a) Brand A has a fluctuating market with no general trend. Brand B has had a steady market with few fluctuations. Brand C has had some of the highest sales, but has had a fairly consistent drop in sales.
- b) Answers will vary but should include things such as:
 - Keep brand A and B because B has a consistent record of sales, and A because, although it is not consistent, it isn't on a downward slide like C.
 - Keep B and C because B has a consistent record of sales and C because, although it has had a drop in sales it seems to be steadying off whereas A is inconsistent.

5. a)



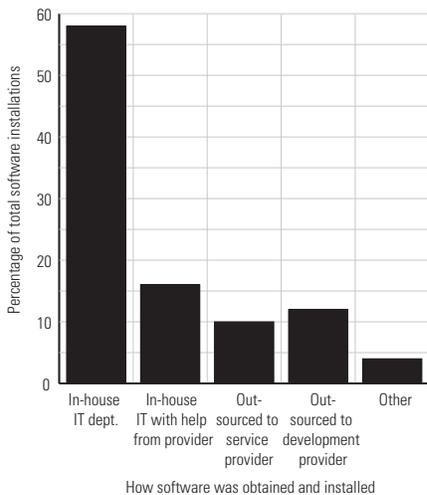
- b) There is a general, slow trend of increase in number of titles available indicating that there are more books being added. The decline would likely mean that the librarian got rid of some titles.

2.2

Bar Graphs

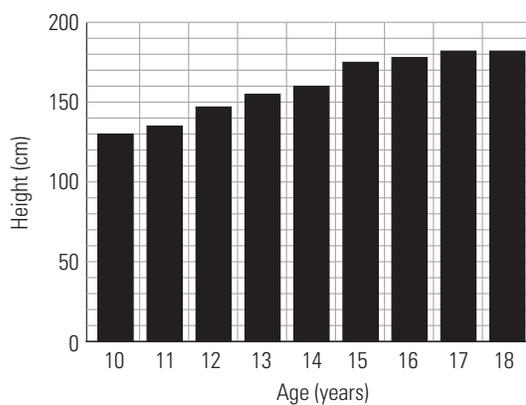
BUILD YOUR SKILLS, p. 82

1. **Software Installations**



2. a) Height is recorded horizontally rather than vertically. Also, his age goes from highest at the bottom to lowest at the top.

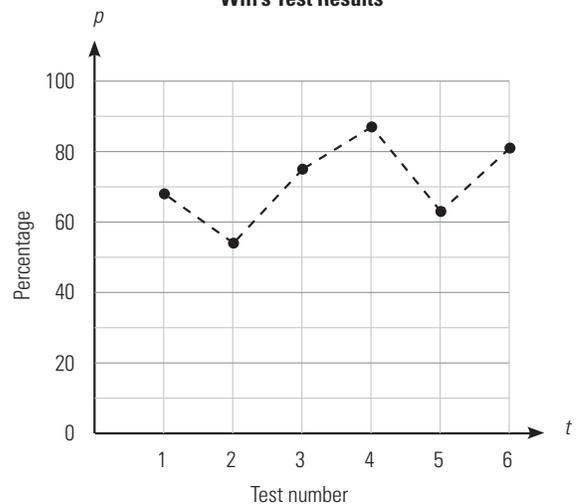
b) **Jamie's Height**



3. The bars are of different widths representing different periods of time. The bars are three dimensional giving a false perspective.

4. a) Students may choose the first graph because it is the more honest representation because it starts at zero.
- b) Approximately 22.5 billion tonnes.
- c) Approximately 27 billion tonnes.
- d) The misleading graph may be chosen because someone might want to deliberately mislead their viewers. Also the second graph does allow people to read the data more precisely which in some situations may be important.

5. **Will's Test Results**



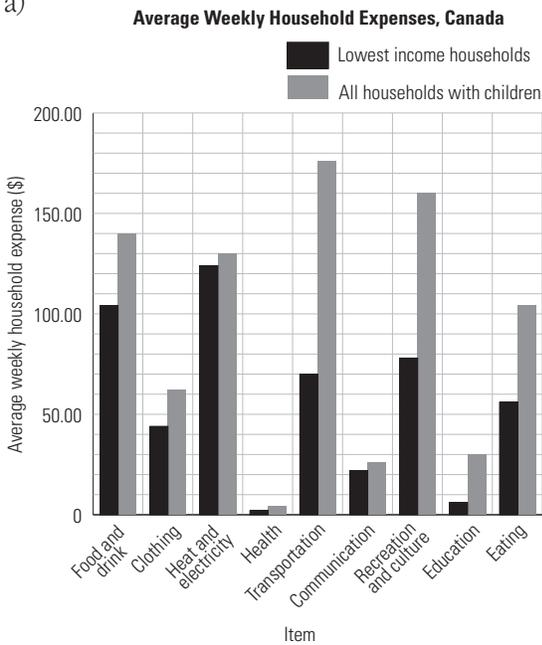
Answers will vary, but should include:

- The broken line graph is better because it is easier to read trends in his test results.
6. a) The number of tickets sold per day tends to decrease. This is probably because fans wanted to be sure that they got tickets as

soon as possible. The people who bought later were maybe not as keen on the band.

- b) The bar graph is the better representation because it more clearly indicates the number of tickets sold. The line graph, on the other hand, makes it look like a continuum.

7. a)



- b) Answers will vary but may include: The lower income families spent less on all items, but heat and electricity they were quite close and also for communications. Health looks fairly close, but it is actually half. The poorer families spent much less on transportation, food and eating out, as well as recreation and culture. They spent only 1/5 of the amount of all families on education.

- 8. a) The most visas were applied for in June and the least in January. Approximately 675 or 680 visas were applied for in July and 125 in January.

- b) The greatest discrepancy between applications and approvals was in July.

Visa applications in July: 535

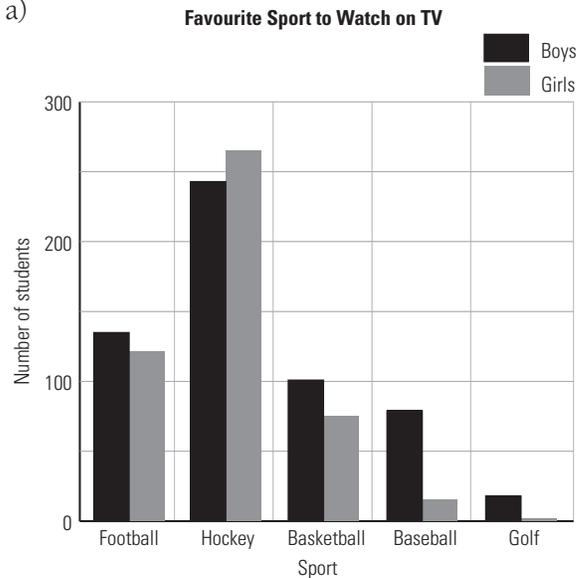
Visa approvals in July: 310

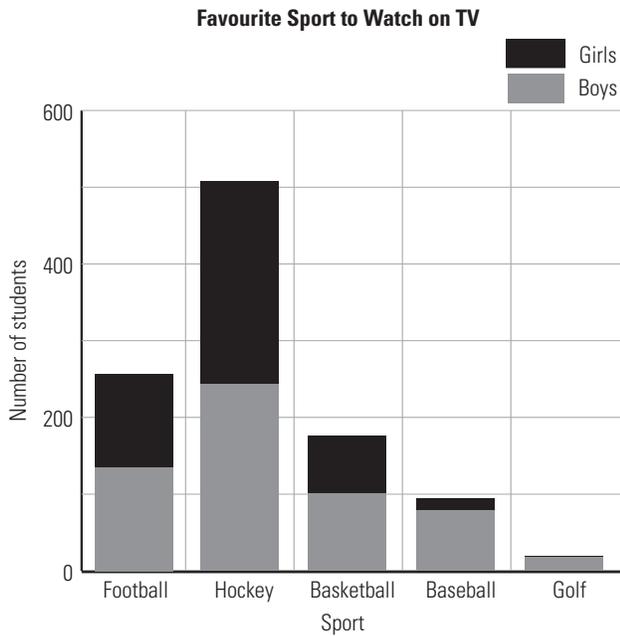
$$535 - 310 = 225$$

The approximate difference between approvals and applications in July was 225.

- 9. a) They are going down because they represent negative numbers. The temperature was below zero.
- b) There is no green bar on Saturday because it represents the high temperature of the day which was zero.
- c) On Wednesday there was no change in temperature. It stayed at -6°C .
- d) It seemed to warm up over the week.

10. a)





b) Add all the values together to find the total.

$$135 + 121 + 243 + 265 + 101 + 75 + 79 + 15 + 18 + 2 = 1054$$

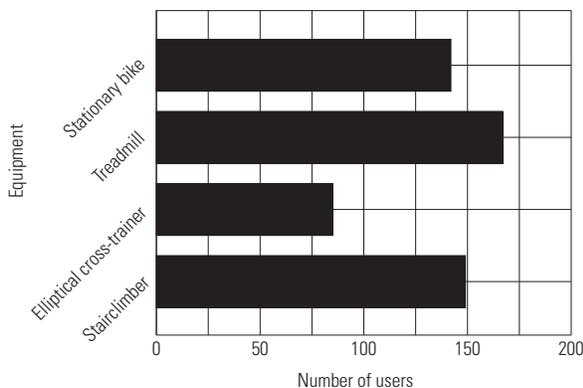
1054 took the survey.

c) Hockey is the most popular sport to watch.

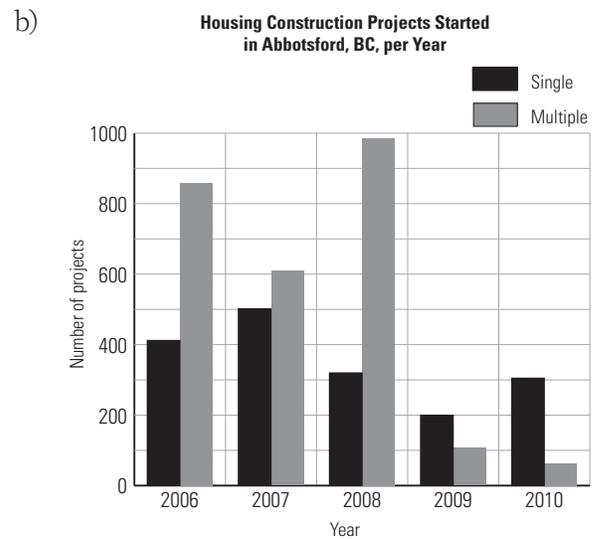
d) The double bar graph indicates the differences in preferences between the genders. The stacked would be more useful if you were interested in totals.

PRACTISE YOUR NEW SKILLS, p. 96

1. **Number of Users of Fitness Equipment per day**



2. a) The most popular movies are number 4 and 5.
- b) The least popular is number 8.
3. More boys have been taking home economics classes but the number of girls has stayed about the same.
4. a) Since 2008, the number of housing starts has dropped considerably, but there is no particular predictable pattern.



Answers will vary but should be along the lines of one of these answers:

The line graph because it shows the rises and falls, and each can be looked at individually.

The bar graph because it is more visual and shows the comparisons side by side as well as across time.

2.3

Histograms

BUILD YOUR SKILLS, p. 101

1. a) Approximately 125 households spend less than \$1000.00
- b) Between \$1000.00 and \$2000.00 is the most common amount spent on renovations.
- c) We cannot be sure because the graph says 5000.00+ but it was over \$5000.00.

2. a) 3800 people spend between 10 and 15 hours on the internet each week.
- b) Add all hours less than 15 hours.

$$3800 + 1700 + 1200 = 6700$$

6700 people spend less than 15 hours a week on the internet.

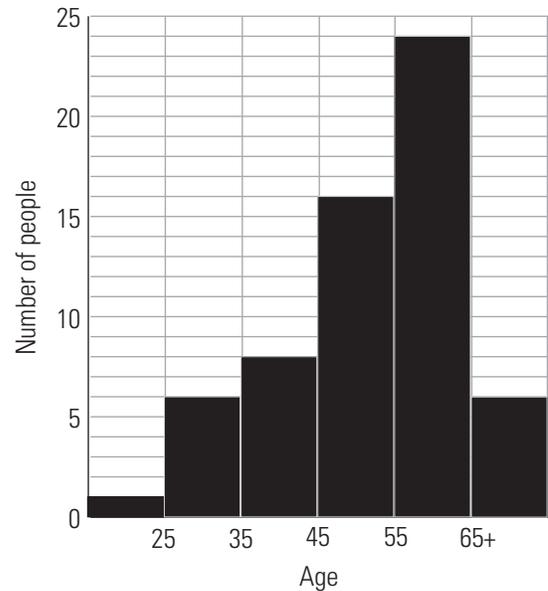
- c) 2600 people spend more than 30 hours on the internet each week.
- d) You can add the total from b) to the rest of the values.

$$6700 + 4900 + 4800 + 3700 + 2600 = 22\ 700$$

Approximately 22 700 people were surveyed.

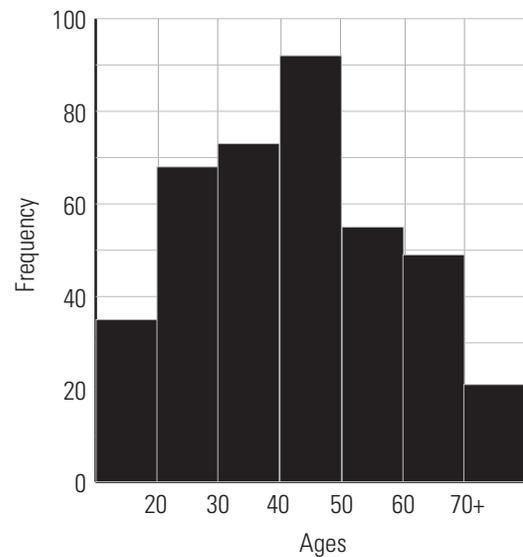
3.

Ages of Employees



4. a)

Ages of Audience Members at Theatre Presentation



- b) Add the values of people below 30.

$$68 + 35 = 103$$

There are 103 people under the age of 30 who attended the presentation.

- c) We cannot tell exactly except that he or she was under the age of 20.

PRACTISE YOUR NEW SKILLS, p. 105

1. a) There are 8 students who received a mark between 70% and 80%.

- b) Add the totals of students below 60%.

$$2 + 1 + 6 = 9$$

There are 9 students who scored below 60%.

- c) We cannot be sure but it was over 90% and less than or equal to 100%.

2. a) There are 2 employees who earn over \$100 000.00

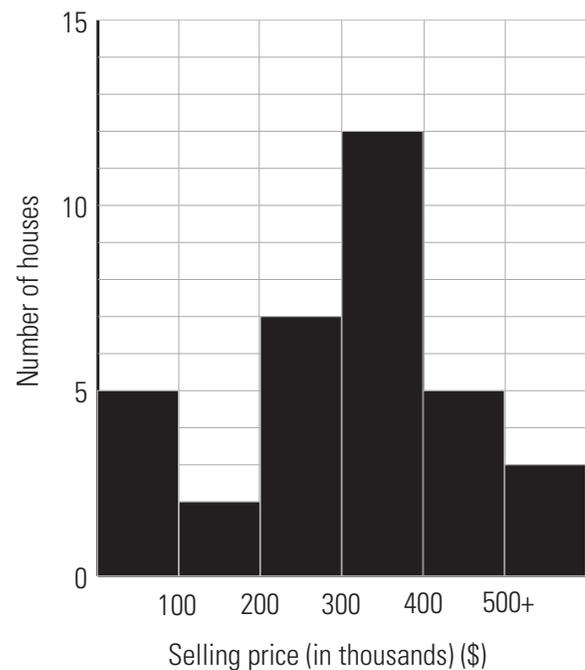
- b) Add the values of employees who earn between \$30 000.00 and \$50 000.00.

$$8 + 12 = 20$$

There are 20 employees who earn between \$30 000.00 and \$50 000.00.

3. a)

Fixed Rate Real Estate Housing Sales

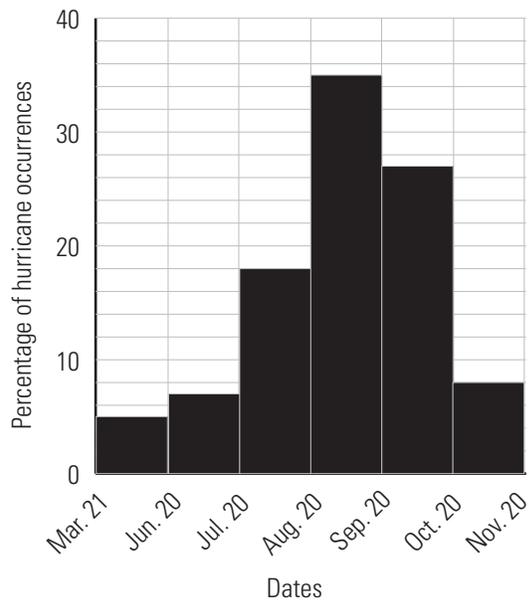


- b) We cannot be sure but it was below \$100 000.

- c) Add the totals of houses that sold for between \$100 000.00 and \$300 000.00.

$$2 + 7 = 9$$

There are 9 houses that sold for between \$100 000.00 and \$300 000.00.

4. a) **Hurricane Occurrences, by Time of Year**

- b) Hurricanes are unlikely to occur during the months from late November of one year until mid March of the next.
- c) A hurricane is most likely to occur from late August until late September.

2.4

Circle Graphs

BUILD YOUR SKILLS, p. 111

1. a) From reading the graph students can see that 14% of students chose green.

Calculate 14% of the total number of students:

$$171 \times 0.14 = 23.94$$

There are 24 students who chose green as their favourite colour.

- b) Students can read the graph and see that 25% of students chose blue.

Calculate 25% of the total number of students:

$$171 \times 0.25 = 42.75$$

The most popular colour is blue. 43 students chose it as their favourite colour.

2. a) Frank spends equal amounts on savings and entertainment at 10% each.

- b) Use algebra to solve for Frank's total savings.

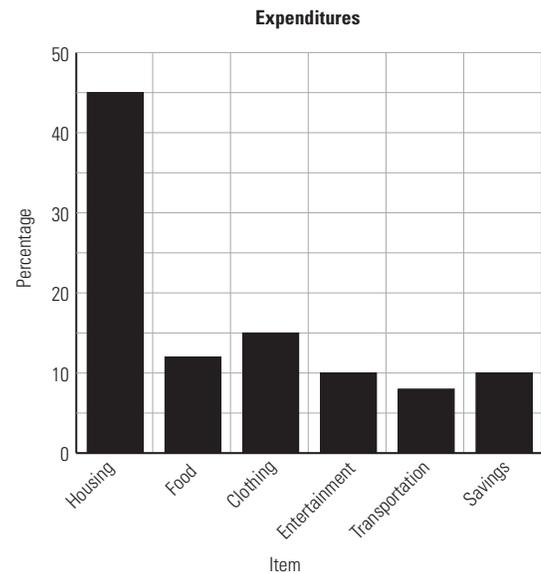
$$250 = 0.10x$$

$$x = \frac{250}{0.10}$$

$$x = 2500$$

He earns \$2500 per month.

c)



Answers may vary, but the circle graph is the better depiction here because it is easier to see what proportion each expenditure accounts for of the whole.

3. a) Add percentages together.

$$15 + 17 + 2 = 34$$

The faucet, shower and bath use 34% of the water.

- b) Add percentages together.

$$2 + 21 = 23$$

The dishwasher and clothes washer use 23% of the water.

- c) Add the two highest percentages together.

$$27 + 21 = 48$$

The toilet and the clothes washer add to about half the water use at 48%.

4. Find the total number of students surveyed.

$$39 + 44 + 21 = 104$$

Find out the percentage each pet represents of the total.

Dog:

$$\frac{39}{104} = 0.375$$

$$0.375 = 37.5\%$$

Cat:

$$\frac{44}{104} = 0.423$$

$$0.423 = 42.3\%$$

Rodent:

$$\frac{21}{104} = 0.202$$

$$0.202 = 20.2\%$$

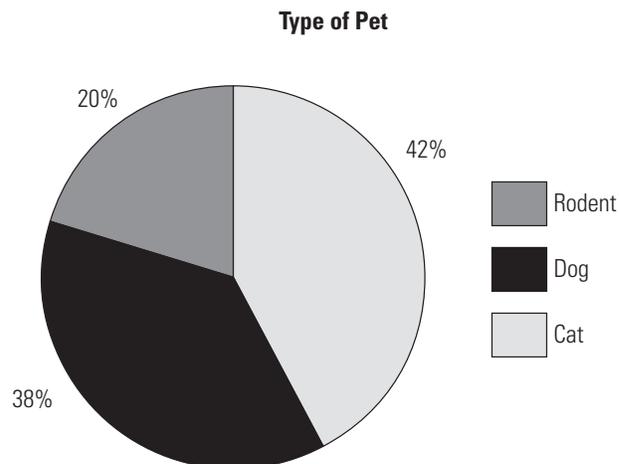
Calculate how many degrees of the circle each pet represents:

$$\text{Dog: } 0.375 \times 360^\circ = 135^\circ$$

$$\text{Cat: } 0.423 \times 360^\circ \approx 152^\circ$$

$$\text{Rodent: } 0.202 \times 360^\circ \approx 73^\circ$$

Use the data calculated to draw the following graph:



5. Find the total number of students surveyed.

$$39 + 44 + 21 + 24 = 128$$

Find out the percentage each pet represents of the total.

Dog:

$$\frac{39}{128} = 0.305$$

$$0.305 = 30.5\%$$

Cat:

$$\frac{44}{128} = 0.344$$

$$0.344 = 34.4\%$$

Rodent:

$$\frac{21}{128} = 0.164$$

$$0.164 = 16.4\%$$

No pet:

$$\frac{24}{128} = 0.188$$

$$0.188 = 18.8\%$$

Calculate how many degrees of the circle each pet represents:

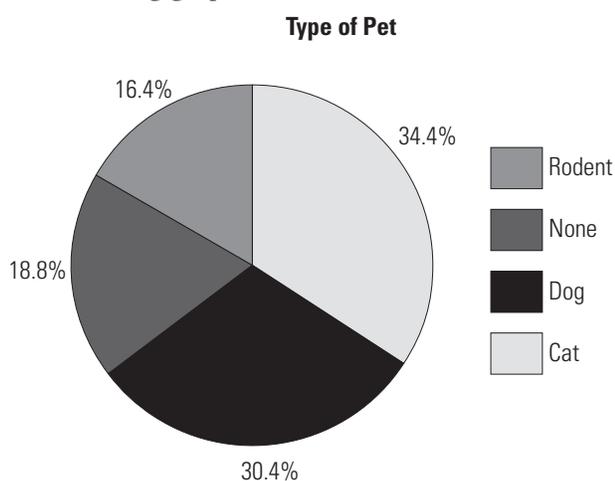
$$\text{Dog: } 0.305 \times 360^\circ \approx 110^\circ$$

$$\text{Cat: } 0.344 \times 360^\circ \approx 124^\circ$$

$$\text{Rodent: } 0.164 \times 360^\circ \approx 59^\circ$$

$$\text{No pet: } 0.188 \times 360^\circ \approx 68^\circ$$

Use the data calculated to draw the following graph:



6. Find the total money fundraised.

$$\$750 + \$325 + \$375 + \$150 + \$100 = \$1700$$

Find out the percentage each fundraising method represents of the total.

Parental donations:

$$\frac{750}{1700} = 0.441$$

$$0.441 = 44.1\%$$

Chocolate bar sales:

$$\frac{325}{1700} = 0.191$$

$$0.191 = 19.1\%$$

Hot lunches:

$$\frac{375}{1700} = 0.221$$

$$0.221 = 22.1\%$$

Concert proceeds:

$$\frac{150}{1700} = 0.088$$

$$0.088 = 8.8\%$$

Classroom donations:

$$\frac{100}{1700} = 0.059$$

$$0.059 = 5.9\%$$

Calculate how many degrees of the circle each fundraising efforts represents.

$$\text{Parental donations: } 0.441 \times 360^\circ \approx 159^\circ$$

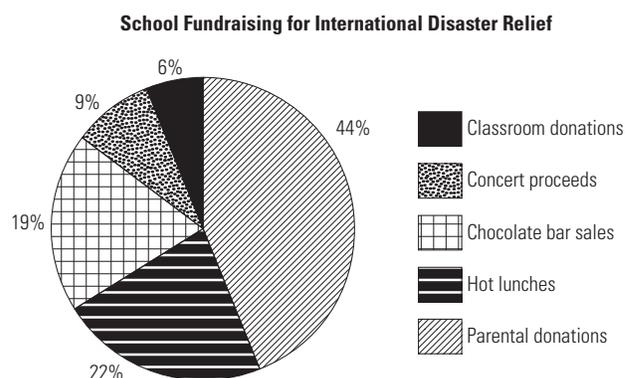
$$\text{Chocolate bar sales: } 0.191 \times 360^\circ \approx 69^\circ$$

$$\text{Hot lunches: } 0.221 \times 360^\circ \approx 80^\circ$$

$$\text{Concert proceeds: } 0.088 \times 360^\circ \approx 32^\circ$$

$$\text{Classroom donations: } 0.059 \times 360^\circ \approx 21^\circ$$

Use the data calculated to draw the following graph.



7. a) Find the total containers sold.

$$68 + 45 + 127 + 93 + 76 + 12 = 421$$

Find out the percentage each brand of hairspray represents of the total.

A:

$$\frac{68}{421} = 0.162$$

$$0.162 = 16.2\%$$

B:

$$\frac{45}{421} = 0.107$$

$$0.107 = 10.7\%$$

C:

$$\frac{127}{421} = 0.302$$

$$0.302 = 30.2\%$$

D:

$$\frac{93}{421} = 0.221$$

$$0.221 = 22.1\%$$

E:

$$\frac{76}{421} = 0.181$$

$$0.181 = 18.1\%$$

F:

$$\frac{12}{421} = 0.029$$

$$0.029 = 2.9\%$$

Calculate how many degrees of the circle each hairspray brand represents.

$$A: 0.162 \times 360^\circ \approx 58^\circ$$

$$B: 0.107 \times 360^\circ \approx 39^\circ$$

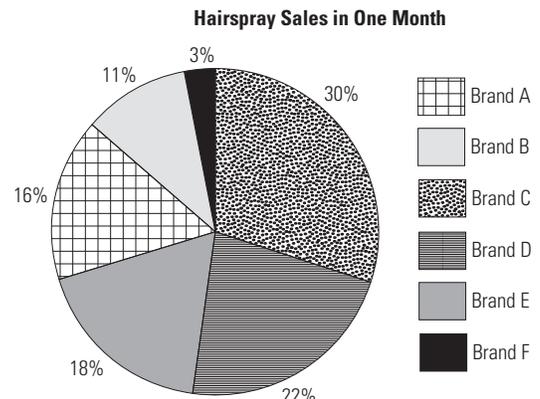
$$C: 0.302 \times 360^\circ \approx 109^\circ$$

$$D: 0.221 \times 360^\circ \approx 80^\circ$$

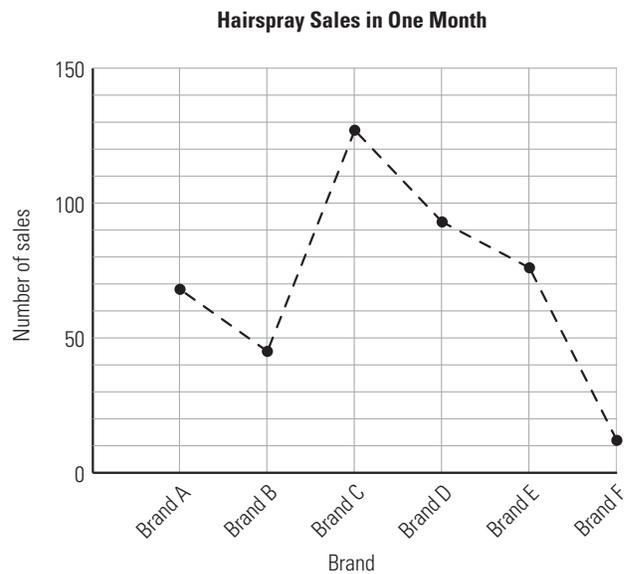
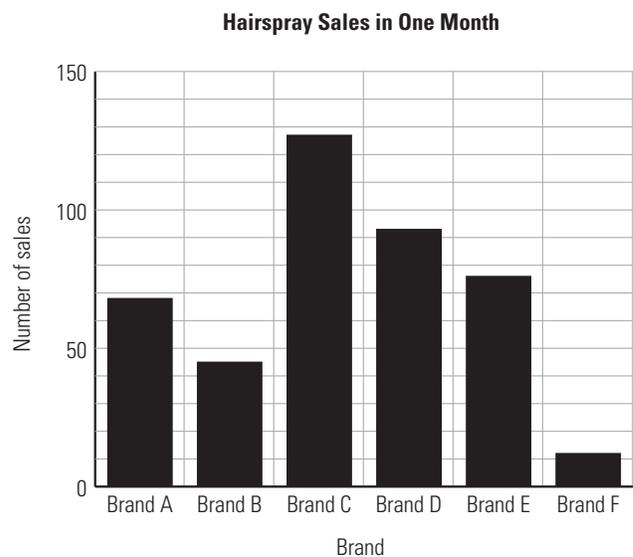
$$E: 0.181 \times 360^\circ \approx 65^\circ$$

$$F: 0.029 \times 360^\circ \approx 10^\circ$$

Use the data calculated to draw the following graph.



b) The information could have been displayed on a bar graph or a broken line graph.



PRACTISE YOUR NEW SKILLS, p. 117

1. a) Add the values in the graph to work out the totals.

$$52 + 14 + 6 + 65 = 137$$

- b) Divide the amount of people who listen to rock by the total to get the percent.

$$\frac{52}{137} = 0.380$$

$$0.380 = 38\%$$

There are 38% of people who prefer rock music.

- c) Calculate the percentage of students who prefer R&B.

$$\frac{14}{137} = 0.102$$

Calculate how many degrees of a circle that percentage represents.

$$0.102 \times 360^\circ \approx 37^\circ$$

R&B represents 37° of the circle.

2. Calculate the number of degrees each percentage represents.

$$\text{Travel package: } 0.60 \times 360^\circ = 216^\circ$$

$$\text{Tourism office: } 0.15 \times 360^\circ = 54^\circ$$

$$\text{Friend recommendation: } 0.10 \times 360^\circ = 36^\circ$$

Subtract the other percentages to work out what percent were walk-ins.

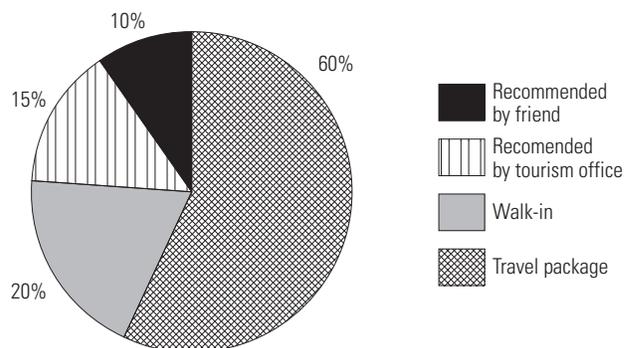
$$100 - 60 - 15 - 10 = 15\%$$

$$15\% = 0.15$$

$$\text{Walk-ins: } 0.15 \times 360^\circ = 54^\circ$$

Use the data calculated to draw the following graph.

Deciding Factors for Choosing Greg's Tour



3. Find the total number of books.

$$21 + 5 + 32 + 10 + 2 + 3 = 73$$

Science fiction:

$$\frac{21}{73} = 0.288$$

$$0.288 = 28.8\%$$

Biography:

$$\frac{5}{73} = 0.068$$

$$0.068 = 6.8\%$$

Historical fiction:

$$\frac{32}{73} = 0.438$$

$$0.438 = 43.8\%$$

Poetry:

$$\frac{10}{73} = 0.137$$

$$0.137 = 13.7\%$$

Autobiography:

$$\frac{2}{73} = 0.027$$

$$0.027 = 2.7\%$$

Self-help:

$$\frac{3}{73} = 0.041$$

$$0.041 = 4.1\%$$

Calculate the number of degrees each percentage represents.

Science fiction: $0.288 \times 360^\circ \approx 104^\circ$

Biography: $0.068 \times 360^\circ \approx 24^\circ$

Historical fiction: $0.438 \times 360^\circ \approx 158^\circ$

Poetry: $0.10 \times 360^\circ \approx 36^\circ$

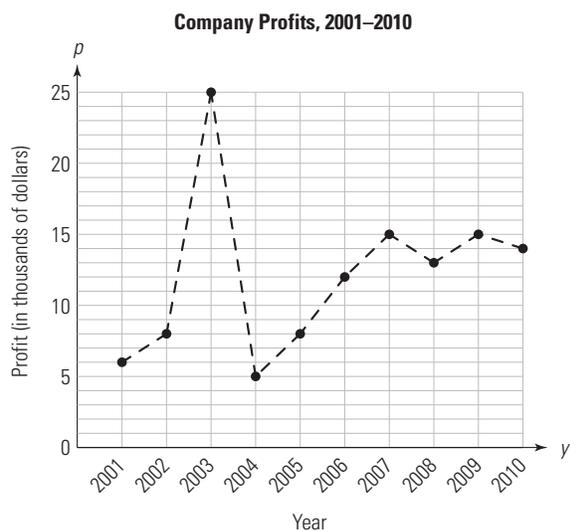
Autobiography: $0.02 \times 360^\circ \approx 7^\circ$

Self-help: $0.041 \times 360^\circ \approx 15^\circ$

Use the data calculated to draw the following graph.

CHAPTER TEST, p. 119

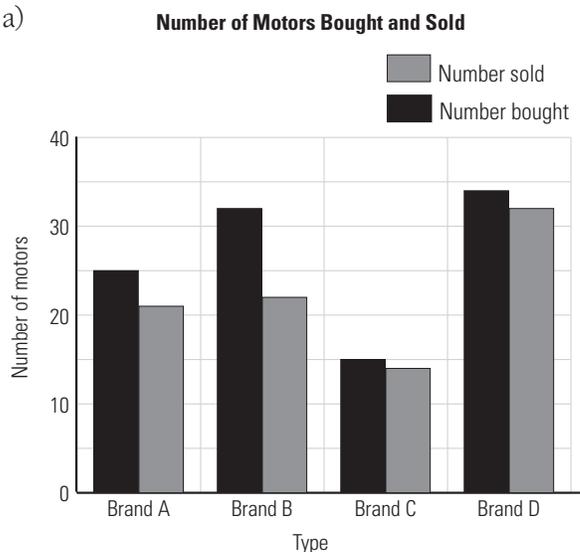
1. a)



b) The profit was highest in 2003 at \$25 000.

c) 2003 was a peak year for the company where their profit margin went way up. It then dropped down to what it had been the previous two years and made a steady increase over 3 years and seems to have leveled off for the last three years.

2. a)



b) Brand C has the smallest difference. Use subtraction to calculate the difference.

$$15 - 14 = 1$$

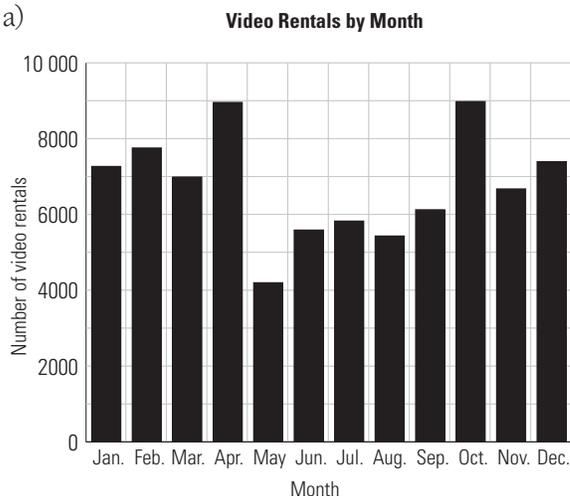
Brand C has the smallest difference, which is 1.

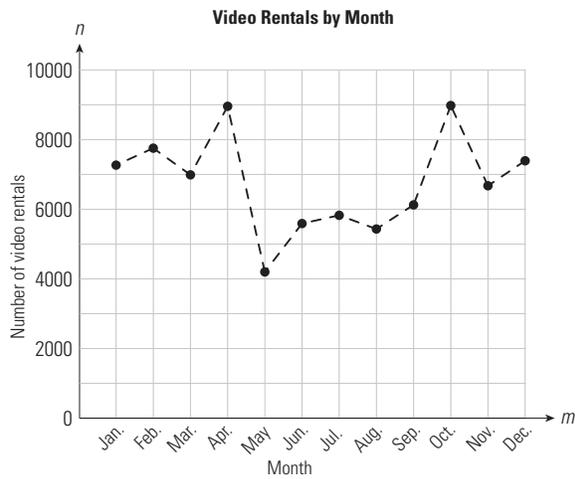
c) Brand B has the largest difference. Use subtraction to calculate the difference.

$$32 - 22 = 10$$

Brand B has the largest difference, which is 10.

3. a)





Either a broken line graph or a bar graph (vertical or horizontal) could be used because it is discrete data.

- b) October had the most rentals at 8978.
- c) Rentals tend to be lower in the summer months, but are otherwise stable.

Chapter 3

Surface Area, Volume, and Capacity

3.1

Surface Area of Prisms

BUILD YOUR SKILLS, p. 127

1. a) It is a circle.

$$A_{\text{circle}} = \pi r^2$$

$$A = \pi \times 44^2$$

$$A = 6082.1 \text{ mm}^2$$

The area of the circle is 6082.1 mm^2 .

- b) It is a triangle.

$$A_{\text{triangle}} = \frac{1}{2}bh$$

$$A = \frac{1}{2}(5.7)(9.3)$$

$$A = 26.5 \text{ m}^2$$

The area of the triangle is 26.5 m^2 .

- c) It is a parallelogram.

$$A_{\text{parallelogram}} = bh$$

$$A = (5.3)(14.8)$$

$$A = 78.4 \text{ cm}^2$$

The area of the parallelogram is 78.4 cm^2 .

- d) It is a triangle.

$$A_{\text{triangle}} = \frac{1}{2}bh$$

$$A = \frac{1}{2}(9)(16)$$

$$A = 72 \text{ in}^2$$

The area of the triangle is 72 in^2 .

2. a) It is a rectangle.

$$A_{\text{rectangle}} = lw$$

$$A = 5.2 \times 8.5$$

$$A = 44.2 \text{ m}^2$$

The area is 44.2 m^2 .

- b) It is a parallelogram.

$$A_{\text{parallelogram}} = bh$$

$$A = 5.2 \times 8.5$$

$$A = 44.2 \text{ m}^2$$

The area is 44.2 m^2 .

- c) It is a triangle.

$$A_{\text{parallelogram}} = bh$$

$$A = (5.3)(14.8)$$

$$A = 78.4 \text{ cm}^2$$

The area is 22.1 m^2 .

- d) It is a triangle.

$$A_{\text{triangle}} = \frac{1}{2}bh$$

$$A = \frac{1}{2}(9)(16)$$

$$A = 72 \text{ in}^2$$

The area is 22.1 m^2 .

- e) It is a triangle.

$$A_{\text{triangle}} = \frac{1}{2}bh$$

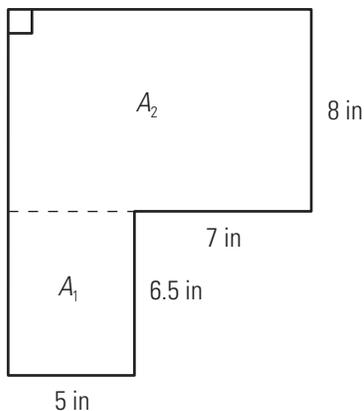
$$A = \frac{1}{2}(8.5)(5.2)$$

$$A = 22.1 \text{ m}^2$$

The area is 22.1 m^2 .

- f) Areas of rectangles and parallelograms are the same if the base and height are the same as the length and width. Areas of triangles are the same if they have the same height and width.

3. a) Divide the shape into two separate rectangles.



Calculate the area of the first rectangle.

$$A_1 = 5 \times 6.5$$

$$A_1 = 32.5 \text{ in}^2$$

Calculate the area of the second rectangle.

$$A_2 = (5 + 7) \times 8$$

$$A_2 = 12 \times 8$$

$$A_2 = 96 \text{ in}^2$$

Add the two areas together to get the total.

$$A = A_1 + A_2$$

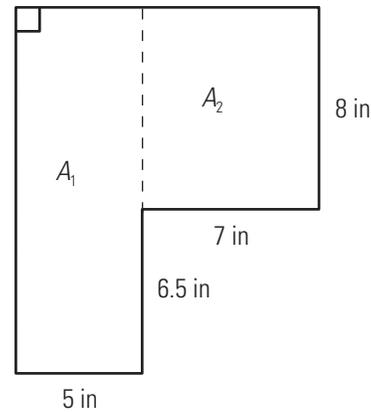
$$A = 32.5 + 96$$

$$A = 128.5 \text{ in}^2$$

The total area is 128.5 in^2 .

ALTERNATE SOLUTION

Divide the shape into two separate rectangles.



Calculate the area of the first rectangle.

$$A_1 = (8 + 6.5) \times 5$$

$$A_1 = 14.5 \times 5$$

$$A_1 = 72.5 \text{ in}^2$$

Calculate the area of the second rectangle.

$$A_2 = 8 \times 7$$

$$A_2 = 56 \text{ in}^2$$

Add the two areas together to get the total.

$$A = A_1 + A_2$$

$$A = 72.5 + 56$$

$$A = 128.5 \text{ in}^2$$

The total area is 128.5 in^2 .

- b) Divide the shape into a rectangle and a triangle.

Calculate the area of the triangle.

$$A_1 = \frac{1}{2}(4.2)(9.6 - 7.8)$$

$$A_1 = \frac{1}{2}(4.2)(1.8)$$

$$A_1 \approx 3.8 \text{ m}^2$$

Calculate the area of the rectangle.

$$A_2 = 7.8 \times 4.2$$

$$A_2 \approx 32.8 \text{ m}^2$$

Add the two areas together to get the total.

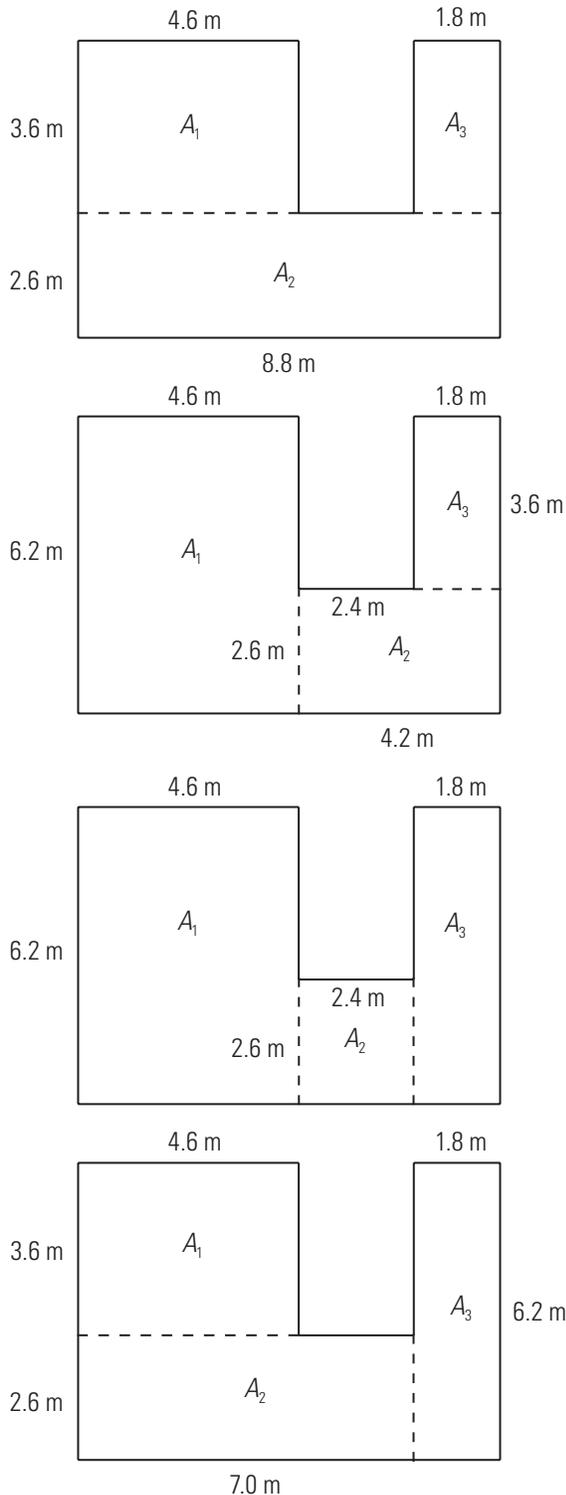
$$A = A_1 + A_2$$

$$A = 3.8 + 32.8$$

$$A = 36.6 \text{ m}^2$$

The total area is 36.6 m^2 .

4. Your answer may look like this.



Calculate the area of the first rectangle.

$$A_1 = 4.6 \times 3.6$$

$$A_1 \approx 16.6 \text{ m}^2$$

Calculate the area of the second rectangle.

$$A_2 = 2.6 \times 8.8$$

$$A_2 \approx 22.9 \text{ m}^2$$

Calculate the area of the third rectangle

$$A_3 = 3.6 \times 1.8$$

$$A_3 \approx 6.5 \text{ m}^2$$

Add the three areas to get the total.

$$A = A_1 + A_2 + A_3$$

$$A = 16.6 + 22.9 + 6.5$$

$$A = 46 \text{ m}^2$$

The area is 46 m^2 .

ALTERNATE SOLUTION 1

$$A_1 \approx 28.5 \text{ m}^2$$

$$A_2 \approx 10.9 \text{ m}^2$$

$$A_3 \approx 6.5 \text{ m}^2$$

$$A = 45.9 \text{ m}^2$$

The area is 45.9 m^2 . This is different from the original solution due to rounding.

ALTERNATE SOLUTION 2

$$A_1 \approx 28.5 \text{ m}^2$$

$$A_2 \approx 6.2 \text{ m}^2$$

$$A_3 \approx 11.2 \text{ m}^2$$

$$A = 45.9 \text{ m}^2$$

The area is 45.9 m^2 . This is different from the original solution due to rounding.

ALTERNATE SOLUTION 3

$$A_1 \approx 16.6 \text{ m}^2$$

$$A_2 \approx 18.2 \text{ m}^2$$

$$A_3 \approx 11.2 \text{ m}^2$$

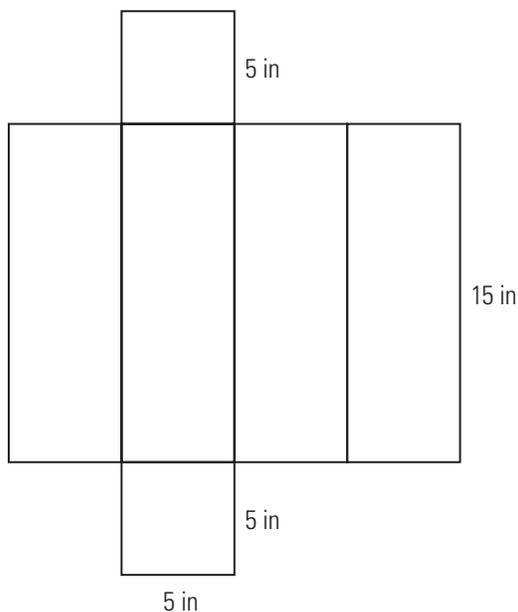
$$A = 46.0 \text{ m}^2$$

The area is 46 m^2 .

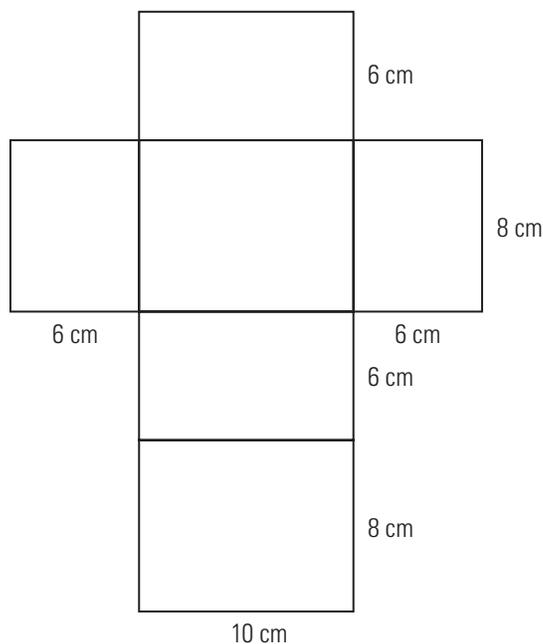
5.

<i>Prism</i>	<i>Shape of base</i>	<i>Right or oblique</i>	<i>Shape of lateral faces</i>	<i>Name of prism</i>
a)	triangular	oblique	parallelogram	oblique triangular prism
b)	rectangular	right	rectangular	right rectangular prism
c)	pentagonal	right	rectangular	right pentagonal prism
d)	trapezoidal	oblique	parallelogram	oblique trapezoidal prism
e)	octagonal	right	rectangular	right octagonal prism
f)	rectangular	oblique	parallelogram	oblique rectangular prism

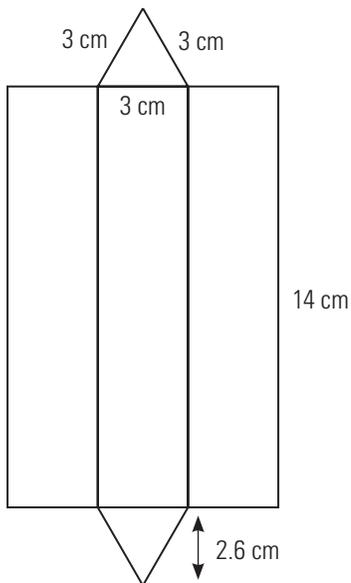
6. a)



8. a)



b)



The formula for the surface area is the total surface area of each face added together.

You can group some of the faces as they are identical.

$$SA = 2 \times A_1 + 2 \times A_2 + 2 \times A_3$$

$$SA = 2(8 \times 6) + 2(6 \times 10) + 2(8 \times 10)$$

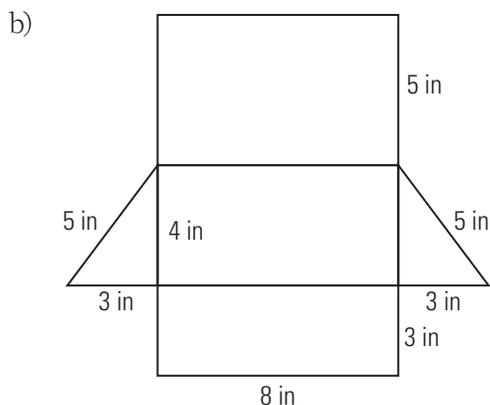
$$SA = 2(48) + 2(60) + 2(80)$$

$$SA = 96 + 120 + 160$$

$$SA = 376 \text{ cm}^2$$

The surface area is 376 cm^2 .

7. Ralph is correct. Manon's net is missing one of the side panels.



The formula for the surface area is the total surface area of each face added together.

You can group the triangular faces as they are identical.

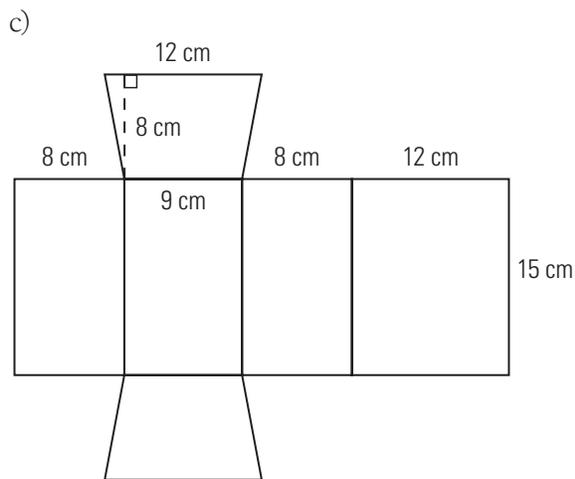
$$SA = 2 \times A_1 + A_2 + A_3 + A_4$$

$$SA = 2 \times \left(\frac{1}{2} \times 4 \times 3 \right) + 4 \times 8 + 3 \times 8 + 5 \times 8$$

$$SA = 12 + 32 + 24 + 40$$

$$SA = 108 \text{ in}^2$$

The surface area is 108 in^2 .



You need to calculate the length of the sides that are the slants of the trapezoid. You can do this using the Pythagorean theorem.

$$x^2 = 1.5^2 + 8^2$$

$$x = \sqrt{1.5^2 + 8^2}$$

$$x = \sqrt{2.25 + 64}$$

$$x = \sqrt{66.25}$$

$$x = 8.1 \text{ cm}$$

Calculate the area of the trapezoidal faces.

$$A_{\text{trapezoid}} = h \frac{(a + b)}{2}$$

$$A = 8 \times \frac{[(1.5 + 1.5 + 9) + 9]}{2}$$

$$A = 8 \times \frac{21}{2}$$

$$A = 84 \text{ cm}^2$$

The formula for the surface area is the total surface area of each face added together.

You can group some of the faces as they are identical.

$$SA = 2 \times A_1 + 2 \times A_2 + A_3 + A_4$$

$$SA = 2 \times (8.1 \times 15) + 2(84) + 9 \times 15 +$$

$$(1.5 + 9 + 1.5) \times 15$$

$$SA = 2(121.5) + 2(84) + 9 \times 15 +$$

$$(1.5 + 9 + 1.5) \times 15$$

$$SA = 243 + 168 + 135 + 180$$

$$SA = 726 \text{ cm}^2$$

The total surface area is 726 cm^2 .

9. The formula for the surface area is the total surface area of each face added together. Students should recognize that the two separate surfaces on the top use the same amount of area as the bottom surface and can be grouped. Also a number of the other faces have the same dimensions and can be grouped.

$$SA = 2 \times A_1 + 4 \times A_2 + 4 \times A_3$$

$$SA = 2 \times (1.5 \times 1.5) + 4 \times (1 \times 1.5) +$$

$$4 \times (2 \times 0.5)$$

$$SA = 2 \times 2.25 + 4 \times 1.5 + 4 \times 1$$

$$SA = 4.5 + 6 + 4$$

$$SA = 14.5 \text{ cm}^2$$

The surface area is 14.5 m².

10. Solve the area of the irregular figure by dividing it into 3 regular shapes.

$$A_1 = 20 \times 5 + 20 \times 5 + (30 - 5 - 5) \times 5$$

$$A_1 = 100 + 100 + 100$$

$$A_1 = 300 \text{ m}^2$$

The total surface area is the surface area of each face added together. Students should recognize that the three separate surfaces on the top use the same amount of area as the bottom surface and can be grouped. Also a number of the other faces have the same dimensions and can be grouped.

$$SA = 2 \times A_1 + 2 \times A_2 + 2 \times A_3 + 2 \times A_4$$

$$SA = 2 \times 300 + 2 \times (20 \times 15) + 2 \times$$

$$[(20 - 5) \times 15] + 2 \times (30 \times 15)$$

$$SA = 2 \times 300 + 2 \times 300 + 2 \times 225 + 2 \times 450$$

$$SA = 600 + 600 + 450 + 900$$

$$SA = 2550 \text{ cm}^2$$

The area of the figure is 2550 cm².

11. a) The total surface area is the surface area of each face added together. A number of the faces have the same dimensions and can be grouped.

$$SA = 2 \times A_1 + 2 \times A_2 + 2 \times A_3$$

$$SA = 2 \times (90 \times 70) + 2 \times (70 \times 60) +$$

$$2 \times (60 \times 90)$$

$$SA = 2 \times 6300 + 2 \times 4200 + 2 \times 5400$$

$$SA = 12\,600 + 8400 + 10\,800$$

$$SA = 31\,800 \text{ cm}^2$$

The surface area of the box is 31 800 cm².

- b) Use the surface area calculated to work out how much wood he will need.

$$w = 31\,800 \times 20\% + 31\,800$$

$$w = 31\,800 \times 0.20 + 31\,800$$

$$w = 6360 + 31\,800$$

$$w = 38\,160 \text{ cm}^2$$

The area of wood he will need to buy is 38 160 cm².

12. Solve the area of the irregular figure by dividing it into 2 regular shapes.

$$A_1 = 6 \times 1.5 + (8 - 6) \times 1.5$$

$$A_1 = 9 + 3$$

$$A_1 = 12 \text{ ft}^2$$

The total surface area is the surface area of each face added together. A number of the faces have the same dimensions and can be grouped.

$$SA = 2 \times A_1 + A_2 + 2 \times A_3 + A_4$$

$$SA = 2 \times 12 + 8 \times 1 + 2 \times (6 \times 1) +$$

$$(6 - 1.5) \times 1$$

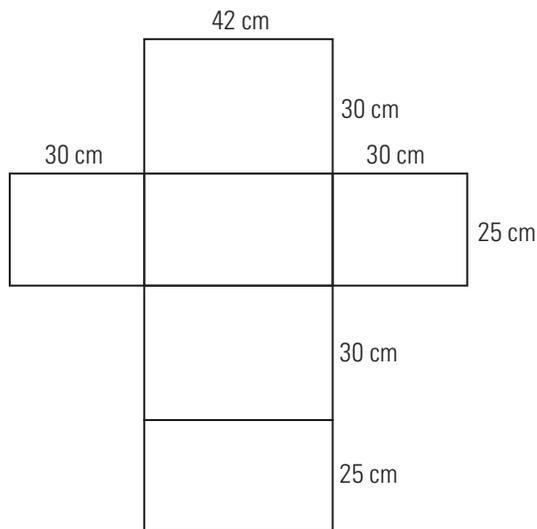
$$SA = 24 + 8 + 12 + 4.5$$

$$SA = 48.5 \text{ ft}^2$$

The total surface area of the duct is 48.5 ft².

PRACTISE YOUR NEW SKILLS, P. 167

1. a)



The total surface is the surface area of each face added together. A number of the faces have the same dimensions and can be grouped.

$$SA = 2 \times A_1 + 2 \times A_2 + 2 \times A_3$$

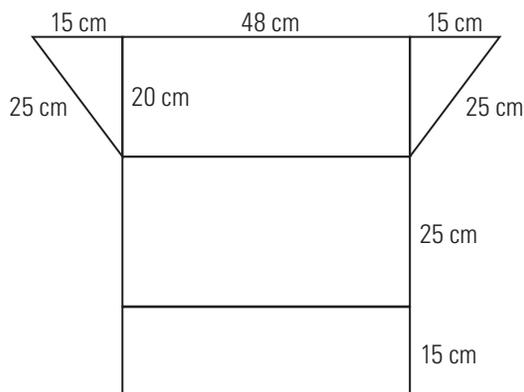
$$SA = 2 \times (42 \times 25) + 2 \times (30 \times 25) + 2 \times (42 \times 30)$$

$$SA = 2100 + 1500 + 2520$$

$$SA = 6120 \text{ cm}^2$$

The surface area of the prism is 6120 cm^2 .

b)



The total surface is the surface area of each face added together. A number of the faces have the same dimensions and can be grouped.

$$SA = 2 \times A_1 + A_2 + A_3 + A_4$$

$$SA = 2 \times (0.5 \times 12 \times 25) + 15 \times 48 + 20 \times 48 + 25 \times 48$$

$$SA = 300 + 720 + 960 + 1200$$

$$SA = 3180 \text{ cm}^2$$

The surface area of the prism is 3180 cm^2 .

2. The total surface is the surface area of each face added together. A number of the faces have the same dimensions and can be grouped.

$$SA = 2 \times A_1 + 2 \times A_2$$

$$SA = 2 \times \left(\frac{1}{2} \times 4 \times 1.5 \right) + 2 \times (12 \times 2.5)$$

$$SA = 6 + 60$$

$$SA = 66 \text{ ft}^2$$

The trough will need 66 ft^2 of wood to build it.

3. The total surface is the surface area of each face added together. A number of the faces have the same dimensions and can be grouped.

$$SA = A_1 + 2 \times A_2 + 2 \times A_3$$

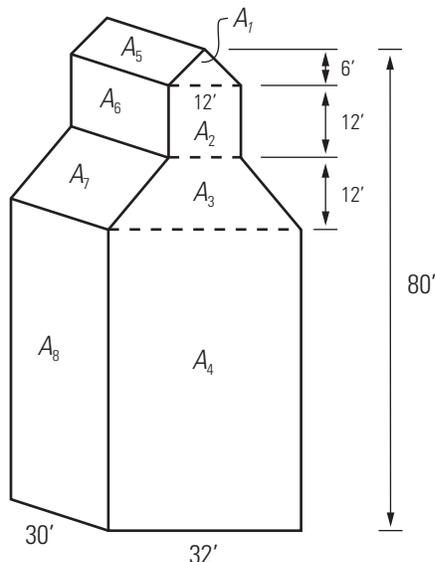
$$SA = 1.2 \times 0.6 + 2 \times (0.6 \times 0.4) + 2 \times (1.2 \times 0.4)$$

$$SA = 0.72 + 0.48 + 0.96$$

$$SA \approx 2.2 \text{ m}^2$$

Aaron will need 2.2 m^2 of glass to make the fish tank.

4. The surface area of the grain elevator can be divided into 8 different sections: the 4 rectangular sections of the side, and the 4 sections on the front of the building.



Note that the back of the elevator and the other side not visible are equal in surface area to the front and side area, so their surface areas will be multiplied by 2 to calculate the total surface area.

You can ignore the area of the bottom of the elevator.

Begin by calculating the areas of the 4 sections of the front face.

$$A_1 = \frac{bh}{2}$$

$$A_1 = \frac{12 \times 6}{2}$$

$$A_1 = 36 \text{ ft}^2$$

$$A_2 = \ell w$$

$$A_2 = 12 \times 12$$

$$A_2 = 144 \text{ ft}^2$$

$$A_3 = \frac{1}{2}(a + b)h$$

$$A_3 = \frac{1}{2}(12 + 32)(12)$$

$$A_3 = 264 \text{ ft}^2$$

$$A_4 = \ell w$$

$$A_4 = 32 \times 50$$

$$A_4 = 1600 \text{ ft}^2$$

Next, calculate the areas of the 4 rectangular sections of the side face.

For the first rectangle, A5, use the Pythagorean theorem to solve for the width.

$$a^2 + b^2 = c^2$$

$$(12 \div 2)^2 + 6^2 = w^2$$

$$\sqrt{6^2 + 6^2} = w$$

$$\sqrt{72} = w$$

$$8.5 \text{ ft} \approx w$$

$$A_5 = \ell w$$

$$A_5 = 30 \times 8.5$$

$$A_5 \approx 254.6 \text{ ft}^2$$

$$A_6 = \ell w$$

$$A_6 = 30 \times 12$$

$$A_6 = 360 \text{ ft}^2$$

For the third rectangle, A7, use the Pythagorean theorem to solve for the width.

$$a^2 + b^2 = c^2$$

$$\left(\frac{32 - 12}{2}\right)^2 + 12^2 = w^2$$

$$\sqrt{10^2 + 12^2} = w$$

$$244 = w$$

$$15.6 \text{ ft} \approx w$$

$$A_7 = \ell w$$

$$A_7 = 30 \times 15.6$$

$$A_7 \approx 468.6 \text{ ft}^2$$

$$A_8 = \ell w$$

$$A_8 = 30 \times (80 - 12 - 12 - 6)$$

$$A_8 = 1500 \text{ ft}^2$$

Finally, add all the areas together and multiply by 2 (because the back and other side faces are the same as the front and side faces shown).

$$SA = 2(A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8)$$

$$SA = 2(36 + 144 + 264 + 1600 + 254.6 + 360 + 468.6 + 1500)$$

$$SA = 9254 \text{ ft}^2$$

$$SA = 3180 \text{ cm}^2$$

The surface area of the grain The surface area of the grain elevator is 9254 ft².

3.2

Surface Area of Pyramids, Cylinders, Spheres & Cones

BUILD YOUR SKILLS, p. 150

1. Divide the irregular figure into a rectangle and a semicircle to calculate two areas that can be added to get the total.

$$A = lw + \frac{1}{2} \pi r^2$$

$$A = 3.5 \times 3.25 + \frac{1}{2} \times \pi \times \left(\frac{3.5}{2}\right)^2$$

$$A = 11.4 + 4.8$$

$$A = 16.2 \text{ ft}^2$$

The area of the figure is 16.2 ft².

2. Divide the irregular figure into a triangle and a semicircle to calculate two areas that can be added to get the total.

$$A = \frac{1}{2} bh + \frac{1}{2} \pi r^2$$

$$A = \frac{1}{2} \times \frac{6.8}{2} \times 6.8 + \frac{1}{2} \times \pi \times \left(\frac{6.8}{2}\right)^2$$

$$A = 11.56 + 18.16$$

$$A \approx 29.7 \text{ cm}^2$$

The area of the figure is 29.7 cm².

3. To find the area of the figure calculate the area of the rectangle and then subtract the semicircle from it.

$$A = lw - \frac{1}{2} \pi r^2$$

$$A = 310 \times 250 - \frac{1}{2} \times \pi \times \left(\frac{90}{2}\right)^2$$

$$A = 77\,500 - 3180.9$$

$$A \approx 74\,319 \text{ mm}^2$$

The area of the shape is 74 319 mm².

4. To find the area of the tank add the area of the circular faces to the area of the side.

$$SA = 2(\pi r^2) + 2\pi rh$$

$$SA = 2 \times \pi \times 1.5^2 + 2 \times \pi \times 1.5 \times 5$$

$$SA \approx 14.1 + 47.1$$

$$SA \approx 61.2 \text{ m}^2$$

The surface area of the tank is 61.2 m².

5. To find the surface area of the pipe find the area of the side. The ends of the pipe are hollow so they can be ignored.

$$SA = 2\pi rh$$

$$SA = 2 \times \pi \times \frac{4.5}{2} \times 18.8$$

$$SA \approx 265.8 \text{ cm}^2$$

The surface area of the pipe is 265.8 cm².

6. To find the surface area of the figure add the areas of each surface. The top two surfaces can be added together to be equivalent to the bottom surface.

$$SA = 2\pi r_1 h_1 + 2\pi r_2 h_2 + 2 \times (\pi r^2)$$

$$SA = 2 \times \pi \times 3.8 \times 9.6 + 2 \times \pi \times (3.8 + 4.2) \times 4.8 + 2 \times \pi \times (3.8 + 4.2)^2$$

$$SA \approx 229.2 + 241.3 + 402.1$$

$$SA \approx 872.6 \text{ mm}^2$$

The surface area of the figure is 872.6 mm².

7. To calculate the surface area calculate the area of the square base.

$$A_1 = lw$$

$$A_1 = 28 \times 28$$

$$A_1 = 784 \text{ cm}^2$$

Calculate the area of one of the triangular sides.

$$A_2 = \frac{1}{2}bh$$

$$A_2 = \frac{1}{2}(28)(21)$$

$$A_2 = 294 \text{ cm}^2$$

Calculate the total surface area.

$$SA = A_1 + 4(A_2)$$

$$SA = 784 + 4(294)$$

$$SA = 1960 \text{ cm}^2$$

The surface area of the pyramid is 1960 cm^2 .

8. Calculate the area of one of the triangles.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(18)(28)$$

$$A = 252 \text{ cm}^2$$

Calculate the total surface area.

$$SA = 5 \times A$$

$$SA = 5 \times 252$$

$$SA = 1260 \text{ cm}^2$$

The exposed area of the pyramid is 1260 cm^2 .

9. Calculate the area one of the triangles.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(12.6)(18.7)$$

$$A \approx 117.8 \text{ cm}^2$$

Calculate the total surface area.

$$SA = 8 \times A$$

$$SA = 8 \times 117.8$$

$$SA = 942.4 \text{ cm}^2$$

The lateral surface area of the pyramid is 942.5 cm^2 .

10. Find the area of the base of the pyramid.

$$A_1 = lw$$

$$A_1 = 12 \times 12$$

$$A_1 = 144 \text{ cm}^2$$

Find the area of one of the triangles.

$$A_2 = \frac{1}{2}bh$$

$$A_2 = \frac{1}{2}(12)(8)$$

$$A_2 = 48 \text{ cm}^2$$

Calculate the total surface area.

$$SA = A_1 + 4A_2$$

$$SA = 144 + 4(48)$$

$$SA = 336 \text{ cm}^2$$

The total surface area of the pyramid is 384 in^2 .

11. Find the height of one of the triangles using the Pythagorean theorem.

$$h^2 = 16^2 - \left(\frac{16}{2}\right)^2$$

$$h = \sqrt{16^2 - 8^2}$$

$$h = \sqrt{256 - 64}$$

$$h = \sqrt{192}$$

$$h \approx 13.9 \text{ cm}$$

Find the area of one of the triangles.

$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(16)(13.9)$$

$$A = 111.2 \text{ cm}^2$$

Calculate the total surface area.

$$SA = 4A$$

$$SA = 4 \times 111.2$$

$$SA = 444.8 \text{ cm}^2$$

The area of the pyramid is 444.8 cm^2 .

12. First use the surface area you know to solve for the slant height.

$$SA = A_1 + 4A_2$$

$$SA = lw + 4\left(\frac{1}{2}bh_s\right)$$

$$680 = 16 \times 16 + 4 \times \left(\frac{1}{2} \times 16 \times h_s\right)$$

$$680 = 256 + 32h_s$$

$$424 = 32h_s$$

$$13.25 = h_s$$

The slant height of the pyramid is 13.25 cm.

Use the slant height and the Pythagorean theorem to solve for the height of the pyramid.

$$h^2 = 13.25^2 - \left(\frac{16}{2}\right)^2$$

$$h = \sqrt{13.25^2 - 8^2}$$

$$h \approx \sqrt{175.6 - 64}$$

$$h \approx \sqrt{111.6}$$

$$h \approx 10.6 \text{ cm}$$

The height of the pyramid was 10.6 cm.

13. Calculate the area of the circular base.

$$A_1 = \pi r^2$$

$$A_1 = \pi(28)^2$$

$$A_1 \approx 2463.0 \text{ cm}^2$$

Calculate the area of the lateral surface.

$$A_2 = \pi rs$$

$$A_2 = \pi \times 28 \times 82$$

$$A_2 = 7213.1 \text{ cm}^2$$

Calculate the total surface area.

$$SA = A_1 + A_2$$

$$SA = 2463 + 7213.1$$

$$SA = 9676.1 \text{ cm}^2$$

The total area of the cone is 9676.1 cm².

14. To calculate the surface area of the cone add the area of the circular base to the area of the lateral surface.

$$SA = \pi r^2 + \pi rs$$

$$SA = \pi\left(\frac{13.6}{2}\right)^2 + \pi\left(\frac{13.6}{2}\right)(9.8)$$

$$SA = 46.24\pi + 66.64\pi$$

$$SA = 112.88\pi$$

$$SA \approx 354.6 \text{ cm}^2$$

The surface area of the cone is 354.6 cm².

15. Calculate the slant height of the cone using the Pythagorean theorem.

$$s^2 = h^2 + r^2$$

$$s^2 = 20^2 + 16^2$$

$$s = \sqrt{20^2 + 16^2}$$

$$s = \sqrt{656}$$

$$s = 25.6 \text{ in}$$

To calculate the surface area of the cone add the area of the circular base to the area of the lateral surface.

$$SA = \pi r^2 + \pi rs$$

$$SA = \pi(16)^2 + \pi(16)(25.6)$$

$$SA = 256\pi + 320\pi$$

$$SA = 2091.0 \text{ in}^2$$

The surface area of the cone is 2091.0 in².

16. Calculate the slant height of the cone using the Pythagorean theorem.

$$s^2 = h^2 + r^2$$

$$s^2 = 39.7^2 + 45.7^2$$

$$s = \sqrt{39.7^2 + 45.7^2}$$

$$s \approx \sqrt{3664.6}$$

$$s \approx 60.5 \text{ mm}$$

To calculate the surface area of the cone add the area of the circular base to the area of the lateral surface.

$$SA = \pi r^2 + \pi rs$$

$$SA = \pi(45.7)^2 + \pi(45.7)(60.5)$$

$$SA \approx 2088.5\pi + 2764.9\pi$$

$$SA \approx 15247 \text{ mm}^2$$

The surface area of the cone is 15 247 mm².

17. Calculate the slant height of the cone using the Pythagorean theorem.

$$s^2 = h^2 + r^2$$

$$s^2 = 32^2 + \left(\frac{28}{2}\right)^2$$

$$s = \sqrt{32^2 + 14^2}$$

$$s \approx \sqrt{1220}$$

$$s \approx 34.9 \text{ cm}$$

To calculate the surface area of the cone add the area of the circular base to the area of the lateral surface.

$$SA = \pi rs$$

$$SA = \pi\left(\frac{28}{2}\right)(34.9)$$

$$SA \approx 488.6\pi$$

$$SA \approx 1535.0 \text{ cm}^2$$

The surface area of the cone is 1535.0 cm².

18. Calculate the surface area of a sphere.

$$SA = 4\pi r^2$$

$$SA = 4 \times \pi \times 1.3^2$$

$$SA = 21.2 \text{ m}^2$$

The surface area of the sphere is 21.2 m².

19. Calculate the surface area of a sphere.

$$SA = 4\pi r^2$$

$$SA = 4 \times \pi \times \left(\frac{24.8}{2}\right)^2$$

$$SA = 1932.2 \text{ mm}^2$$

The surface area of the sphere is 1932.2 mm².

20. To get the surface area of a hemisphere calculate half the surface area of a sphere and the area of the base.

$$SA = \frac{1}{2} \times 4\pi r^2 + \pi r^2$$

$$SA = \frac{1}{2} \times 4 \times \pi \times 18.5^2 + \pi \times 18.5^2$$

$$SA \approx 684.5\pi + 342.3\pi$$

$$SA \approx 3225.8 \text{ cm}^2$$

The surface area of the hemisphere is 2150.4 cm².

21. The surface area of the figure is the surface area of a hemisphere, the lateral surface area of a cylinder and a lateral surface area of a cone.

Calculate the surface area of a hemisphere.

$$SA_1 = \frac{1}{2} \times 4\pi r^2$$

$$SA_1 = \frac{1}{2} \times 4 \times \pi \times \left(\frac{0.8}{2}\right)^2$$

$$SA_1 \approx 1.01 \text{ m}^2$$

Calculate the surface area of the lateral cylinder.

$$SA_2 = 2\pi rh$$

$$SA_2 = 2 \times \pi \times \frac{0.8}{2} \times 2.2$$

$$SA_2 \approx 5.53 \text{ m}^2$$

Calculate the surface area of the lateral sides of the cone.

$$SA_3 = \pi rs$$

$$SA_3 = \pi \times \frac{0.8}{2} \times 1.4$$

$$SA_3 \approx 1.76 \text{ m}^2$$

Calculate the total surface area.

$$SA = SA_1 + SA_2 + SA_3$$

$$SA = 1.01 + 5.53 + 1.76$$

$$SA = 8.3 \text{ m}^2$$

The total surface area of the figure is 8.3 m^2 .

22. Find the lateral surface area of the top cylinder.

$$SA_1 = 2\pi rh$$

$$SA_1 = 2 \times \pi \times 2 \times 2$$

$$SA_1 \approx 25.1 \text{ ft}^2$$

Find the lateral surface area of the middle cylinder.

$$SA_2 = 2\pi rh$$

$$SA_2 = 2 \times \pi \times 3 \times 2$$

$$SA_2 \approx 37.7 \text{ ft}^2$$

Find the lateral surface area of the bottom cylinder.

$$SA_3 = 2\pi rh$$

$$SA_3 = 2 \times \pi \times 4 \times 2$$

$$SA_3 \approx 50.3 \text{ ft}^2$$

Find the surface area of the circle on the bottom.

$$SA_4 = \pi r^2$$

$$SA_4 = \pi \times 4^2$$

$$SA_4 \approx 50.3 \text{ ft}^2$$

The three exposed surfaces on the top if added together are equivalent to the surface area of the circle on the bottom. Calculate the total surface area.

$$SA = SA_1 + SA_2 + SA_3 + 2 \times SA_4$$

$$SA = 25.1 + 37.7 + 50.3 + 2 \times 50.3$$

$$SA = 213.7 \text{ ft}^2$$

The total exposed surface area of the cylinder is 213.6 ft^2 .

23. Find the surface area of the hemisphere.

$$SA_1 = \frac{1}{2} \times 4\pi r^2$$

$$SA_1 = \frac{1}{2} \times 4 \times \pi \times \left(\frac{18}{2}\right)^2$$

$$SA_1 \approx 508.9 \text{ cm}^2$$

Find the length of the slant of the cone.

$$s^2 = 12^2 + \left(\frac{18}{2}\right)^2$$

$$s = \sqrt{12^2 + 9^2}$$

$$s = \sqrt{144 + 81}$$

$$s = \sqrt{225}$$

$$s = 15 \text{ cm}$$

Find the lateral surface area of the cone.

$$SA_2 = \pi rs$$

$$SA_2 = \pi \times \frac{18}{2} \times 15$$

$$SA_2 \approx 424.1 \text{ cm}^2$$

Add the totals together.

$$SA = SA_1 + SA_2$$

$$SA = 508.9 + 424.1$$

$$SA = 933.0 \text{ cm}^2$$

The surface area of the figure is 933.0 cm^2 .

PRACTICE YOUR SKILLS, p. 167

1. Calculate the area of the bottom of the cake pan by calculating the area of a circle.

$$SA_1 = \pi r^2$$

$$SA_1 = \pi \times \left(\frac{9}{2}\right)^2$$

$$SA_1 \approx 63.6 \text{ in}^2$$

Calculate the area of the side of the cake pan by calculating the lateral surface of a cylinder.

$$SA_2 = 2\pi rh$$

$$SA_2 = 2 \times \pi \times \frac{9}{2} \times 1.5$$

$$SA_2 \approx 42.4 \text{ in}^2$$

Calculate the total area.

$$SA = SA_1 + SA_2$$

$$SA = 63.6 + 42.4$$

$$SA = 106 \text{ in}^2$$

He will use 106 in^2 to make the cake tin.

2. Find the surface area of one cup by calculating the lateral surface area of the cone. Convert the values from mm to cm.

$$SA_c = \pi rs$$

$$SA_c = \pi \times \frac{3}{2} \times 6$$

$$SA_c \approx 28.3 \text{ cm}^2$$

Use that value to find how much are needed to produce 50 cups.

$$SA = 28.3 \times 50$$

$$SA = 1415 \text{ cm}^2$$

To make 50 conical cups you would need 1415 cm^2 of paper.

3. Calculate the area of the front and back of the hut by calculating the area of a circle. If you ignore the door at this point you can do this by calculating the area of one whole circle because the front and back are each equal to half.

$$SA_1 = \pi r^2$$

$$SA_1 = \pi \left(\frac{20}{2} \right)^2$$

$$SA_1 \approx 314.2 \text{ ft}^2$$

Calculate the area of the door.

$$SA_2 = 7 \times 8$$

$$SA_2 = 56 \text{ ft}^2$$

Calculate the area of the half of a lateral cylinder.

$$SA_3 = \frac{1}{2} \times 2\pi rh$$

$$SA_3 = \frac{1}{2} \times 2 \times \pi \times \frac{20}{2} \times 48$$

$$SA_3 \approx 1508.0 \text{ ft}^2$$

Calculate the total area by adding the area of the front, back and the sides and subtracting the door.

$$SA = SA_1 - SA_2 + SA_3$$

$$SA = 314.2 - 56 + 1508.0$$

$$SA = 1766.2 \text{ ft}^2$$

There would need to be 1766.2 ft^2 of corrugated steel.

4. Find the surface area of one of the triangles.

$$A_t = \frac{1}{2}bh$$

$$A_t = \frac{1}{2}(3.5)(12.4)$$

$$A_t = 21.7 \text{ ft}^2$$

Use the area calculated to solve for the lateral surface area.

$$SA = 6 \times A_t$$

$$SA = 6 \times 21.7$$

$$SA = 130.2 \text{ ft}^2$$

The lateral surface area of the hexagonal pyramid is 130.2 ft^2 .

5. Calculate the surface area of the cone by adding the base to the lateral surface area.

$$SA = \pi r^2 + \pi rs$$

$$SA = \pi(12)^2 + \pi(12)(15)$$

$$SA \approx 452.4 + 565.5$$

$$SA \approx 1017.9 \text{ ft}$$

The surface area of the cone is 1017.9 ft².

6. Calculate the surface area of the tennis ball.

$$SA = 4\pi r^2$$

$$SA = 4 \times \pi \times \left(\frac{6.7}{2}\right)^2$$

$$SA = 141.0 \text{ cm}^2$$

The surface area of the tennis ball is 141.0 cm².

3.3

Volume and Capacity of Prisms and Cylinders

BUILD YOUR SKILLS, p. 171

1. a) Use the formula for volume.

$$V = A_{\text{base}} \times h$$

$$V = lwh$$

$$V = 15.7 \times 18.8 \times 12.5$$

$$V \approx 3690 \text{ cm}^3$$

The volume of the prism is 3690 cm^3 .

Since 1 L equals 1000 cm^3 , divide the volume by 1000.

$$3690 \div 1000 = 3.69 \text{ L}$$

The prism has a capacity of 3.69 L.

- b) Use the formula for volume.

$$V = 2.75 \times 2.75 \times 4.5$$

$$V \approx 34.0 \text{ m}^3$$

The volume is 34 m^3 .

Since we only know the conversion of cm^3 to L we must convert m^3 to cm^3 .

There are 100 centimetres in a metre. So 1 m^3 is equal to $(100 \text{ cm})^3$, which equals $1\,000\,000 \text{ cm}^3$. So multiply the amount in m^3 by $1\,000\,000 \text{ cm}^3/\text{m}^3$. Then since 1 L equal 1000 cm^3 , divide the volume by 1000.

$$34 \times 1\,000\,000 = 34\,000\,000 \text{ cm}^3$$

$$34\,000\,000 \div 1000 = 34000 \text{ L}$$

The capacity of prism is 34 000 L.

- c) Convert the units from inches to centimetres.

$$1.5 \text{ in} = (2.54 \times 1.5) \text{ cm}$$

$$1.5 \text{ in} = 3.81 \text{ cm}$$

$$3.75 \text{ in} = (2.54 \times 3.75) \text{ cm}$$

$$3.75 \text{ in} \approx 9.53 \text{ cm}$$

$$2.25 \text{ in} = (2.54 \times 2.25) \text{ cm}$$

$$2.25 \text{ in} \approx 5.72 \text{ cm}$$

Use the formula for volume.

$$V = 3.81 \times 9.53 \times 5.72$$

$$V \approx 208 \text{ cm}^3$$

Since 1 mL equals 1 cm^3 the capacity is 208 mL.

ALTERNATE SOLUTION

If you know the conversions you could also do the calculations in imperial.

Use the formula for volume.

$$V = 1.5 \times 3.75 \times 2.25$$

$$V \approx 12.7 \text{ in}^3$$

Since 1 US gal is 231 in^3 , divide the volume by 231.

$$12.7 \div 231 \approx 0.055 \text{ US gal}$$

The capacity is 0.055 US gal.

2. Use the volume formula to solve for the height.

$$V = A_{\text{base}} \times h$$

$$V = lwh$$

$$142 = 5.2 \times 7.8 \times$$

$$3.5 \text{ m} \approx h$$

The height of the prism is 3.5 m.

3. We can use the volume formula to solve for the missing height.

$$l_1 w_1 h_1 = l_2 w_2 h_2$$

$$18 \times 12 \times 32 = 14 \times 20 \times h_2$$

$$6912 = 280h_2$$

$$24.7 \text{ cm} \approx h_2$$

The height of the second prism must be 24.7 cm.

4. Calculate the total volume of the hole.

$$V = lwh$$

$$V = 35 \times 25 \times 12$$

$$V = 10500 \text{ m}^3$$

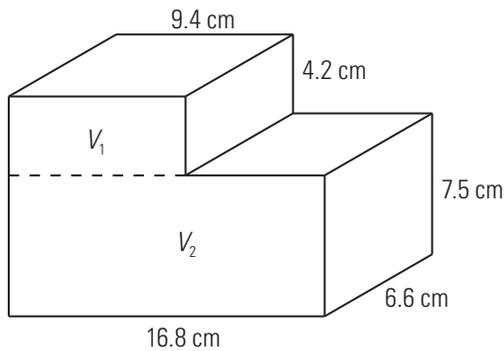
To determine how many trips must be made divide the total volume by the volume the trailer can carry.

$$t = 10500 \div 15$$

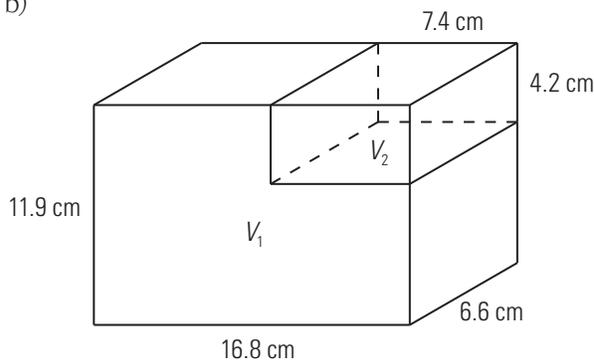
$$t = 700$$

The excavation will take 700 trips.

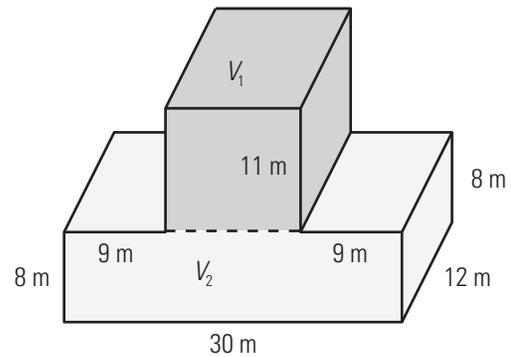
5. a)



- b)



6. Divide the figure into two regular sections.



Calculate the volume of the top section of the shape.

$$V_1 = lwh$$

$$V_1 = (30 - 9 - 9) \times 12 \times 11$$

$$V_1 = 12 \times 12 \times 11$$

$$V_1 = 1584 \text{ m}^3$$

Calculate the volume of the bottom section of the shape.

$$V_2 = lwh$$

$$V_2 = 30 \times 12 \times 8$$

$$V_2 = 2880 \text{ m}^3$$

Calculate the total volume.

$$V = V_1 + V_2$$

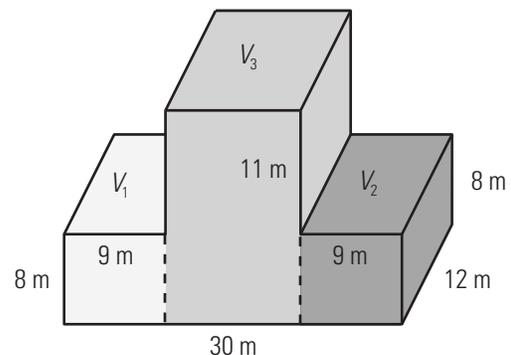
$$V = 1584 + 2880$$

$$V = 4464 \text{ m}^3$$

The volume of the figure is 4464 m³.

ALTERNATE SOLUTION

Divide the figure into three regular sections.



Calculate the volume of the left section of the shape.

$$V_1 = lwh$$

$$V_1 = 9 \times 12 \times 8$$

$$V_1 = 864 \text{ m}^3$$

Calculate the volume of the middle section of the shape.

$$V_2 = lwh$$

$$V_2 = (30 - 9 - 9) \times 12 \times (11 + 8)$$

$$V_2 = 12 \times 12 \times 19$$

$$V_2 = 2736 \text{ m}^3$$

The left and right sections have the same dimensions so at this point calculate the total volume.

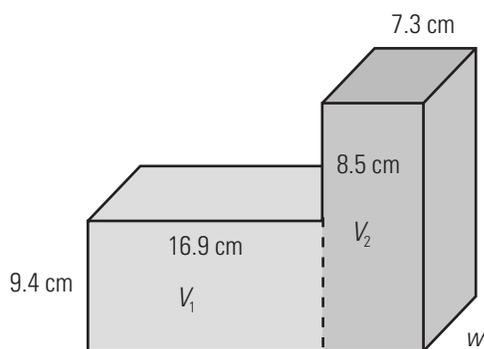
$$V = 2(V_1) + V_2$$

$$V = 2(864) + 2736$$

$$V = 4464 \text{ m}^3$$

The volume of the figure is 4464 m^3 .

7. Divide the figure into two sections.



Calculate w as an expression of the left section of the shape.

$$V_1 = lwh$$

$$V_1 = 16.9 \times w \times 9.4$$

$$V_1 = 158.86w$$

Calculate w as an expression of the right section of the shape.

$$V_2 = lwh$$

$$V_2 = 7.3 \times w \times (8.5 + 9.4)$$

$$V_2 = 130.67w$$

Since 1 L equals 1000 cm^3 , multiply the capacity by 1000 to get the volume.

$$1.8 \times 1000 = 1800 \text{ cm}^3$$

$$V = 1800 \text{ cm}^3$$

Use the values calculated to solve for w .

$$V = V_1 + V_2$$

$$1800 = 158.86w + 130.67$$

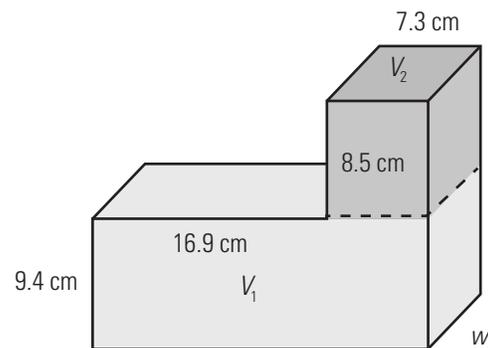
$$1800 = 289.53w$$

$$6.2 \text{ cm} \approx w$$

The depth of the prism is 6.2 cm.

ALTERNATE SOLUTION

Divide the figure into two sections.



Calculate w as an expression of the bottom section of the shape.

$$V_1 = lwh$$

$$V_1 = (16.9 + 7.3) \times w \times 9.4$$

$$V_1 = 227.48w$$

Calculate w as an expression of the top section of the shape.

$$V_2 = lwh$$

$$V_2 = 7.3 \times w \times 8.5$$

$$V_2 = 62.05w$$

As above convert the capacity of the container to an expression of volume to get 1800 cm^3 .

Use the values calculated to solve for w .

$$V = V_1 + V_2$$

$$1800 = 227.48w + 62.05$$

$$1800 = 289.53w$$

$$6.2 \text{ cm} \approx w$$

The depth of the prism is 6.2 cm.

8. Calculate the volume of the cylinder.

$$V = A_{\text{base}} \times h$$

$$V = \pi r^2 h$$

$$V = \pi \times \left(\frac{15}{2}\right)^2 \times 36$$

$$V \approx 6362 \text{ cm}^3$$

The volume of the cylinder is 6362 cm^3 .

Since 1 L equals 1000 cm^3 , divide the volume by 1000 to get the capacity.

$$6362 \div 1000 \approx 6.4 \text{ L}$$

The capacity of the cylinder is approximately 6.4 L.

9. Calculate the volume of the top cylinder.

$$V_t = \pi r^2 h$$

$$V_t = \pi \times \left(\frac{10}{2}\right)^2 \times 20$$

$$V_t \approx 1570.8 \text{ cm}^3$$

Calculate the volume of the middle cylinder.

$$V_m = \pi r^2 h$$

$$V_m = \pi \times \left(\frac{20}{2}\right)^2 \times 20$$

$$V_m \approx 6283.2 \text{ cm}^3$$

Calculate the volume of the bottom cylinder.

$$V_b = \pi r^2 h$$

$$V_b = \pi \times \left(\frac{40}{2}\right)^2 \times 20$$

$$V_b \approx 25132.7 \text{ cm}^3$$

Calculate the total volume.

$$V = V_t + V_m + V_b$$

$$V = 1570.8 + 6283.2 + 25132.7$$

$$V \approx 32987 \text{ cm}^3$$

The volume of the cylinders is $32\,987 \text{ cm}^3$.

Since 1 L equals 1000 cm^3 , divide the volume by 1000 to get the capacity.

$$32987 \div 1000 \approx 33.0 \text{ L}$$

The capacity of the cylinders is approximately 33 L.

10. Convert the capacity of the can into an expression of volume.

$$V = 3.24 \times 1000$$

$$V = 3240 \text{ cm}^3$$

Use the formula for volume to solve for the height of the can.

$$V = A_{\text{base}} \times h$$

$$V = \pi r^2 h$$

$$3240 = \pi \times \left(\frac{15.56}{2}\right)^2 \times h$$

$$3240 \approx 190.16h$$

$$17.0 \text{ cm} \approx h$$

The height of the can is 17.0 cm.

PRACTICE YOUR SKILLS, p. 179

1. a) Calculate the volume of the figure.

$$V = A_{\text{base}} \times h$$

$$V = lwh$$

$$V = 6.8 \times 4.2 \times 3.9$$

$$V \approx 111.4 \text{ cm}^3$$

The volume is 111.4 cm^3 .

- b) Calculate the volume of the figure.

$$V = A_{\text{base}} \times h$$

$$V = \pi r^2 h$$

$$V = \pi \times 2.1^2 \times 2.5$$

$$V \approx 34.6 \text{ cm}^3$$

The volume is 34.6 cm^3 .

- c) Calculate the volume of the cylinder.

$$V_c = \pi r^2 h$$

$$V_c = \pi \times 0.8^2 \times 5.2$$

$$V_c \approx 10.5 \text{ m}^3$$

Calculate the volume of the rectangular prism.

$$V_r = lwh$$

$$V_r = 4.5 \times 4.5 \times 3.5$$

$$V_r \approx 70.9 \text{ m}^3$$

Calculate the total volume.

$$V = V_c + V_r$$

$$V = 10.5 + 70.9$$

$$V = 81.4 \text{ m}^3$$

The volume of the figure is 81.4 m^3 .

2. It should be apparent that a rectangular prism is bigger to prove this fact it is necessary to calculate volume of each. Calculate the volume of the rectangular prism.

$$V_r = lwh$$

$$V_r = 16 \times 16 \times 25$$

$$V_r = 6400 \text{ cm}^3$$

Calculate the volume of the cylinder.

$$V_c = \pi r^2 h$$

$$V_c = \pi \times \left(\frac{16}{2}\right)^2 \times 25$$

$$V_c \approx 5026.5 \text{ cm}^3$$

The rectangular prism is bigger.

3. The silo is shaped like a cylinder so use the formula for the volume of a cylinder.

$$V = \pi r^2 h$$

$$V = \pi \times \left(\frac{24}{2}\right)^2 \times 70$$

$$V \approx 31\,667 \text{ ft}^3$$

The silo can hold $31\,667 \text{ ft}^3$ of grain.

4. a) Calculate the volume. Convert all values to metres.

$$V = lwh$$

$$V = 6 \times 4.5 \times 0.15$$

$$V \approx 4.1 \text{ m}^3$$

The volume of concrete needed is 4.1 m^3 .

- b) Use the conversion rate provided to calculate the weight.

$$W = 2400 \times 4.1$$

$$W = 9840 \text{ kg}$$

The patio will weigh 9840 kg .

5. Convert the capacity of the water to volume, 1 L equals
- 1000 cm^3
- .

$$V = 15 \times 1000$$

$$V = 15\,000 \text{ cm}^3$$

Use the formula for volume to solve for the depth.

$$V = lwh$$

$$15\,000 = 30 \times 20 \times h$$

$$15\,000 = 600h$$

$$25 \text{ cm} = h$$

The depth of water in the tank is 25 cm.

6. Convert the capacity of the tank to volume.

$$V = 20\,000 \times 1000$$

$$V = 20\,000\,000 \text{ cm}^3$$

Convert the value to m^3 . There are 100 centimetres in a metre. So 1 m^3 is equal to $(100 \text{ cm})^3$, which equals $1\,000\,000 \text{ cm}^3$.

Convert to metres by dividing the value by 1 000 000.

$$20\,000\,000 \div 1\,000\,000 = 20 \text{ m}^3$$

Use the formula for volume to solve for the height of the tank.

$$V = \pi r^2 h$$

$$20 = \pi \times \left(\frac{2.4}{2}\right)^2 \times h$$

$$20 \approx 4.52h$$

$$4.4 \text{ m} \approx h$$

The height of the tank is approximately 4.4 m.

ALTERNATE SOLUTION

Instead of converting the volume from cm^3 to m^3 you could have instead converted the diameter from metres to centimetres. Use the formula for volume to solve for the height of the tank.

$$V = \pi r^2 h$$

$$20\,000\,000 = \pi \times \left(\frac{240}{2}\right)^2 \times h$$

$$20\,000\,000 \approx 45\,239h$$

$$442 \text{ cm} \approx h$$

The height of the tank is approximately 442 cm or 4.4 m.

3.4

Volume and Capacity of Spheres, Cones, & Pyramids

BUILD YOUR SKILLS, p. 183

1. a) Calculate the volume of the sphere.

$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \pi \times 8.5^3$$

$$V = 2572 \text{ cm}^3$$

The volume of the sphere is 2572 cm³.

- b) Calculate the volume of the sphere.

$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \pi \times \left(\frac{78}{2}\right)^3$$

$$V \approx 248\,475 \text{ cm}^3$$

The volume of the sphere is 248 475 cm³.

2. a) Calculate the volume of the outer sphere.

$$V_o = \frac{4}{3} \pi r^3$$

$$V_o = \frac{4}{3} \pi \times 76^3$$

$$V_o \approx 1\,838\,778 \text{ cm}^3$$

Calculate the volume of the inner sphere.

$$V_i = \frac{4}{3} \pi r^3$$

$$V_i = \frac{4}{3} \pi \times 46^3$$

$$V_i \approx 407\,720 \text{ cm}^3$$

Subtract the volume of the inner sphere from the volume of the outer sphere.

$$V = V_o - V_i$$

$$V = 1\,838\,778 - 407\,720$$

$$V = 1\,431\,058 \text{ cm}^3$$

The volume of the space between the spheres is 1 431 058 cm³.

- b) Convert the volume to capacity, 1 L equals 1000 cm
- ³
- .

$$1\,431\,058 \div 1000 \approx 1431 \text{ L}$$

The capacity of the space between the two spheres is approximately 1431 L.

3. a) Use the formula for calculating the volume of a pyramid.

$$V = \frac{1}{3} lwh$$

$$V = \frac{1}{3} \times 15 \times 17 \times 19$$

$$V = 1615 \text{ in}^3$$

The volume of the pyramid is 1615 in³.

- b) Use the formula for calculating the volume of a pyramid.

$$V = \frac{1}{3} lwh$$

$$V = \frac{1}{3} \times 5 \times 4 \times 6$$

$$V = 40 \text{ ft}^3$$

The volume of the pyramid is 40 ft³.

- c) Calculate the volume of the top pyramid.

$$V_1 = \frac{1}{3} lwh$$

$$V_1 = \frac{1}{3} \times 9.8 \times 4.3 \times 6.2$$

$$V_1 \approx 87.1 \text{ cm}^3$$

Calculate the volume of the bottom pyramid.

$$V_2 = \frac{1}{3}lwh$$

$$V_2 = \frac{1}{3} \times 9.8 \times 4.3 \times 4.8$$

$$V_2 \approx 67.4 \text{ cm}^3$$

Calculate the total volume.

$$V = V_1 + V_2$$

$$V = 87.1 + 67.4$$

$$V = 154.5 \text{ cm}^3$$

The volume of the two pyramids is
154.5 cm³.

ALTERNATE SOLUTION

Students may have noticed that since the two pyramids share a base you could also have calculated the volume as if the two pyramids were just one big pyramid.

$$V = \frac{1}{3}lw(h_1 + h_2)$$

$$V = \frac{1}{3} \times 9.8 \times 4.3 \times (6.2 + 4.8)$$

$$V = \frac{1}{3} \times 9.8 \times 4.3 \times 11$$

$$V \approx 154.5 \text{ cm}^3$$

The volume of the two pyramids is 154.5 cm³.

4. Convert the units from feet to metres.

$$20 \text{ ft} = (20 \times 0.305) \text{ m}$$

$$20 \text{ ft} = 6.10 \text{ m}$$

$$9 \text{ ft} = (9 \times 0.305) \text{ m}$$

$$9 \text{ ft} \approx 2.75 \text{ m}$$

$$22 \text{ ft} = (22 \times 0.305) \text{ m}$$

$$22 \text{ ft} = 6.71 \text{ m}$$

$$15 \text{ ft} = (15 \times 0.305) \text{ m}$$

$$15 \text{ ft} \approx 4.58 \text{ m}$$

Calculate the volume of the pyramid.

$$V_1 = \frac{1}{3}lwh$$

$$V_1 = \frac{1}{3} \times 6.71 \times 6.10 \times (4.58 - 2.75)$$

$$V_1 = \frac{1}{3} \times 6.71 \times 6.10 \times 1.83$$

$$V_1 = 25.0 \text{ m}^3$$

Calculate the volume of the rectangular prism.

$$V_2 = lwh$$

$$V_2 = 6.71 \times 6.10 \times 2.75$$

$$V_2 = 112.6 \text{ m}^3$$

Calculate the total volume.

$$V = V_1 + V_2$$

$$V = 25.0 + 112.6$$

$$V = 137.6 \text{ m}^3$$

The volume of the figure is 137.6 m³.

Convert from volume to capacity.

$$137.6 \times 1000 = 137\,600 \text{ L}$$

The capacity of the figure is 137 600 L.

ALTERNATE SOLUTION

If you know the conversion rates from imperial volume to capacity you could have also calculated the solution as follows using imperial measures.

Calculate the volume of the pyramid.

$$V_1 = \frac{1}{3}lwh$$

$$V_1 = \frac{1}{3} \times 22 \times 20 \times (15 - 9)$$

$$V_1 = \frac{1}{3} \times 22 \times 20 \times 6$$

$$V_1 = 880 \text{ ft}^3$$

Calculate the volume of the rectangular prism.

$$V_2 = lwh$$

$$V_2 = 22 \times 20 \times 9$$

$$V_2 = 3960 \text{ ft}^3$$

Calculate the total volume.

$$V = V_1 + V_2$$

$$V = 880 + 3960$$

$$V = 4840 \text{ ft}^3$$

The volume of the figure is 4840 ft^3 .

Convert from volume to capacity, 1 ft^3 equals
7.48 US gallons.

$$4840 \times 7.48 \approx 36\,203 \text{ US gal}$$

The capacity of the figure is 36 203 US gallons.

5. Calculate the volume of the pyramid.

$$V_p = \frac{1}{3}lwh$$

$$V_p = \frac{1}{3} \times 28 \times 16 \times 12$$

$$V_p = 1792 \text{ in}^3$$

Calculate the volume of the rectangular prism.

$$V_r = lwh$$

$$V_r = 28 \times 16 \times 12$$

$$V_r = 5376 \text{ in}^3$$

Calculate the difference in the two volumes.

$$x = V_r - V_p$$

$$x = 5376 - 1792$$

$$x = 3584 \text{ in}^3$$

The difference in the volumes is 3584 in^3 .

6. a) Calculate the height of the pyramid using the Pythagorean theorem.

$$h^2 = 25^2 - \left(\frac{35}{2}\right)^2$$

$$h = \sqrt{25^2 - 17.5^2}$$

$$h = \sqrt{318.75}$$

$$h \approx 17.9 \text{ mm}$$

Calculate the volume of the pyramid.

$$V = \frac{1}{3}lwh$$

$$V = \frac{1}{3} \times 35 \times 35 \times 17.9$$

$$V \approx 7309 \text{ mm}^3$$

The volume of the pyramid is 7309 mm^3 .

- b) Calculate the height of the pyramid using the Pythagorean theorem.

$$h^2 = 19^2 - \left(\frac{23}{2}\right)^2$$

$$h = \sqrt{19^2 - 11.5^2}$$

$$h = \sqrt{228.75}$$

$$h \approx 15.1 \text{ in}$$

Calculate the volume of the pyramid.

$$V = \frac{1}{3}lwh$$

$$V = \frac{1}{3} \times 23 \times 23 \times 15.1$$

$$V \approx 2663 \text{ in}^3$$

The volume of the pyramid is 2663 in^3 .

- c) Calculate the height of the pyramid using the Pythagorean theorem.

$$h^2 = (2 \times 12)^2 - \left(\frac{9}{2}\right)^2$$

$$h = \sqrt{24^2 - 4.5^2}$$

$$h = \sqrt{555.75}$$

$$h \approx 23.6 \text{ in}$$

Calculate the volume of the pyramid.

$$V = \frac{1}{3}lwh$$

$$V = \frac{1}{3} \times 9 \times 9 \times 23.6$$

$$V \approx 637 \text{ in}^3$$

The volume of the pyramid is 637 in^3 .

- d) Calculate the height of the pyramid using the Pythagorean theorem.

$$h^2 = 16^2 - \left(\frac{14}{2}\right)^2$$

$$h = \sqrt{16^2 - 7^2}$$

$$h = \sqrt{207}$$

$$h \approx 14.4 \text{ mm}$$

Calculate the volume of the pyramid.

$$V = \frac{1}{3}lwh$$

$$V = \frac{1}{3} \times 14 \times 12 \times 14.4$$

$$V \approx 806 \text{ mm}^3$$

The volume of the pyramid is 806 mm^3 .

7. a) Calculate the volume of the cone.

$$V = \frac{1}{3} \times A_{\text{base}} \times h$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \times 5^2 \times 14.5$$

$$V \approx 379.6 \text{ in}^3$$

The volume of the cone is 379.6 in^3 .

- b) Calculate the volume of the first cone.

$$V_1 = \frac{1}{3} \pi r^2 h$$

$$V_1 = \frac{1}{3} \pi \times 6^2 \times 18$$

$$V_1 \approx 678.6 \text{ cm}^3$$

Calculate the volume of the second cone.

$$V_2 = \frac{1}{3} \pi r^2 h$$

$$V_2 = \frac{1}{3} \pi \times 6^2 \times 24$$

$$V_2 \approx 904.8 \text{ cm}^3$$

Calculate the total volume.

$$V = V_1 + V_2$$

$$V = 678.6 + 904.8$$

$$V \approx 1583 \text{ cm}^3$$

The total volume of the figure is approximately 1583 cm^3 .

- c) Calculate the volume of the cone.

$$V_1 = \frac{1}{3} \pi r^2 h$$

$$V_1 = \frac{1}{3} \pi \times 3^2 \times 6$$

$$V_1 \approx 56.5 \text{ m}^3$$

Calculate the volume of the cylinder.

$$V_2 = A_{\text{base}} \times h$$

$$V_2 = \pi r^2 h$$

$$V_2 = \pi \times 3^2 \times 5$$

$$V_2 \approx 141.4 \text{ m}^3$$

Calculate the total volume.

$$V = V_1 + V_2$$

$$V = 56.5 + 141.4$$

$$V \approx 198 \text{ m}^3$$

The total area of the figure is 198 m^3 .

8. Use the formula for the volume of a cone to solve for the height.

$$V = \frac{1}{3} \pi r^2 h$$

$$4071.5 = \frac{1}{3} \pi \times 12^2 \times h$$

$$4071.5 \approx 150.8h$$

$$27 \text{ mm} \approx h$$

The height of the cone is 27 mm .

9. Use the Pythagorean theorem to calculate the height of the cone.

$$h^2 = 15^2 - 8^2$$

$$h = \sqrt{15^2 - 8^2}$$

$$h = \sqrt{161}$$

$$h \approx 12.7 \text{ cm}$$

Calculate the volume of the cone.

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \times 8^2 \times 12.7$$

$$V \approx 851 \text{ cm}^3$$

The volume of the cone is 851 cm^3 .

PRACTICE YOUR SKILLS, p. 192

1. a) Calculate the volume of the sphere.

$$V_{\text{sphere}} = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \pi \times 6.3^3$$

$$V \approx 1047 \text{ mm}^3$$

The volume of the sphere is 1047 mm^3 .

- b) Calculate the volume of the cone.

$$V_{\text{cone}} = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \times 6.3^2 \times 12.4$$

$$V \approx 515 \text{ mm}^3$$

The volume of the cone is 515 mm^3 .

- c) Calculate the volume of the pyramid.

$$V_{\text{pyramid}} = \frac{1}{3} lwh$$

$$V = \frac{1}{3} \times 3.6 \times 5.2 \times 4.9$$

$$V \approx 30.6 \text{ mm}^3$$

The volume of the pyramid is 30.6 mm^3 .

- d) Calculate the volume of the hemisphere.

$$V_1 = \frac{1}{2} \times \frac{4}{3} \pi r^3$$

$$V_1 = \frac{1}{2} \times \frac{4}{3} \pi \times 4.3^3$$

$$V_1 \approx 166.5 \text{ mm}^3$$

Calculate the volume of the cone.

$$V_2 = \frac{1}{3} \pi r^2 h$$

$$V_2 = \frac{1}{3} \pi \times 4.3^2 \times 5.4$$

$$V_2 \approx 104.6 \text{ mm}^3$$

Calculate the total volume.

$$V = V_1 + V_2$$

$$V = 166.5 + 104.6$$

$$V \approx 271 \text{ mm}^3$$

The volume of the figure is 271 mm^3 .

2. Calculate the volume of the water tower.

$$V = \frac{4}{3} \pi r^3$$

$$V = \frac{4}{3} \pi \times \left(\frac{31.2}{2}\right)^3$$

$$V \approx 15\,902 \text{ ft}^3$$

Convert the volume into capacity.

$$15\,902 \times 7.48 \approx 118\,947 \text{ US gal.}$$

The capacity of the spherical water tower is $118\,947 \text{ US gal.}$

3. a) Calculate the volume of the pyramid.

$$V_p = \frac{1}{3} lwh$$

$$V_p = \frac{1}{3} \times 14 \times 14 \times 45$$

$$V_p = 2940 \text{ cm}^3$$

Calculate the volume of the cone.

$$V_c = \frac{1}{3} \times \pi r^2 h$$

$$V_c = \frac{1}{3} \times \pi \times \left(\frac{14}{2}\right)^2 \times 45$$

$$V_c \approx 2309 \text{ cm}^3$$

Calculate the difference in volumes.

$$x = 2940 - 2309$$

$$x = 631 \text{ cm}^3$$

The difference in volumes is 631 cm^3 .

- b) The difference in capacity is the difference in volume converted to capacity. One centimetre cubed is equal to one millilitre.

The difference in capacity is 631 mL.

4. Calculate the volume of the pile of grain.

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \times \left(\frac{4.5}{2}\right)^2 \times 1.2$$

$$V \approx 6.4 \text{ m}^3$$

There are 6.4 m³ of grain in the pile.

5. Calculate the volume of the first sphere.

$$V_1 = \frac{4}{3} \pi r^3$$

$$V_1 = \frac{4}{3} \pi \times 3.4^3$$

$$V_1 \approx 164.6 \text{ cm}^3$$

Calculate the volume of the second sphere.

$$V_2 = \frac{4}{3} \pi r^3$$

$$V_2 = \frac{4}{3} \pi \times 2.8^3$$

$$V_2 \approx 92.0 \text{ cm}^3$$

Calculate the volume of the third sphere.

$$V_3 = \frac{4}{3} \pi r^3$$

$$V_3 = \frac{4}{3} \pi \times 4.6^3$$

$$V_3 \approx 407.7 \text{ cm}^3$$

Calculate the total volume.

$$V = V_1 + V_2 + V_3$$

$$V = 164.6 + 92.0 + 407.7$$

$$V \approx 664 \text{ cm}^3$$

The volume of water displaced is 664 cm^3 .

CHAPTER TEST, p. 194

1. a) Calculate the slant height of the triangle using the Pythagorean theorem.

$$h^2 = 8^2 - \left(\frac{6}{2}\right)^2$$

$$h = \sqrt{8^2 - 3^2}$$

$$h = \sqrt{55}$$

$$h \approx 7.4 \text{ cm}$$

Calculate the area of one of the triangles.

$$A_1 = \frac{1}{2} bh$$

$$A_1 = \frac{1}{2} \times 6 \times 7.4$$

$$A_1 = 22.2 \text{ cm}^2$$

Calculate the area of the base.

$$A_2 = lw$$

$$A_2 = 6 \times 6$$

$$A_2 = 36 \text{ cm}^2$$

Calculate the total surface area.

$$SA = 4A_1 + A_2$$

$$SA = 4 \times 22.2 + 36$$

$$SA = 124.8 \text{ cm}^2$$

The total surface area is 124.8 cm^2 .

- b) Calculate the surface area of each face.

$$A_1 = 15 \times 8$$

$$A_1 = 120 \text{ cm}^2$$

$$A_2 = 8 \times 8$$

$$A_2 = 64 \text{ cm}^2$$

Calculate the total surface area.

$$SA = 4A_1 + 2A_2$$

$$SA = 4 \times 120 + 2 \times 64$$

$$SA = 608 \text{ cm}^2$$

The total surface area is 608 cm^2 .

- c) Calculate the surface area of the triangular face.

$$A_1 = \frac{1}{2}bh$$

$$A_1 = \frac{1}{2} \times 5 \times 8$$

$$A_1 = 20 \text{ cm}^2$$

Calculate the slant height of the triangle using the Pythagorean theorem.

$$s^2 = 8^2 + 5^2$$

$$s = \sqrt{8^2 + 5^2}$$

$$s = \sqrt{89}$$

$$s \approx 9.4 \text{ cm}$$

Calculate the area of each rectangular surface.

$$A_2 = 5 \times 18$$

$$A_2 = 90 \text{ cm}^2$$

$$A_3 = 8 \times 18$$

$$A_3 = 144 \text{ cm}^2$$

$$A_4 = 9.4 \times 18$$

$$A_4 \approx 169 \text{ cm}^2$$

Calculate the total surface area.

$$SA = 2A_1 + A_2 + A_3 + A_4$$

$$SA = 2 \times 20 + 90 + 144 + 169$$

$$SA = 443 \text{ cm}^2$$

The surface area of triangular prism is 443 cm^2 .

2. Calculate the area of each face of the box.

$$A_1 = 3 \times 2$$

$$A_1 = 6 \text{ m}^2$$

$$A_2 = 2 \times 7.5$$

$$A_2 = 15 \text{ m}^2$$

$$A_3 = 3 \times 7.5$$

$$A_3 = 22.5 \text{ m}^2$$

Calculate the total surface area.

$$SA = 2A_1 + 2A_2 + 2A_3$$

$$SA = 2 \times 6 + 2 \times 15 + 2 \times 22.5$$

$$SA = 12 + 30 + 45$$

$$SA = 87 \text{ m}^2$$

The surface area of the box is 87 m^2 .

Calculate the volume of the box.

$$V = lwh$$

$$V = 3 \times 2 \times 7.5$$

$$V = 45 \text{ m}^3$$

The volume of the box is 45 m^3 .

3. a) Calculate the area of circular base.

$$SA_1 = \pi r^2$$

$$SA_1 = \pi \times 4.5^2$$

$$SA_1 \approx 63.6 \text{ cm}^2$$

Calculate the area of the lateral side of the can.

$$SA_2 = 2\pi rh$$

$$SA_2 = 2\pi \times 4.5 \times 5$$

$$SA_2 = 141.4 \text{ cm}^2$$

$$SA = 2SA_1 + SA_2$$

$$SA = 2 \times 63.6 + 141.4$$

$$SA = 268.6 \text{ cm}^2$$

The surface area of the can is 268.6 cm^2 .

- b) Calculate the volume of the can.

$$V = A_{\text{base}} \times h$$

$$V = \pi r^2 h$$

$$V = \pi \times 4.5^2 \times 5$$

$$V \approx 318.1 \text{ cm}^3$$

The volume of the can is 318.1 cm^3 .

- c) Use the volume of the can to calculate the capacity of the can. Each centimetre cubed is equal to 1 mL so the capacity of the can is 318.1 mL.

4. a) Calculate the surface area of the ball.

$$SA = 4\pi r^2$$

$$SA = 4\pi \times 56^2$$

$$SA = 39\,408 \text{ mm}^2$$

The surface area of the ball is $39\,408 \text{ mm}^2$.

- b) Calculate the surface area of the ball.

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi \times 56^3$$

$$V \approx 735\,619 \text{ mm}^3$$

The volume of the sphere is $735\,619 \text{ mm}^3$.

5. Calculate the slant height the pyramid.

$$s^2 = 9^2 + \left(\frac{11}{2}\right)^2$$

$$s = \sqrt{9^2 + 5.5^2}$$

$$s = \sqrt{111.25}$$

$$s \approx 10.5 \text{ cm}$$

Calculate the area of the triangular faces of the pyramid.

$$A_1 = \frac{1}{2}bh$$

$$A_1 = \frac{1}{2} \times 11 \times 10.5$$

$$A_1 \approx 57.8 \text{ cm}^2$$

Calculate the area of the lateral faces of the top prism.

$$A_2 = 11 \times 16$$

$$A_2 = 176 \text{ cm}^2$$

Calculate the area of the large front face on the bottom.

$$A_3 = 25 \times 14$$

$$A_3 = 350 \text{ cm}^2$$

Calculate the area of the side face on the bottom.

$$A_4 = 11 \times 14$$

$$A_4 = 154 \text{ cm}^2$$

Calculate the area of the bottom of the figure.

$$A_5 = 25 \times 11$$

$$A_5 = 275 \text{ cm}^2$$

Calculate the area of the space on top of the larger rectangle that is filled by the smaller figure.

$$A_6 = 11 \times 11$$

$$A_6 = 121 \text{ cm}^2$$

Calculate the total surface area.

$$SA = 4A_1 + 4A_2 + 2A_3 + 2A_4 + 2A_5 - 2A_6$$

$$SA = 4 \times 57.8 + 4 \times 176 + 2 \times 350 + 2 \times 154 + 2 \times 275 - 121$$

$$SA = 231.2 + 704 + 700 + 308 + 550 - 121$$

$$SA = 2372.2 \text{ cm}^2$$

The surface area of the figure is 2372.2 cm^2 .

Calculate the volume of the pyramid.

$$V_1 = \frac{1}{3}lwh$$

$$V_1 = \frac{1}{3} \times 11 \times 11 \times 9$$

$$V_1 = 363 \text{ cm}^3$$

Calculate the volume of the top rectangular prism.

$$V_2 = lwh$$

$$V_2 = 11 \times 11 \times 16$$

$$V_2 = 1936 \text{ cm}^3$$

Calculate the volume of the bottom rectangular prism.

$$V_3 = lwh$$

$$V_3 = 25 \times 11 \times 14$$

$$V_3 = 3850 \text{ cm}^3$$

Calculate the total volume.

$$V = V_1 + V_2 + V_3$$

$$V = 363 + 1936 + 3850$$

$$V = 6149 \text{ cm}^3$$

The volume of the figure is 6149 cm^3 .

6. Calculate the volume of the pile.

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \times \left(\frac{6.8}{2}\right)^2 \times 2.8$$

$$V \approx 33.9 \text{ m}^3$$

The volume of the pile is 33.9 m^3 .

7. Calculate the area of one face of the pyramid.

$$A_1 = \frac{1}{2}bh$$

$$A_1 = \frac{1}{2} \times 18 \times 12$$

$$A_1 = 108 \text{ m}^2$$

Calculate the total surface area.

$$SA = 4A_1$$

$$SA = 4 \times 108$$

$$SA = 432 \text{ m}^2$$

Calculate the cost of the roofing material.

$$432 \times 5.75 = \$2484$$

The cost of the roofing material is \$2484.00.

Chapter 4

Trigonometry of Right Triangles

4.1

Solving for Angles, Lengths, and Distances

BUILD YOUR SKILLS, p. 200

1. a) $\tan \theta = \frac{7.9}{12.3}$

$$\tan \theta = 0.6423$$

$$\theta = \tan^{-1}(0.6423)$$

$$\theta = 32.7^\circ$$

The trigonometric ratio equals 0.6423 and θ is 32.7° .

b) $\sin \theta = \frac{1.8}{2.3}$

$$\sin \theta = 0.7826$$

$$\theta = \sin^{-1}(0.7826)$$

$$\theta = 51.5^\circ$$

The trigonometric ratio equals 0.7826 and θ is 51.5° .

c) $\cos \theta = \frac{5.5}{7.75}$

$$\cos \theta = 0.7097$$

$$\theta = \cos^{-1}(0.7097)$$

$$\theta = 44.8^\circ$$

The trigonometric ratio equals 0.7097 and θ is 44.8° .

2. Find the value of h using the Pythagorean theorem.

$$c^2 = a^2 + b^2$$

$$19.8^2 = 10.6^2 + h^2$$

$$19.8^2 - 10.6^2 = h^2$$

$$h = \sqrt{19.8^2 - 10.6^2}$$

$$h = \sqrt{392.04 - 112.36}$$

$$h = \sqrt{279.68}$$

$$h = 16.7 \text{ cm}$$

The value of h is 16.7 cm.

3. Use the tan ratio to solve for the value of the shared side.

$$\tan 35^\circ = \frac{y}{19}$$

$$y = \tan 35^\circ \times 19$$

$$y \approx 13.3 \text{ cm}$$

Use the Pythagorean theorem to solve for x .

$$x^2 = 33^2 + 13.3^2$$

$$x = \sqrt{33^2 + 13.3^2}$$

$$x = \sqrt{1089 + 176.89}$$

$$x = \sqrt{1265.89}$$

$$x = 35.6 \text{ cm}$$

The value of x is 35.6 cm.

4. Use the tan ratio to solve for the other angle in the triangle.

$$\tan \alpha = \frac{8}{11}$$

$$\alpha = \tan^{-1}\left(\frac{8}{11}\right)$$

$$\alpha \approx 36^\circ$$

Use the angle calculated to solve for θ .

$$\theta = 180^\circ - 90^\circ - 36^\circ$$

$$\theta = 54^\circ$$

The value of θ is 54° .

5. Use the cos ratio to solve for x .

$$\cos 28^\circ = \frac{6.3}{x}$$

$$x = \frac{6.3}{\cos 28^\circ}$$

$$x = 7.1 \text{ m}$$

Use the value of x and the sin ratio to solve for y .

$$\sin 28^\circ = \frac{y}{0.8 + x}$$

$$\sin 28^\circ = \frac{y}{0.8 + 7.1}$$

$$\sin 28^\circ = \frac{y}{7.9}$$

$$y = 7.9 \times \sin 28^\circ$$

$$y = 3.7 \text{ m}$$

The value of x is 7.1 m and the value of y is 3.7 m.

6. Use the tan ratio to solve for the angle of offset.

$$\tan \theta = \frac{12.6}{14.9}$$

$$\theta = \tan^{-1}\left(\frac{12.6}{14.9}\right)$$

$$\theta = 40.2^\circ$$

Use the angle of offset and the sin ratio to find the length of the travel pipe.

$$\sin 40.6^\circ = \frac{12.6}{t}$$

$$t = \frac{12.6}{\sin 40.6^\circ}$$

$$t = 19.4 \text{ cm}$$

The angle of offset is 40.2° and the length of the travel pipe is 19.4 cm.

ALTERNATE SOLUTION

Use the Pythagorean theorem to find the length of the travel pipe.

$$t^2 = 12.6^2 + 14.9^2$$

$$t = \sqrt{12.6^2 + 14.9^2}$$

$$t = \sqrt{158.76 + 222.01}$$

$$t = \sqrt{380.77}$$

$$t = 19.5 \text{ cm}$$

7. Use the tan ratio to solve for the depth of the ravine.

$$\tan 52^\circ = \frac{d}{120}$$

$$d = 120 \times \tan 52^\circ$$

$$d = 153.6 \text{ m}$$

The depth of the ravine is 153.6 m.

8. Use the tan ratio to solve for the width of the river.

$$\tan 28^\circ = \frac{AB}{500}$$

$$AB = 500 \times \tan 28^\circ$$

$$AB = 265.9 \text{ m}$$

The width of the river is 265.9 m.

9. Use the sin ratio to solve for the left side.

$$\sin 68.1^\circ = \frac{1.75}{x}$$

$$x = \frac{1.75}{\sin 68.1^\circ}$$

$$x = 1.89 \text{ m}$$

Use the sin ratio to solve for the right side.

$$\sin 49.4^\circ = \frac{1.75}{y}$$

$$y = \frac{1.75}{\sin 49.4^\circ}$$

$$y = 2.30 \text{ m}$$

Use the values calculated to find the perimeter.

$$\ell = 5 + 2.75 + 1.89 + 2.30$$

$$\ell = 11.94 \text{ m}$$

Pauline will need to build 11.94 m of fence.

10. Find the angle of the right side of the triangle.

$$\theta = 131.6^\circ - 90^\circ$$

$$\theta = 41.6^\circ$$

Use the angle calculated and the sin ratio to find the height of the triangle.

$$\sin 41.6^\circ = \frac{h}{3}$$

$$h = 2.00 \text{ m}$$

Calculate the area of the triangle.

$$a_t = \frac{1}{2}(2)(8)$$

$$a_t = 8 \text{ m}^2$$

Calculate the area of the square.

$$a_s = 8 \times 8$$

$$a_s = 64 \text{ m}^2$$

Calculate the total area.

$$a = 8 + 64$$

$$a = 72 \text{ m}^2$$

Soo-Jin will need to buy 72 m².

PRACTISE YOUR NEW SKILLS, p. 210

1. a) Use the tan ratio to solve for x .

$$\tan 62^\circ = \frac{x}{9.8}$$

$$x = 18.4 \text{ m}$$

b) Use the cos ratio to solve for x .

$$\cos 78^\circ = \frac{x}{25.9}$$

$$x = 5.4 \text{ m}$$

c) Use the Pythagorean theorem to solve for x .

$$x^2 = 8.5^2 + 19.5^2$$

$$x = \sqrt{8.5^2 + 19.5^2}$$

$$x = \sqrt{72.25 + 380.25}$$

$$x = \sqrt{452.5}$$

$$x = 21.3 \text{ cm}$$

d) Use the tan ratio to solve for x .

$$\tan x = \frac{1.5}{19.5}$$

$$x = \tan^{-1}\left(\frac{1.5}{19.5}\right)$$

$$x = 4.4^\circ$$

e) Use the sin ratio to solve for x .

$$\sin x = \frac{96.8}{98.1}$$

$$x = \sin^{-1}\left(\frac{96.8}{98.1}\right)$$

$$x = 80.7^\circ$$

f) Use the sin ratio to solve for x .

$$\sin 48^\circ = \frac{14.3}{x}$$

$$x = \frac{14.3}{\sin 48^\circ}$$

$$x = 19.2 \text{ cm}$$

g) Use the cos ratio to solve for x .

$$\cos 66^\circ = \frac{3.5}{x}$$

$$x = \frac{3.5}{\cos 66^\circ}$$

$$x = 8.6 \text{ m}$$

h) Use the cos ratio to solve for x .

$$\cos x = \frac{3.1}{5.8}$$

$$x = \cos^{-1}\left(\frac{3.1}{5.8}\right)$$

$$x = 57.7^\circ$$

2. Use the cos ratio to find the length of the ladder.

$$\begin{aligned}\cos 65^\circ &= \frac{1.8}{\ell} \\ \ell &= \frac{1.8}{\cos 65^\circ} \\ \ell &= 4.3 \text{ m}\end{aligned}$$

The length of the ladder is 4.3 m.

3. Use the tan ratio to find the height of the tree.

$$\begin{aligned}\tan 38^\circ &= \frac{t}{12.4} \\ t &= 12.4 \times \tan 38^\circ \\ t &= 9.7 \text{ m}\end{aligned}$$

The height of the tree is 9.7 m.

4. Use the sin ratio to find the angle of depression.

$$\begin{aligned}\sin \theta &= \frac{200}{350} \\ \theta &= \sin^{-1}\left(\frac{200}{350}\right) \\ \theta &= 34.8^\circ\end{aligned}$$

Use the cos ratio to find the horizontal distance.

$$\begin{aligned}\cos 34.8^\circ &= \frac{h}{350} \\ h &= 350 \times \cos 34.8^\circ \\ h &= 287.4 \text{ m}\end{aligned}$$

The angle of depression is 34.8° and the horizontal distance travelled is 287.4 m.

5. We know that the roof of the playhouse is an isosceles triangle because each length of the roof is equivalent. When you divide an isosceles triangle into two even triangle you know that each will have the same dimensions.

The base of the whole triangle is 4 ft and each legs of the smaller triangles will be equal to half of the base of the whole triangle.

$$4 \div 2 = 2$$

Solve for the angle of the base of the smaller triangle.

$$127^\circ - 90^\circ = 37^\circ$$

Use the values calculated and the cos ratio to solve for a .

$$\begin{aligned}\cos 37^\circ &= \frac{2}{a} \\ a &= \frac{2}{\cos 37^\circ} \\ a &= 2.5 \text{ ft}\end{aligned}$$

Use the tan ratio to solve for b .

$$\begin{aligned}\tan 37^\circ &= \frac{b}{2} \\ b &= 2 \times \tan 37^\circ \\ b &= 1.5 \text{ ft}\end{aligned}$$

Use the knowledge of the properties of triangles to solve for c .

$$\begin{aligned}c &= 180^\circ - 90 - 37^\circ \\ c &= 53^\circ\end{aligned}$$

Use the value we calculated for b to solve for d .

$$\begin{aligned}d &= 5 - 1.5 \\ d &= 3.5 \text{ ft}\end{aligned}$$

4.2

Solving Complex Problems in the Real World

BUILD YOUR SKILLS, p. 215

1. Use the tan ratio for the base of the larger triangle.

$$\begin{aligned}\tan 37^\circ &= \frac{4.9}{y} \\ y &= \frac{4.9}{\tan 37^\circ} \\ y &= 6.5 \text{ m}\end{aligned}$$

Use the value you calculated and the cos ratio to solve for x .

$$\begin{aligned}\cos 37^\circ &= \frac{x}{6.5} \\ x &= 6.5 \times \cos 37^\circ \\ x &= 5.2 \text{ m}\end{aligned}$$

The value of x is 5.2 m.

2. a) Use the Pythagorean theorem to solve for x .

$$\begin{aligned}x^2 &= 9.4^2 + 8.6^2 \\ x &= \sqrt{9.4^2 + 8.6^2} \\ x &= \sqrt{88.36 + 73.96} \\ x &= \sqrt{162.32} \\ x &= 12.7 \text{ cm}\end{aligned}$$

Use the value calculated for x and the cos ratio to solve for y .

$$\begin{aligned}\cos 48^\circ &= \frac{12.7}{y} \\ y &= \frac{12.7}{\cos 48^\circ} \\ y &= 19.0 \text{ cm}\end{aligned}$$

- b) Use the tan ratio to solve for both parts of x .

$$\begin{aligned}\tan 32^\circ &= \frac{x_1}{6.7} \\ x_1 &= 6.7 \times \tan 32^\circ \\ x_1 &= 4.19 \\ \tan 48^\circ &= \frac{x_2}{6.7} \\ x_2 &= 6.7 \times \tan 48^\circ \\ x_2 &= 7.44 \\ x &= x_1 + x_2 \\ x &= 4.19 + 7.44 \\ x &= 11.6 \text{ m}\end{aligned}$$

Use the cos ratio to solve for y .

$$\begin{aligned}\cos 48^\circ &= \frac{6.7}{y} \\ y &= \frac{6.7}{\cos 48^\circ} \\ y &= 10.0 \text{ m}\end{aligned}$$

3. Use the sin ratio to solve for h .

$$\begin{aligned}\sin 53^\circ &= \frac{h}{21.4} \\ h &= 21.4 \times \sin 53^\circ \\ h &= 17.1 \text{ cm}\end{aligned}$$

Solve for the hypotenuse of the lower triangle using the sin ratio.

$$\begin{aligned}\sin 53^\circ &= \frac{8.6}{H_1} \\ H_1 &= \frac{8.6}{\sin 53^\circ} \\ H_1 &= 10.8\end{aligned}$$

Use the value calculated to find the hypotenuse of the upper triangle.

$$H_2 = 21.4 - 10.8$$

$$H_2 = 10.6 \text{ cm}$$

Use the properties of triangles to calculate the value of the top angle in the top triangle.

$$\theta = 180^\circ - 90^\circ - 53^\circ$$

$$\theta = 37^\circ$$

Use the angle calculated, the length of the hypotenuse and the cos ratio to solve for x .

$$\cos 37^\circ = \frac{x}{10.6}$$

$$x = 10.6 \times \cos 37^\circ$$

$$x = 8.5 \text{ cm}$$

Using the sin ratio solve for y .

$$\sin 37^\circ = \frac{y}{10.6}$$

$$y = 10.6 \times \sin 37^\circ$$

$$y = 6.4 \text{ cm}$$

ALTERNATE SOLUTION

Alternately you could calculate the other angle in the upper triangle. Start by calculating the other angle in the lower triangle by using the properties of triangles.

$$\alpha = 180^\circ - 90^\circ - 53^\circ$$

$$\alpha = 37^\circ$$

Use the properties of straight lines to calculate the lower angle of the upper triangle.

$$\beta = 180^\circ - 90^\circ - 37^\circ$$

$$\beta = 53^\circ$$

Use the value of the lower angle and the sin ratio to calculate the value of x .

$$\sin 53^\circ = \frac{x}{10.6}$$

$$x = 10.6 \times \sin 53^\circ$$

$$x = 8.5 \text{ cm}$$

Use the value of the lower angle and the cos ratio to calculate the value of y .

$$\cos 53^\circ = \frac{y}{10.6}$$

$$y = 10.6 \times \cos 53^\circ$$

$$y = 6.4 \text{ cm}$$

4. Use the tan ratio to calculate the horizontal distance between buildings.

$$\tan 25^\circ = \frac{200}{x}$$

$$x = \frac{200}{\tan 25^\circ}$$

$$x = 428.9 \text{ m}$$

Use the tan ratio and the horizontal distance to calculate the difference in heights of B_1 and B_2 .

$$\tan 40^\circ = \frac{h}{428.9}$$

$$h = 428.9 \times \tan 40^\circ$$

$$h = 359.9 \text{ m}$$

Calculate the height of B_2 .

$$\text{Height of } B_2 = 359.9 + 200$$

$$\text{Height of } B_2 \approx 560 \text{ m}$$

The height of B_2 is approximately 560 m.

5. a) Find the height of the ladder at its shortest.

$$\sin 65^\circ = \frac{h_1}{18}$$

$$h_1 = 18 \times \sin 65^\circ$$

$$h_1 = 16.3 \text{ ft}$$

Find the height of the ladder at its longest.

$$\sin 65^\circ = \frac{h_2}{32}$$

$$h_2 = 32 \times \sin 65^\circ$$

$$h_2 = 29.0 \text{ ft}$$

Find the differences in the two heights.

$$x = 29.0 - 16.3$$

$$x = 12.7 \text{ ft}$$

The ladder reaches 12.7 ft further up the building when it is fully extended.

- b) Find the distance of the ladder at its shortest.

$$\cos 65^\circ = \frac{d_1}{18}$$

$$d_1 = 18 \times \cos 65^\circ$$

$$d_1 = 7.6 \text{ ft}$$

Find the distance of the ladder at its longest.

$$\cos 65^\circ = \frac{d_2}{32}$$

$$d_2 = 32 \times \cos 65^\circ$$

$$d_2 = 13.5 \text{ ft}$$

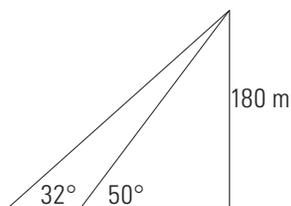
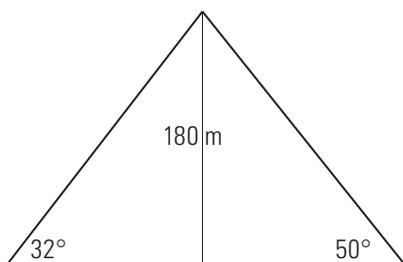
Find the differences in the two distances.

$$x = 13.5 - 7.6$$

$$x = 5.9 \text{ ft}$$

The base of the ladder must be 5.9 ft further from the building when it is fully extended.

6. There are two situations.



Solve for the distance Zola is from the cell phone tower.

$$\tan 32^\circ = \frac{180}{z}$$

$$z = \frac{180}{\tan 32^\circ}$$

$$z \approx 288 \text{ m}$$

Solve for the distance Naeem is from the cell phone tower.

$$\tan 50^\circ = \frac{180}{n}$$

$$n = \frac{180}{\tan 50^\circ}$$

$$n \approx 151 \text{ m}$$

Calculate their distance if they are on the same side of the tower.

$$d_1 = 288 - 151$$

$$d_1 = 137 \text{ m}$$

If they are on the same side of the tower they are 137 m apart.

Calculate their distance if they are on opposite side of the tower.

$$d_2 = 288 + 151$$

$$d_2 = 439 \text{ m}$$

If they are on the opposite side of the tower they are 439 m apart.

7. a) Solve for the first segment using the tan ratio.

$$\tan 25^\circ = \frac{14.9}{x_1}$$

$$x_1 = \frac{14.9}{\tan 25^\circ}$$

$$x_1 = 32.0 \text{ m}$$

The second segment is 8 m.

Solve for the third segment using the tan ratio.

$$\tan 47^\circ = \frac{26.8}{x_3}$$

$$x_3 = \frac{26.8}{\tan 47^\circ}$$

$$x_3 = 25.0 \text{ m}$$

Add each horizontal distance together to find the total.

$$d = 32.0 + 8 + 25.0$$

$$d = 65.0 \text{ m}$$

The total horizontal distance covered by this portion of the track is 65.0 m.

- b) The distance travelled by the car over the first segment of track is the hypotenuse. Use the sin ratio to calculate the hypotenuse.

$$\sin 25^\circ = \frac{14.9}{h_1}$$

$$h_1 = \frac{14.9}{\sin 25^\circ}$$

$$h_1 = 35.3 \text{ m}$$

The second segment of track covered is 8 m.

The distance travelled by the car over the third segment of track is the hypotenuse.

Use the sin ratio to calculate the hypotenuse.

$$\sin 47^\circ = \frac{26.8}{h_3}$$

$$h_3 = \frac{26.8}{\sin 47^\circ}$$

$$h_3 = 39.3 \text{ m}$$

Add each distance of track the car travels.

$$d = 35.3 + 8 + 39.3$$

$$d = 82.6 \text{ m}$$

The car travels 82.6 m along his portion of the track.

8. a) Use the Pythagorean theorem to calculate the straight-line distance.

$$d^2 = 185^2 + 100^2$$

$$d = \sqrt{185^2 + 100^2}$$

$$d = \sqrt{34225 + 10000}$$

$$d = \sqrt{44225}$$

$$d \approx 210.3 \text{ km}$$

- b) Use the tan ratio and the straight-line distance calculated to calculate the angle of elevation.

$$\tan \theta = \frac{7}{210.3}$$

$$\theta = \tan^{-1}\left(\frac{7}{210.3}\right)$$

$$\theta = 1.9^\circ$$

9. Use the sin ratio to calculate the horizontal distance from Sylvie to the tree, which is the same as the hypotenuse of the triangle between Mathieu, Sylvie and the tree.

$$\sin 73^\circ = \frac{89}{d}$$

$$d = \frac{89}{\sin 73^\circ}$$

$$d = 93.1 \text{ m}$$

Use the tan ratio and the distance calculated to calculate the height of the nest.

$$\tan 35^\circ = \frac{h}{93.1}$$

$$h = 93.1 \times \tan 35^\circ$$

$$h = 65.2 \text{ m}$$

The height of the nest is 65.2 m.

PRACTISE YOUR NEW SKILLS, p. 223

1. a) Use the cos ratio to solve the value of
- x
- .

$$\cos 32^\circ = \frac{x}{8.4}$$

$$x = 8.4 \times \cos 32^\circ$$

$$x = 7.1 \text{ cm}$$

Use the sin ratio to solve for y .

$$\sin 54^\circ = \frac{6.8}{y}$$

$$y = \frac{6.8}{\sin 54^\circ}$$

$$y = 8.4 \text{ cm}$$

- b) Use the property of right angles to solve for
- θ
- .

$$\theta = 90^\circ - 57^\circ$$

$$\theta = 33^\circ$$

Use the cos ratio to solve for x .

$$\cos 57^\circ = \frac{21.8}{x}$$

$$x = \frac{21.8}{\cos 57^\circ}$$

$$x = 40 \text{ cm}$$

Use the cos ratio to solve for y .

$$\cos 33^\circ = \frac{40}{y}$$

$$y = \frac{40}{\cos 33^\circ}$$

$$y = 47.7 \text{ cm}$$

- c) Use the tan ratio to solve for
- x
- .

$$\tan 15^\circ = \frac{21}{x}$$

$$x = \frac{21}{\tan 15^\circ}$$

$$x = 78.4 \text{ ft}$$

Use the sin ratio to solve for y .

$$\sin 15^\circ = \frac{21}{y}$$

$$y = \frac{21}{\sin 15^\circ}$$

$$y = 81.1 \text{ ft}$$

Use the tan ratio to solve for θ .

$$\tan \theta = \frac{70.6}{78.4}$$

$$\theta = \tan^{-1}\left(\frac{70.6}{78.4}\right)$$

$$\theta = 42^\circ$$

2. Calculate the base of the triangle of which
- x
- is the hypotenuse.

$$2 \times 2 = 4 \text{ ft}$$

Use the Pythagorean theorem to calculate the value of x .

$$x^2 = 4^2 + 7.5^2$$

$$x = \sqrt{4^2 + 7.5^2}$$

$$x = \sqrt{16 + 56.25}$$

$$x = \sqrt{72.25}$$

$$x = 8.5 \text{ ft}$$

Calculate the base of the triangle of which y is the hypotenuse.

$$2 \times 4 = 8 \text{ ft}$$

Use the Pythagorean theorem to calculate the value of y .

$$x^2 = 7.5^2 + 8^2$$

$$x = \sqrt{7.5^2 + 8^2}$$

$$x = \sqrt{56.25 + 64}$$

$$x = \sqrt{120.25}$$

$$x \approx 11.0 \text{ ft}$$

Use the tan ratio to solve for G .

$$\tan G = \frac{7.5}{8}$$

$$G = \tan^{-1}\left(\frac{7.5}{8}\right)$$

$$G \approx 43.2^\circ$$

3. a) Calculate the difference in heights of the buildings.

$$147 - 109 = 38 \text{ ft}$$

Use the difference in height and the tan ratio to calculate the distance between the buildings.

$$\tan 41^\circ = \frac{38}{x}$$

$$x = \frac{38}{\tan 41^\circ}$$

$$x = 43.7 \text{ m}$$

The buildings are 43.7 m apart.

- b) Use the tan ratio to calculate the angle of depression.

$$\tan \theta = \frac{109}{43.7}$$

$$\theta = \tan^{-1}\left(\frac{109}{43.7}\right)$$

$$\theta = 68.2^\circ$$

The angle of depression is 68.2° .

4. a) Use the tan ratio to calculate the height of the tree.

$$\tan 20^\circ = \frac{h}{100}$$

$$h = 100 \times \tan 20^\circ$$

$$h = 36.4 \text{ m}$$

The height of the tree is 36.4 m.

- b) Calculate the distance he is from the tree using the tan ratio.

$$\tan 36^\circ = \frac{36.4}{x}$$

$$x = \frac{36.4}{\tan 36^\circ}$$

$$x = 50.1 \text{ m}$$

Subtract this value from his original distance to calculate how far he paddled.

$$d = 100 - 50.1$$

$$d = 49.9 \text{ m}$$

He paddled 49.9 m closer to the tree.

5. Use the tan ratio to calculate the distance from Point 1 to the base of the cliff.

$$\tan 38^\circ = \frac{200}{x}$$

$$x = \frac{200}{\tan 38^\circ}$$

$$x \approx 256 \text{ m}$$

Use the distance calculated and the tan ratio to determine the height of the cliff.

$$\tan 32^\circ = \frac{h}{256}$$

$$h = 256 \times \tan 32^\circ$$

$$h \approx 160 \text{ m}$$

The height of the cliff is 160 m.

CHAPTER TEST, p. 226

1. a) Use the sin ratio to solve for
- x
- .

$$\sin 47^\circ = \frac{x}{3.8}$$

$$x = 3.8 \times \sin 47^\circ$$

$$x = 2.8 \text{ m}$$

- b) Use the sin ratio to solve for
- X
- .

$$\sin X = \frac{4.3}{7.5}$$

$$X = \sin^{-1}\left(\frac{4.3}{7.5}\right)$$

$$X = 35^\circ$$

- c) Use the tan ratio to solve for
- X
- .

$$\tan X = \frac{3.8}{1.1}$$

$$X = \tan^{-1}\left(\frac{3.8}{1.1}\right)$$

$$X = 73.9^\circ$$

- d) Use the cos ratio to solve for
- x
- .

$$\cos 54^\circ = \frac{23.4}{x}$$

$$x = \frac{23.4}{\cos 54^\circ}$$

$$x = 39.8 \text{ in}$$

2. a) Use the cos ratio to solve for
- x
- .

$$\cos 42^\circ = \frac{7.5}{x}$$

$$x = \frac{7.5}{\cos 42^\circ}$$

$$x = 10.1 \text{ cm}$$

Use the tan ratio to solve for Y .

$$\tan Y = \frac{10.1}{9.2}$$

$$Y = \tan^{-1}\left(\frac{10.1}{9.2}\right)$$

$$Y = 47.7^\circ$$

- b) Use the tan ratio to solve for
- X
- .

$$\tan X = \frac{4.2}{3.1}$$

$$X = \tan^{-1}\left(\frac{4.2}{3.1}\right)$$

$$X = 53.6^\circ$$

Use the tan ratio to solve for y .

$$\tan 53.6^\circ = \frac{y}{6.2}$$

$$y = 6.2 \times \tan 53.6^\circ$$

$$y = 8.4 \text{ cm}$$

Use the Pythagorean theorem to solve for z .

$$z^2 = 8.4^2 + 6.2^2$$

$$z = \sqrt{8.4^2 + 6.2^2}$$

$$z = \sqrt{70.56 + 38.44}$$

$$z = \sqrt{109}$$

$$z = 10.4 \text{ cm}$$

- c) Use the Pythagorean theorem to solve for
- x
- .

$$x^2 = 4.4^2 + 3.9^2$$

$$x = \sqrt{4.4^2 + 3.9^2}$$

$$x = \sqrt{19.36 + 15.21}$$

$$x = \sqrt{34.57}$$

$$x = 5.9 \text{ cm}$$

Use the tan ratio to solve for Y .

$$\tan Y = \frac{4.4}{3.9}$$

$$Y = \tan^{-1}\left(\frac{4.4}{3.9}\right)$$

$$Y = 48.4^\circ$$

Use the Pythagorean theorem to solve for z .

$$z^2 = 4.4^2 - 2.9^2$$

$$z = \sqrt{4.4^2 - 2.9^2}$$

$$z = \sqrt{19.36 - 8.41}$$

$$z = \sqrt{10.95}$$

$$z = 3.3 \text{ cm}$$

3. Divide the patio into a rectangle and a triangle.

Calculate the area of the rectangle.

$$a_r = 8 \times 19$$

$$a_r = 152 \text{ m}^2$$

To calculate the area of the triangle you will need to know the base and the height of the triangle. Calculate the base of the triangle.

$$b = 19 - 6.5$$

$$b = 12.5 \text{ m}$$

The angle on the bottom of the triangle can be determined by subtracting the value of a straight angle from the reflex angle.

$$\theta = 205^\circ - 180^\circ$$

$$\theta = 25^\circ$$

Use that angle and the sin ratio to calculate the height of the triangle.

$$\sin 25^\circ = \frac{h}{8.3}$$

$$h = 8.3 \times \sin 25^\circ$$

$$h = 3.5 \text{ m}$$

Use the base and height of the triangle to calculate its area.

$$a_t = \frac{1}{2}bh$$

$$a_t = \frac{1}{2}(12.5)(3.5)$$

$$a_t \approx 21.9 \text{ m}^2$$

Calculate the total area.

$$a = a_t + a_r$$

$$a = 152 + 21.9$$

$$a = 173.9 \text{ m}^2$$

The area of Paolo's patio is 173.9 m^2 .

4. To solve students will need to calculate the length of the top of the kite and the length of the bottom of the kite and add the results together.

Use the cos ratio to solve for the length of the top of the kite.

$$\cos\left(\frac{74^\circ}{2}\right) = \frac{\ell_1}{50}$$

$$\ell_1 = 50 \times \cos 37^\circ$$

$$\ell_1 = 39.9 \text{ cm}$$

Use the cos ratio to solve for the length of the bottom of the kite.

$$\cos\left(\frac{80^\circ}{2}\right) = \frac{\ell_2}{62}$$

$$\ell_2 = 62 \times \cos 40^\circ$$

$$\ell_2 = 47.5 \text{ cm}$$

Add the two lengths together for the total.

$$\ell = \ell_1 + \ell_2$$

$$\ell = 39.9 + 47.5$$

$$\ell = 87.4 \text{ cm}$$

The length of the kite is 87.4 cm .

5. Use the angle of depression to the top of the building to calculate the difference in height of the building.

$$\tan 71^\circ = \frac{x}{31}$$

$$x = 31 \times \tan 71^\circ$$

$$x = 90 \text{ m}$$

Subtract the difference in heights of the buildings from the height of the building Chung-ho is standing on.

$$h = 135 - 90$$

$$h = 45 \text{ m}$$

The height of the other building is 45 m .

6. a) Use the tan ratio to calculate the height of the nest.

$$\tan 42^\circ = \frac{h}{75}$$

$$h = 75 \times \tan 42^\circ$$

$$h = 67.5 \text{ m}$$

The nest is 67.5 m high.

- b) Find how far Bernhardt is from the tree.

$$d = 75 + 30$$

$$d = 105 \text{ m}$$

Use the tan ratio to find the angle of elevation.

$$\tan \theta = \frac{67.5}{105}$$

$$\theta = \tan^{-1} \left(\frac{67.5}{105} \right)$$

$$\theta = 32.7^\circ$$

The angle of elevation he sees the nest at is 32.7° .

7. a) Use the Pythagorean theorem to find the straight-line distance.

$$d^2 = 75^2 + 35^2$$

$$d = \sqrt{75^2 + 35^2}$$

$$d = \sqrt{5625 + 1225}$$

$$d = \sqrt{6850}$$

$$d = 82.8 \text{ km}$$

The straight-line distance to the airport is 82.8 km.

- b) Use the tan ratio to solve for the angle of elevation.

$$\theta = \tan^{-1} \left(\frac{6}{82.8} \right)$$

$$\theta = 4.1^\circ$$

The angle of elevation is 4.1° .

Chapter 5

Scale Representations

5.1

Scale Drawings and Models

BUILD YOUR SKILLS, p. 233

1. a) Solve for x .

$$\frac{3}{8} = \frac{x}{168}$$

$$168 \times \frac{3}{8} = \frac{x}{168} \times 168$$

$$168 \times \frac{3}{8} = x$$

$$63 = x$$

b) Solve for x .

$$\frac{x}{13} = \frac{7}{91}$$

$$13 \times \frac{x}{13} = \frac{7}{91} \times 13$$

$$x = \frac{7}{91} \times 13$$

$$x = 1$$

c) Solve for x .

$$\frac{x}{7} = \frac{30}{105}$$

$$7 \times \frac{x}{7} = \frac{30}{105} \times 7$$

$$x = \frac{30}{105} \times 7$$

$$x = 2$$

d) Solve for x .

$$\frac{408}{x} = \frac{4}{9}$$

$$x \times \frac{408}{x} = \frac{4}{9} \times x$$

$$408 = \frac{4}{9}x$$

$$\frac{9}{4} \times 408 = \frac{4}{9}x \times \frac{9}{4}$$

$$\frac{9}{4} \times 408 = x$$

$$918 = x$$

e) Solve for x .

$$\frac{90}{198} = \frac{5}{x}$$

$$x \times \frac{90}{198} = \frac{5}{x} \times x$$

$$\frac{90}{198}x = 5$$

$$\frac{198}{90} \times \frac{90}{198}x = 5 \times \frac{198}{90}$$

$$x = 5 \times \frac{198}{90}$$

$$x = 11$$

f) Solve for x .

$$\frac{34}{289} = \frac{2}{x}$$

$$x \times \frac{34}{289} = \frac{2}{x} \times x$$

$$\frac{34}{289} x = 2$$

$$\frac{289}{34} \times \frac{34}{289} x = 2 \times \frac{289}{34}$$

$$x = 2 \times \frac{289}{34}$$

$$x = 17$$

2. a) Solve for x .

$$\frac{5}{6} = \frac{12}{x}$$

$$x \times \frac{5}{6} = \frac{12}{x} \times x$$

$$\frac{5}{6} x = 12$$

$$\frac{6}{5} \times \frac{5}{6} x = 12 \times \frac{6}{5}$$

$$x = 12 \times \frac{6}{5}$$

$$x = 14.4$$

b) Solve for k .

$$\frac{7}{15} = \frac{9}{k}$$

$$k \times \frac{7}{15} = \frac{9}{k} \times k$$

$$\frac{7}{15} k = 9$$

$$\frac{15}{7} \times \frac{7}{15} k = 9 \times \frac{15}{7}$$

$$k = 9 \times \frac{15}{7}$$

$$k \approx 19.3$$

c) Solve for m .

$$\frac{1.2}{4.9} = \frac{m}{7.3}$$

$$7.3 \times \frac{1.2}{4.9} = \frac{m}{7.3} \times 7.3$$

$$7.3 \times \frac{1.2}{4.9} = m$$

$$1.8 = m$$

d) Solve for p .

$$\frac{p}{85} = \frac{76}{39}$$

$$85 \times \frac{p}{85} = \frac{76}{39} \times 85$$

$$p = \frac{76}{39} \times 85$$

$$p = 165.6$$

3. Set up a ratio for Tom and Mary's age given the information provided.

$$\frac{3}{4} = \frac{15}{M}$$

$$M \times \frac{3}{4} = \frac{15}{M} \times M$$

$$\frac{3}{4} M = 15$$

$$\frac{4}{3} \times \frac{3}{4} M = 15 \times \frac{4}{3}$$

$$M = 20 \text{ years}$$

Mary is 20 years old

4. Set up a ratio showing the relationship between the distances and the time travelled.

$$\frac{355}{d} = \frac{7}{8.5}$$

$$d \times \frac{355}{d} = \frac{7}{8.5} \times d$$

$$355 = \frac{7}{8.5} d$$

$$\frac{8.5}{7} \times 355 = \frac{7}{8.5} d \times \frac{8.5}{7}$$

$$\frac{8.5}{7} \times 355 = d$$

$$431 \text{ km} \approx d$$

Georgina will have travelled 431 km in 8.5 hours.

5. Convert both distances to the same units.

$$1850 \text{ km} = 185\,000\,000 \text{ cm}$$

Use this to determine the scale statement.

$$\text{scale statement} = \text{map:original}$$

$$\text{scale statement} = 3.7:185\,000\,000$$

$$\text{scale statement} = 1:50\,000\,00$$

6. a) The scale statement is 200:1.
b) Determine the scale factor.

$$\text{scale factor} = \frac{\text{model}}{\text{original}}$$

$$\text{scale factor} = \frac{200}{1}$$

$$\text{scale factor} = 200$$

Use the scale factor to calculate the width of the original.

$$\text{scale factor} = \frac{\text{model}}{\text{original}}$$

$$200 = \frac{2}{x}$$

$$x \times 200 = \frac{2}{x} \times x$$

$$200x = 2$$

$$x = 0.01 \text{ cm}$$

The actual width of the hair is 0.01 cm.

7. Convert the heights to be the same units.

$$1.93 \text{ m} = 193 \text{ cm}$$

Determine the scale statement.

$$\text{scale statement} = \text{card:original}$$

$$\text{scale statement} = 5.4:193$$

$$\text{scale statement} = 1:35.7$$

The scale used to print the card was 1:35.7.

8. Use the relationship about the height of the man and the heights in the image to set up a ratio to solve for the height of the flagpole.

$$\frac{2.3 \text{ cm}}{1.78 \text{ m}} = \frac{7.6 \text{ cm}}{f}$$

$$f \times \frac{2.3}{1.78} = \frac{7.6}{f} \times f$$

$$\frac{2.3}{1.78} f = 7.6$$

$$\frac{1.78}{2.3} \times \frac{2.3}{1.78} f = 7.6 \times \frac{1.78}{2.3}$$

$$f \approx 5.9 \text{ m}$$

The height of the flagpole is 5.9 m.

9. a) The picture is approx. 4.75 cm long.

Convert the size of the beluga into centimetres.

$$4.2 \text{ m} = 420 \text{ cm}$$

Determine the scale statement.

The scale statement is = picture:original

$$\text{scale statement} = 5:420$$

$$\text{scale statement} = 1:84$$

The scale statement for the picture is 1:84, which is 47:420

- b) Use the scale statement to set up a ratio to determine the length of the actual alligator.

$$\frac{1}{84} = \frac{5.9}{a}$$

$$a \times \frac{1}{84} = \frac{5.9}{a} \times a$$

$$\frac{a}{84} = 5.9$$

$$a \approx 496 \text{ cm or } 5.0 \text{ m}$$

The length of the alligator is 5.0 m.

- c) Convert the size of the ostrich into centimetres.

$$1.9 \text{ m} = 190 \text{ cm}$$

Use the scale statement to set up a ratio to determine the height of the ostrich in the picture.

$$\frac{1}{84} = \frac{o}{190}$$

$$190 \times \frac{1}{84} = \frac{o}{190} \times 190$$

$$\frac{190}{84} = o$$

$$2.3 \text{ cm} \approx o$$

The height of the ostrich in the picture is 2.3 cm

10. a) Use the scale statement to determine the size of a mite.

$$\frac{12.5}{1} = \frac{3.8}{m}$$

$$m \times 12.5 = \frac{3.8}{m} \times m$$

$$12.5m = 3.8$$

$$12.5m \div 12.5 = 3.8 \div 12.5$$

$$m \approx 0.30 \text{ cm}$$

The actual size of the mite is 0.30 cm or 3 mm.

- b) Use the scale statement to determine the height of the drawing of the cat.

$$\frac{12.5}{1} = \frac{c}{30}$$

$$30 \times 12.5 = \frac{c}{30} \times 30$$

$$375 \text{ cm} = c$$

The height of the cat would be 375 cm or 3.75 m if drawn at that scale.

- c) It is unlikely that the same scale would be used as the sizes are very different. For the cat, you would probably use a scale of 1:c. That is, you would represent it smaller than it actually is. The only time you might use the same scale for them is if, for some reason, you needed to compare how small the mite would be on the cat.

PRACTICE YOUR NEW SKILLS, p. 241

1. Convert the object to centimetres so the scale can be determined.

$$7.8 \text{ m} = 780 \text{ cm}$$

Determine the scale factor.

$$\text{scale factor} = \frac{\text{picture}}{\text{original}}$$

$$\text{scale factor} = \frac{1.5}{780}$$

$$\text{scale factor} = \frac{1}{520}$$

The scale factor used is $\frac{1}{520}$.

2. Use the school to solve for the length of the shoreline.

$$\frac{3 \text{ cm}}{100 \text{ km}} = \frac{x \text{ cm}}{2719 \text{ km}}$$

$$2719 \times \frac{3}{100} = \frac{x}{2719} \times 2719$$

$$2719 \times \frac{3}{100} = x$$

$$81.6 \text{ cm} \approx x$$

The length of the shoreline on the map is.

3. a) Measure the master bedroom to get approximately 1.69 in.

Convert the measure from feet to centimetres.

$$12.5 \times 12 = 150 \text{ in}$$

Calculate the scale using the measurement.

$$\text{scale statement} = 1.69 \text{ in}:150 \text{ in}$$

$$\text{scale statement} = 1:88.8$$

The scale used to create the floor plan is 1:88.8 or 13:1200.

- b) Measure the family room to determine the approximate dimensions, which are 1.375 in by 2.125 in. Use the scale statement to calculate the dimensions.

$$1.375 \times 88.8 = 122.1 \text{ in or } 10.2 \text{ ft}$$

$$2.125 \times 88.8 = 188.7 \text{ in or } 15.7 \text{ ft}$$

The dimensions of the family room are 122.1 in \times 188.7 in or 10.2 ft \times 15.7 ft.

- c) Measure the bedroom to determine the approximate dimensions, which are 1.313 in by 1.125 in. Use the scale statement to calculate the dimensions.

$$1.313 \times 88.8 = 116.6 \text{ in or } 9.7 \text{ ft}$$

$$1.125 \times 88.8 = 99.9 \text{ in or } 8.3 \text{ ft}$$

The dimensions of the smaller bedroom are 116.6 in \times 99.9 in or 9.7 ft \times 8.3 ft.

4. a) Convert the measure in metres to centimetres.

$$298 \text{ m} = 29\,800 \text{ cm}$$

Determine the scale statement.

$$\text{scale statement} = 11.9 : 29\,800$$

$$\text{scale statement} = 1 : 2504$$

The scale used to build the model was 2504:4b. Calculate the height of the antenna.

$$355 - 298 = 57 \text{ m}$$

Convert the measure in metres to centimetres.

$$57 \text{ m} : 5700 \text{ cm}$$

Use the scale calculated to determine how long the model antenna be.

$$5700 \div 2504 \approx 2.3 \text{ cm}$$

The height of the antenna in the model will be 2.3 cm.

5. Use the scale to calculate the actual dimensions of the bookcase.

$$7.8 \times 30 = 234 \text{ cm}$$

$$5.4 \times 30 = 162 \text{ cm}$$

$$1 \times 30 = 30 \text{ cm}$$

The dimensions of the bookcase is 234 cm \times 162 cm \times 30 cm.

6. Calculate the actual height of the TD Canada Trust Building.

$$16.6 \times 10 = 166 \text{ m}$$

Calculate the actual height of the Nexen Building.

$$15.0 \times 10 = 150 \text{ m}$$

Calculate the difference in heights.

$$166 - 150 = 16 \text{ m}$$

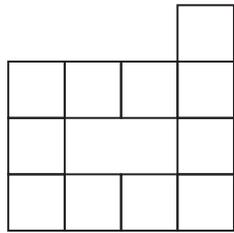
The actual difference in heights of the two buildings is 16 m.

5.2

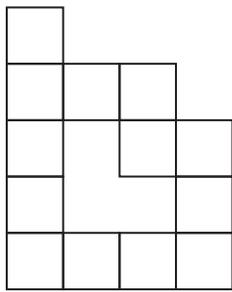
Two-Dimensional Representations

BUILD YOUR SKILLS, p. 246

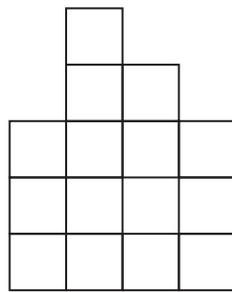
1.



Top

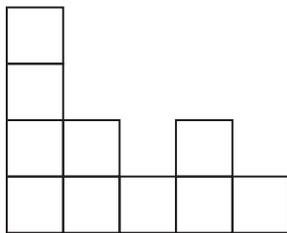


Front



Side

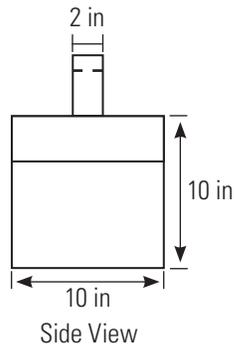
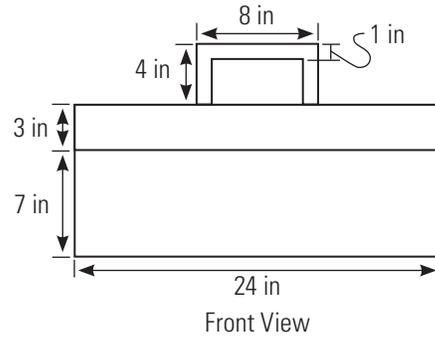
2. a)



Front

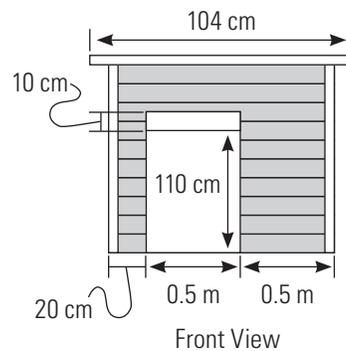
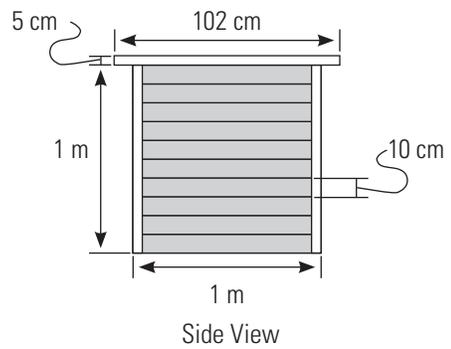
b) No, there might be some blocks we cannot see, hidden by the tower.

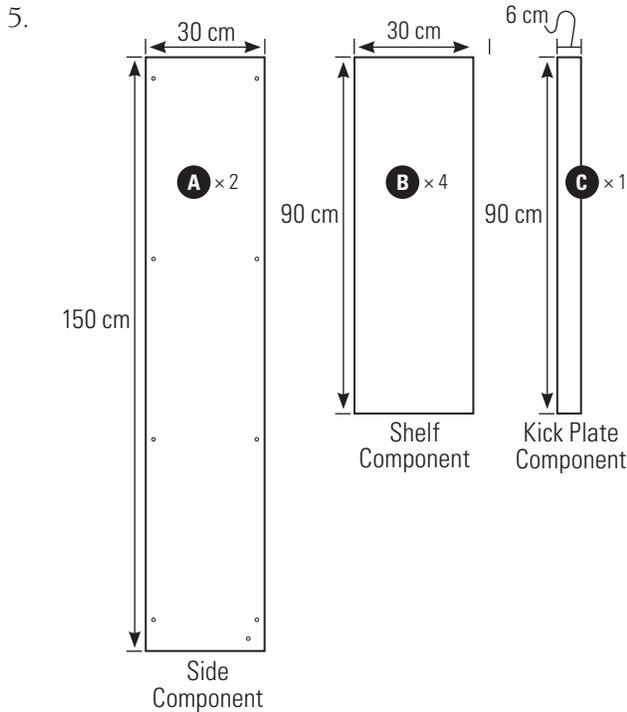
3.



4.

Scale 1:40

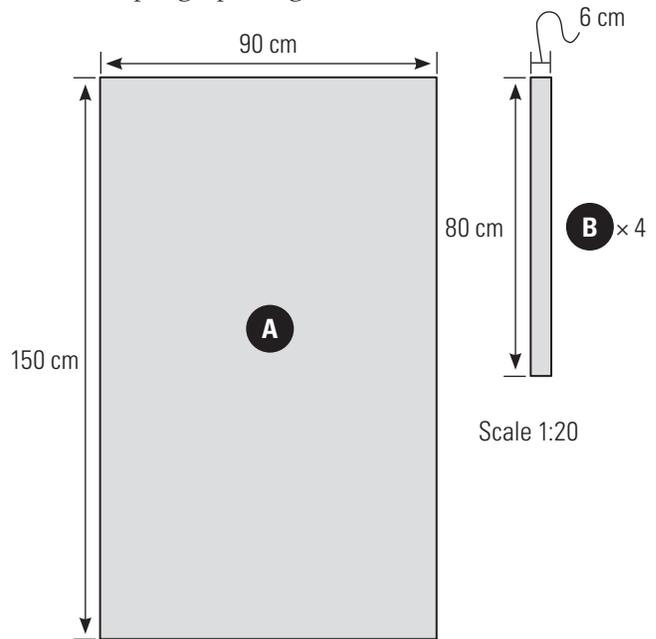




She must cut each block to be

$$12 \frac{3}{4} \text{ in} \times 12 \frac{3}{4} \text{ in}$$

7. A sample graph might look like this.



6. a) Measure the pattern to determine the scale. When measuring you should get approximately 2 in. Convert the measure in feet to inches.

$$1 \text{ foot} = 12 \text{ inches}$$

Determine the scale.

$$2 \text{ in}:12 \text{ in}$$

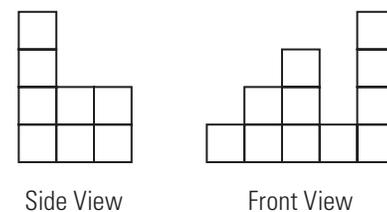
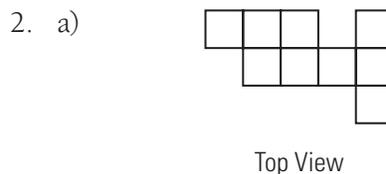
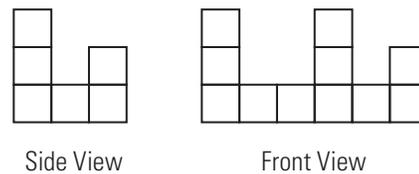
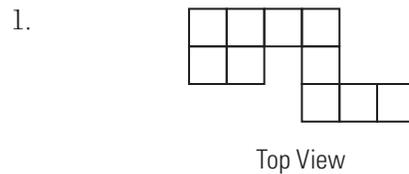
$$1:6$$

The scale used to draw the diagram is 1:6.

- b) She can make a quilt which will have square patches, but it will be smaller than she planned because she forgot about the seam allowance.
- c) Calculate the length and width of each block by adding the seam allowance on either end.

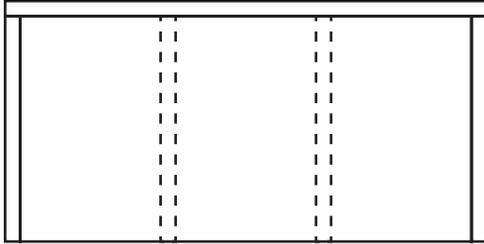
$$12 + \frac{3}{8} + \frac{3}{8} = 12 \frac{3}{4} \text{ in}$$

PRACTISE YOUR NEW SKILLS, p. 252

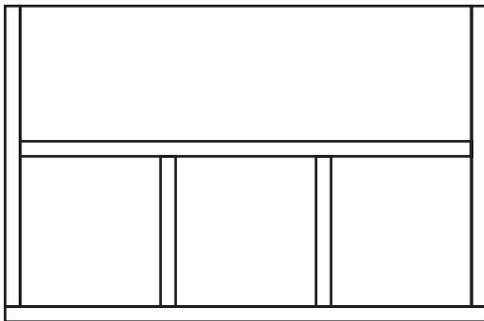


- b) No, there could be additional blocks or gaps that we cannot see from the current diagram.

3. Scale 1:6



Top View



Front View

4. a) The width of the buildings in the diagram is approximately 2 cm and the height is approximately 4 cm. Convert the actual width and height to centimetres.

$$7 \text{ m} = 700 \text{ cm}$$

$$14 \text{ m} = 1400 \text{ cm}$$

Determine the scale using the width.

$$\text{scale statement} = 2:700$$

$$\text{scale statement} = 1:350$$

Though not absolutely necessary you can confirm your calculations by recalculating the scale using the height.

$$\text{scale statement} = 4:1400$$

$$\text{scale statement} = 1:350$$

The scale used to draw the diagram is 1:350.

- b) Calculate the dimensions of the model houses.

$$700 \div 100 = 7 \text{ cm}$$

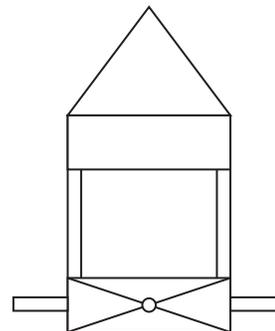
$$1400 \div 100 = 14 \text{ cm}$$

The models will be 7 cm wide by 14 cm tall high.

- c) No, we do not know what the back parts of the houses look like.

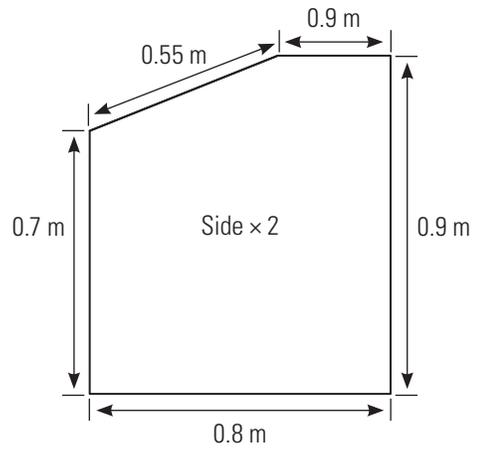
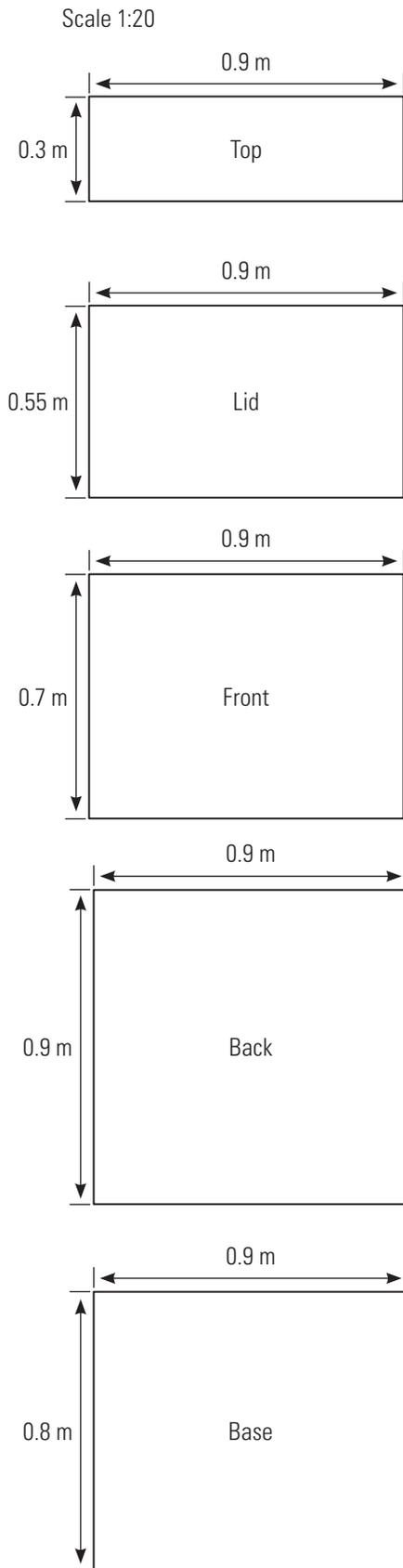
5. a) She will need to provide a front and a top view. She does not need to provide side views because all the sides are the same. She does not need to provide a top view, because the roof can be seen on the front view, since all the sides are the same.

b)

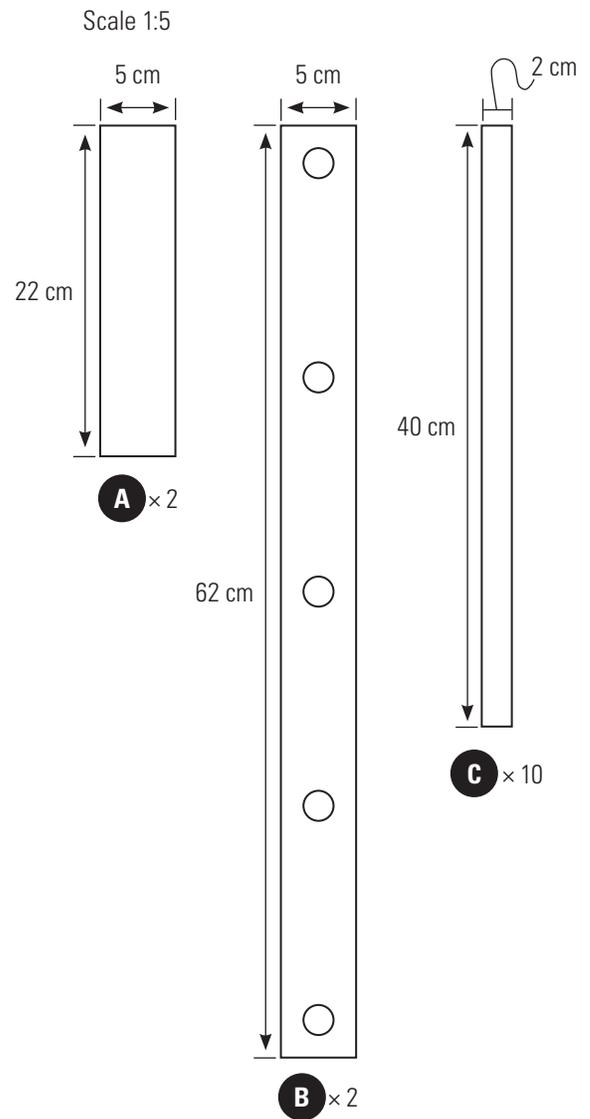


Scale statements will vary based on the measurements.

6.



7.



5.3

Three-Dimensional Representations

BUILD YOUR SKILLS, p. 260

1. Use the measurement provided to determine the scale. There are 9 dots that equal 18 feet.

$$9 \text{ dots} : 18 \text{ ft}$$

$$1 : 2$$

The scale is 1 dot : 2 feet.

Count all the desired lengths.

$$x = 1$$

$$y = 5$$

$$z = 8$$

$$h = 4$$

Using the scale calculate the lengths.

$$x = 1 \times 2$$

$$x = 2 \text{ ft}$$

$$y = 5 \times 2$$

$$y = 10 \text{ ft}$$

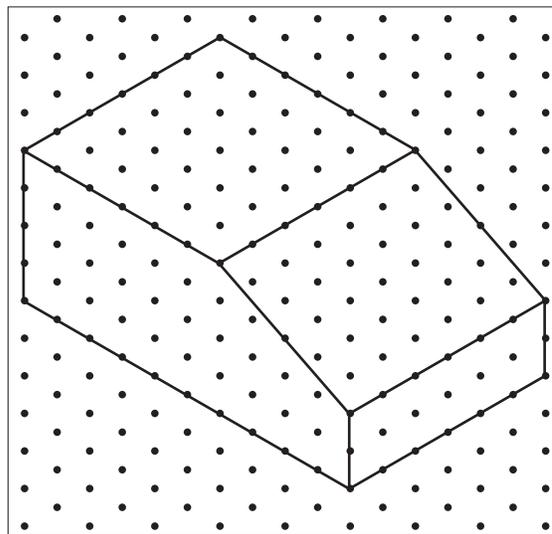
$$z = 8 \times 2$$

$$z = 16 \text{ ft}$$

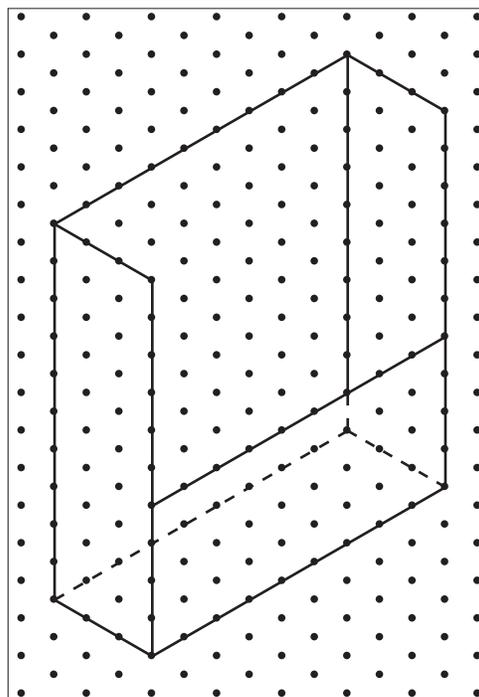
$$h = 4 \times 2$$

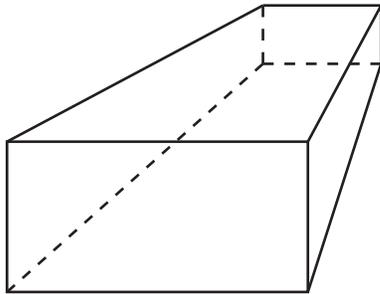
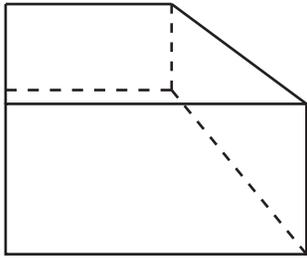
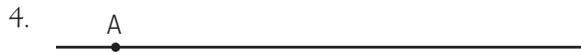
$$h = 8 \text{ ft}$$

2. Scale 1:20

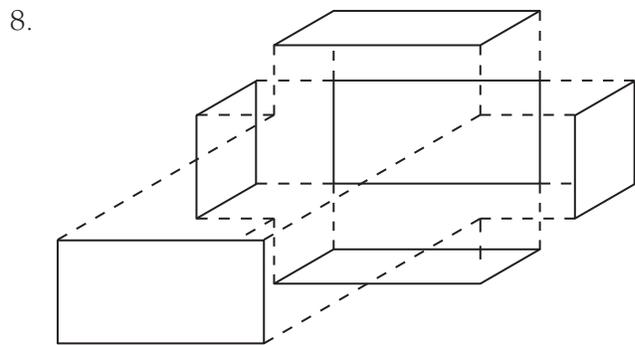
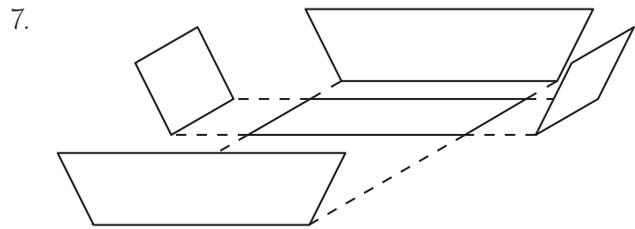
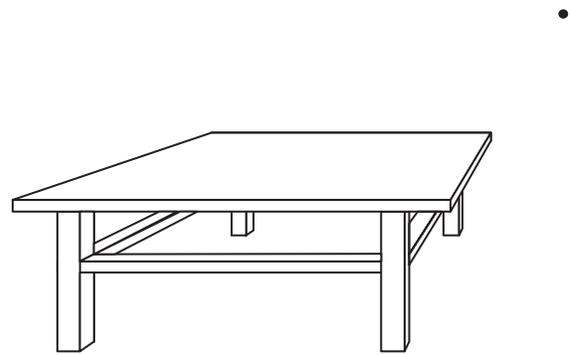
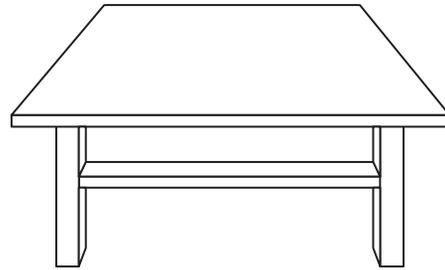
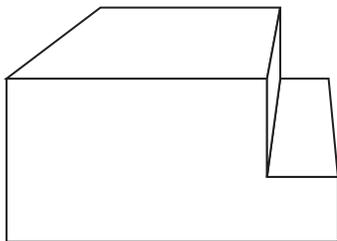


3. Scale 1:60

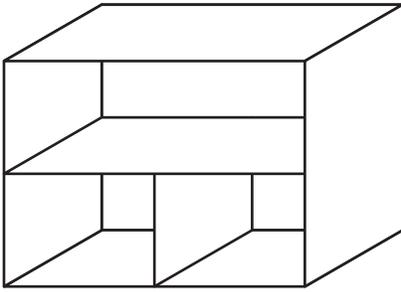




The two will not look exactly alike because it depends on where you place your vanishing point and how deep you choose to make your box. Thus, even if you choose to make them the same depth, they will look slightly different because of the angle to the vanishing point.



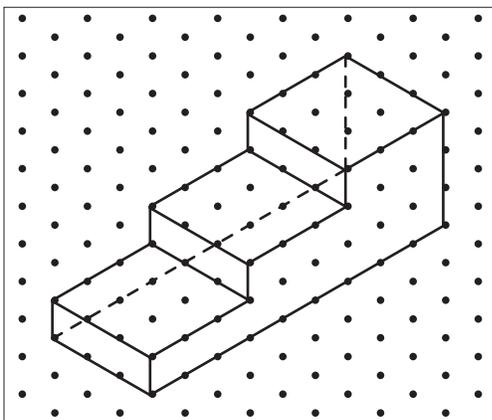
9.



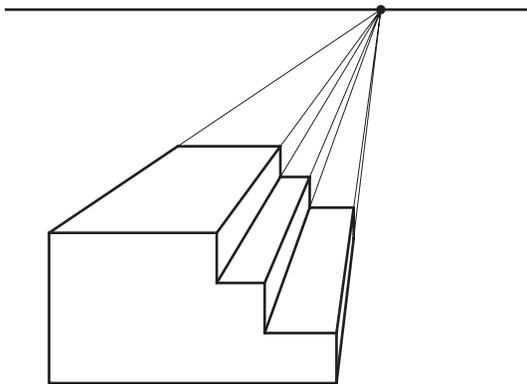
PRACTISE YOUR NEW SKILLS, p. 269

1. See diagram on the next page.

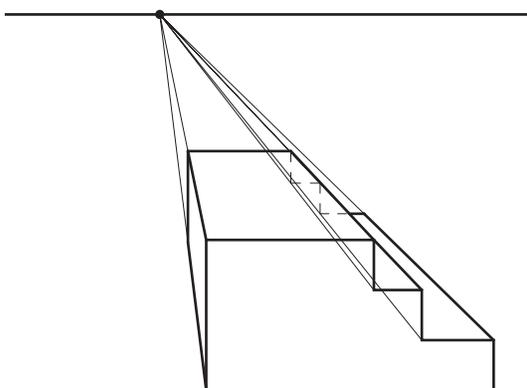
2.



3. a)

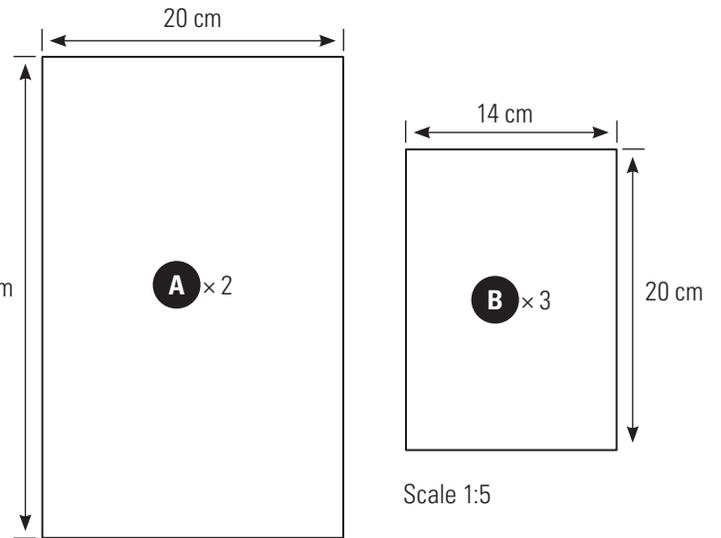


b)

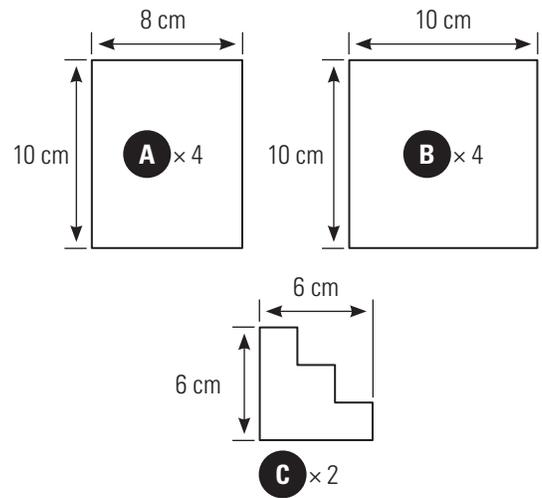


c) The perspective and the angles to the vanishing point are different.

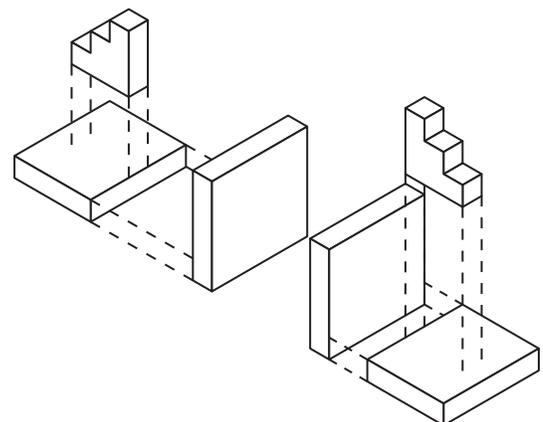
4.



5. a) Scale 1:5

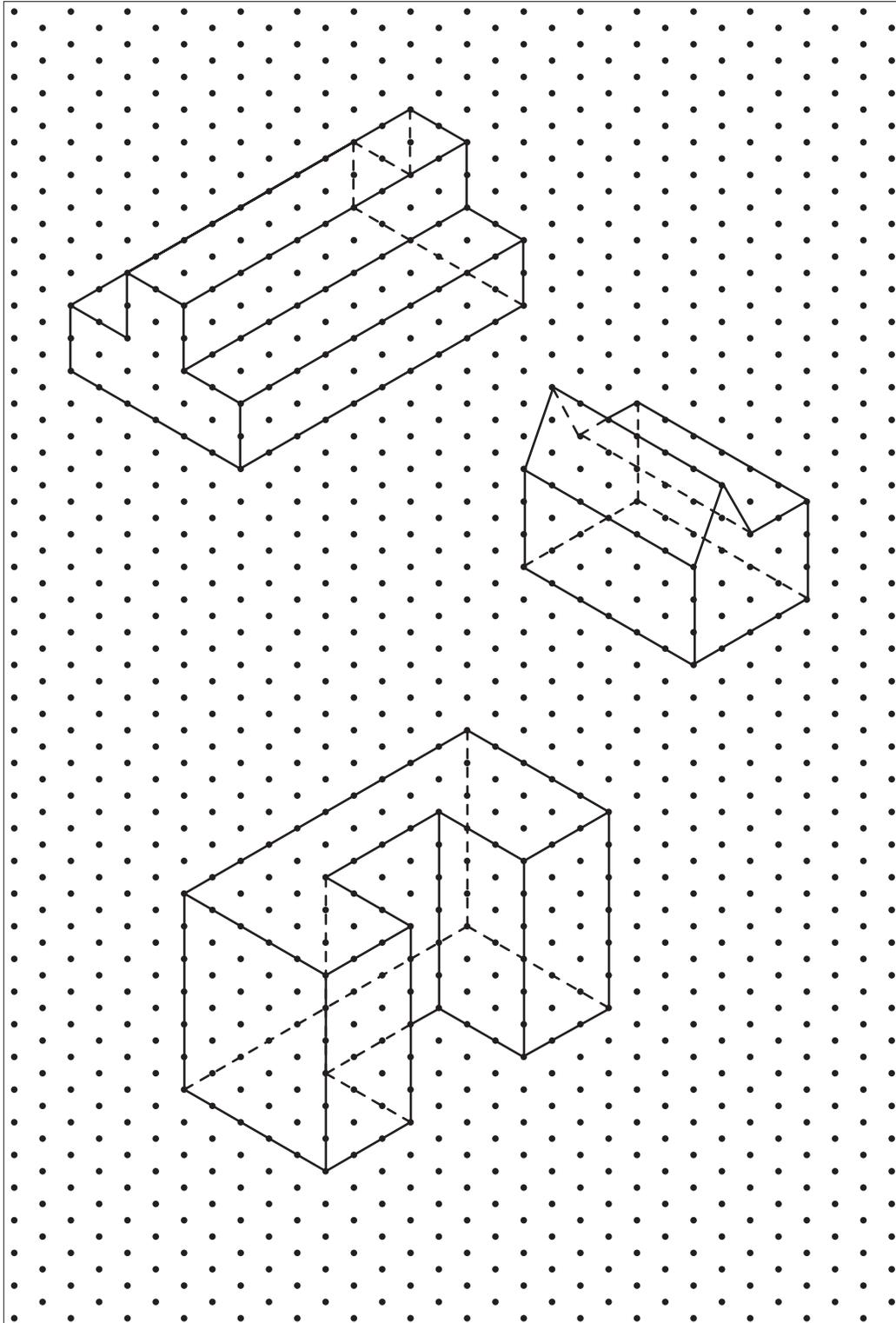


b)



5.3 Three-Dimensional Representations, Practise Your New Skills, Question 1.

Scale 1:1



CHAPTER TEST, p. 273

1. a) $\frac{1}{128}$ Use the scale to determine the scale factor.

$$\text{scale factor} = \frac{\text{model}}{\text{original}}$$

$$\text{scale factor} = \frac{1}{128}$$

The scale factor of the model is $\frac{1}{128}$.

- b) Use the scale to determine the wingspan of the actual airplane.

$$30 \times 128 = 3840 \text{ cm or } 38.4 \text{ m}$$

The wingspan of the actual airplane is 3840 cm or 38.4 m.

2. Convert the kilometres to centimetres.

$$350 \times 100\,000 = 35\,000\,000 \text{ cm}$$

Simplify to determine the scale.

$$7 \text{ cm} : 35\,000\,000 \text{ cm}$$

$$7 : 35\,000\,000$$

$$1 : 5\,000\,000$$

7:35 000 000 The simplest expression of the scale is 1:5 000 000.

3. Set up a ratio to solve for the height of the flagpole.

$$\frac{4.8}{1.3} = \frac{h}{190}$$

$$190 \times \frac{4.8}{1.3} = \frac{h}{190} \times 190$$

$$190 \times \frac{4.8}{1.3} = h$$

$$702 \text{ cm} \approx h$$

The height of the flagpole is 702 cm.

ALTERNATE SOLUTION

It is possible to get the same solution if you set the ratios up differently.

$$\frac{h}{4.8} = \frac{190}{1.3}$$

$$4.8 \times \frac{h}{4.8} = \frac{190}{1.3} \times 4.8$$

$$h = \frac{190}{1.3} \times 4.8$$

$$h = 702 \text{ cm}$$

The height of the flagpole is 702 cm.

702 cm or approx 7 m

4. a) Use the measurements provided to determine the scale statement.

$$\text{scale statement} = 7 \text{ cm} : 105 \text{ cm}$$

$$\text{scale statement} = 1 : 15$$

The scale of the dollhouse is 1:15.

- b) $\frac{1}{15}$ Use the scale statement to determine the scale factor.

$$\text{scale factor} = \frac{\text{dollhouse}}{\text{original}}$$

$$\text{scale factor} = \frac{1}{15}$$

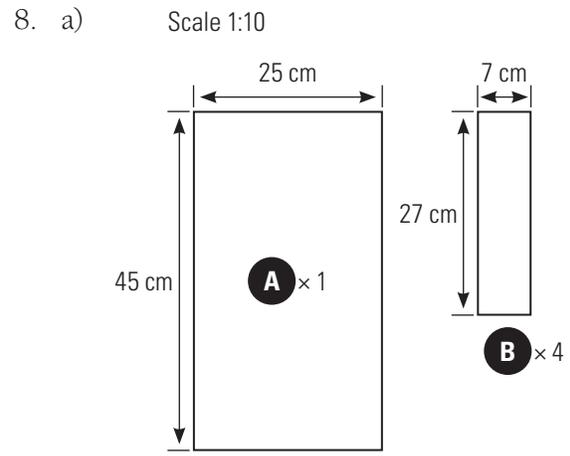
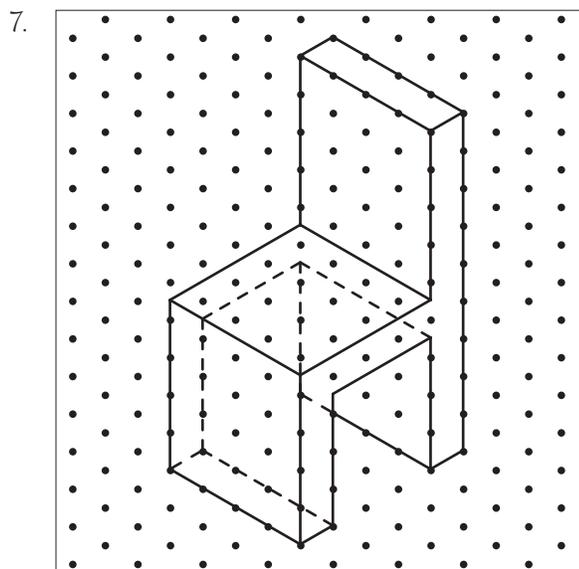
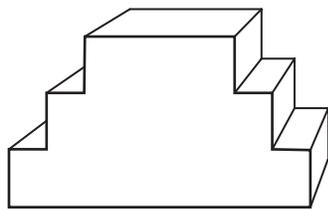
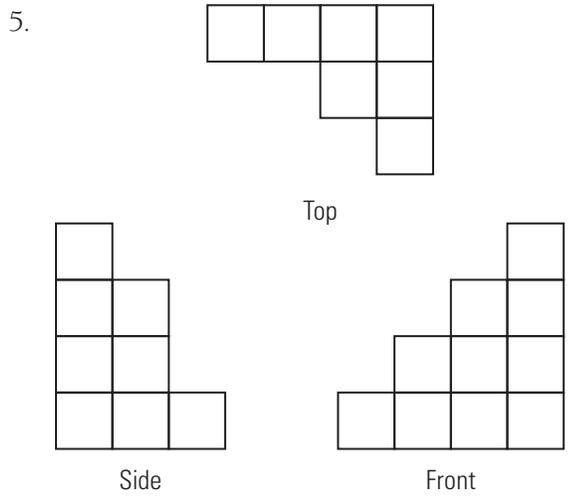
The scale factor Charlotte is using is $\frac{1}{15}$.

- c) Multiply the actual height of the window to determine the height of the window in the dollhouse.

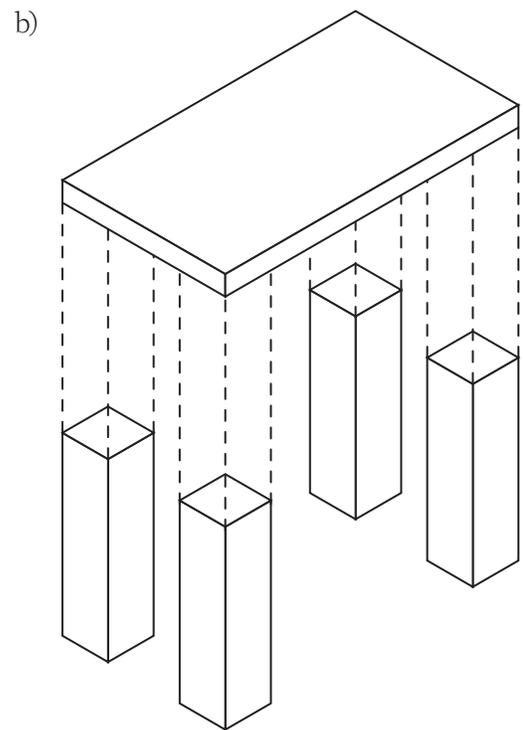
$$h = 126 \times \frac{1}{15}$$

$$h = 8.4 \text{ cm}$$

The height of the window is 8.4 cm.



Scale statements will differ depending on the diagrams.



Chapter 6

Financial Services

6.1

Choosing an Account

BUILD YOUR SKILLS, p. 282

1. a) The Value Account allows 10 self-service transactions. Additional self-service transactions cost \$0.50 each, and all teller-assisted transactions cost \$1.00 each.

Calculate Frédéric's total number of self-service transactions.

$$2 + 4 + 5 + 10 = 21$$

Ten of those transactions will be free. He will be charged \$0.50 each for the other 11 transactions.

$$11 \times \$0.50 = \$5.50$$

He maintains the required minimum monthly balance, so his account fee is waived. He will be charged only \$5.50.

- b) The Self-service Account allows 25 free self-service transactions. Frédéric makes fewer transactions than this, so he will not have to pay any transaction fees. He also maintains the required minimum monthly balance, so he will not be charged the account fee.

He will not have to pay service charges.

2. a) The Bonus Savings Account allows only 2 debit transactions. All other transactions cost \$1.25 each. Two of Thom's transactions will be free, and all 27 others will be charged at \$1.25 each.

$$27 \times \$1.25 = \$33.75$$

There is no account fee, so Thom will owe \$33.75 in service charges.

- b) The Full-service Account allows 40 free transactions, so Thom will not have to pay any transaction fees. However, he does not maintain the minimum monthly balance, so he will have to pay the account fee of \$24.50.

3. The Value Account allows 10 self-service transactions. Additional self-service transactions cost \$0.50 each, and all teller-assisted transactions cost \$1.00 each.

Li Ping's 8 self-service transactions are free. Her 2 teller-assisted transactions will cost \$1.00 each. She does not maintain the required minimum monthly balance, so she will have to pay the \$3.90 account fee. Li Ping will therefore owe a total of \$5.90 in fees.

4.

<i>Transaction</i>	<i>Description</i>	<i>Withdrawal</i>	<i>Deposit</i>	<i>Balance</i>
				\$1798.53
Direct deposit	Paycheque		\$1432.51	\$3231.04
ATM	Cash	\$200.00		\$3031.04
Bank card	Groceries	\$63.95		\$2967.09
Bank card	Clothes	\$75.32		\$2891.77
Bank card	Movie	\$24.50		\$2867.27
Teller	Hydro bill	\$89.56		\$2777.71
Direct deposit	Paycheque		\$1432.50	\$4210.21
ATM	Cash	\$100.00		\$4110.21
Auto-withdrawal	Loan payment	\$375.86		\$3734.35
Bank card	Groceries	\$154.32		\$3580.03
Bank card	Gas	\$56.23		\$3523.80
Bank card	Dinner	\$25.38		\$3498.42
Auto-withdrawal	Rent	\$575.00		\$2923.42
Bank card	Books	\$123.45		\$2799.97
Auto-withdrawal	Monthly banking fee	\$24.50		\$2775.47

Mitra maintains the minimum monthly balance of \$2000.00, so she will not have to pay the account fee. Her end of month balance is \$2775.47.

5.

<i>Transaction</i>	<i>Description</i>	<i>Withdrawal</i>	<i>Deposit</i>	<i>Balance</i>
				\$532.98
ATM	Cash	\$50.00		$\$532.98 - \$50.00 = \$482.98$
Bank card	Movie/snack	\$18.54		$\$482.98 - \$18.54 = \$464.44$
Auto-deposit	Paycheque		\$238.21	$\$464.44 + \$238.21 = \$702.65$
Auto-payment	Loan	\$385.21		$\$702.65 - \$385.21 = \$317.44$
Bank card	Groceries	\$115.87		$\$317.44 - \$115.87 = \$201.57$
ATM	Cash	\$100.00		$\$201.57 - \$100.00 = \$101.57$
Auto-withdrawal	Account fee	\$3.90		$\$101.57 - \$3.90 = \$97.67$

Marek's account allows 10 free self-service transactions, so he does not have to pay any transaction fees. However, he does not maintain the required minimum monthly balance, so he does have to pay the account fee of \$3.90. His final balance will be \$97.67.

6.

<i>Transaction</i>	<i>Description</i>	<i>Withdrawal</i>	<i>Deposit</i>	<i>Balance</i>
				\$785.53
ATM	Cash	\$40.00		$\$785.53 - \$40.00 = \$745.53$
ATM	Cash	\$100.00		$\$745.53 - \$100.00 = \$645.53$
Bank card	Groceries	$\$45.98 + \1.25 service charge		$\$645.53 - \$45.98 - \$1.25 =$ $\$598.30$
Bank card	Movie	$\$12.95 + \1.25 service charge		$\$598.30 - \$12.95 - \$1.25 =$ $\$584.10$
ATM	Babysitting	\$1.25 service charge	\$30.00	$\$584.10 + \$30.00 - \$1.25 =$ $\$612.85$
Bank card	Snacks	$\$9.59 + \1.25 service charge		$\$612.85 - \$9.59 - \$1.25 =$ $\$602.01$
Direct deposit	Paycheque	\$1.25 service charge	\$187.37	$\$602.01 + \$187.37 - \$1.25 =$ $\$788.13$
Auto-withdrawal	Telephone	$\$58.82 + \1.25 service charge		$\$788.13 - \$58.82 - \$1.25 =$ $\$728.06$
ATM	Cash	$\$100.00 + \1.25 service charge		$\$728.06 - \$100.00 - \$1.25 =$ $\$626.81$
Auto-withdrawal	Account fee	–	–	\$626.81

With the Bonus Savings Account, only Noah's first two transactions are free. Each additional transaction costs \$1.25. However, there is no monthly account fee. Noah's final balance is \$626.81.

PRACTISE YOUR NEW SKILLS, p. 287

1. Calculate Georgina's fees.

$$(\$0.50 \times 8) + (\$2.00 \times 3) = \$10.00$$

2. Rolfe's Self-service Account allows 25 free self-service transactions, so he will not have to pay any transaction fees.

Calculate Rolfe's minimum monthly balance. Since you don't know the order of his transactions, assume he made all the withdrawals before depositing any money.

$$\$2675.32 - (6 \times \$100.00) = \$2075.32$$

The required minimum monthly balance is \$1500.00, and Rolfe definitely had that much in his account at all times so he will not have to pay the monthly account fee.

Rolfe will not have to pay any fees this month.

3. If Patrice maintains his current spending habits, he will probably have a minimum monthly balance of at least \$2000.00. He makes about 15 self-service transactions a month.

Use a table to show Patrice's fees with each type of account. (See table on next page)

Patrice should get a Full-service Account or a Self-service Account. The Full-service Account gives him the most free transactions so it would give him the most flexibility, but he would need to continue to maintain a minimum monthly balance of \$2000.00.

	<i>Value Account</i>	<i>Self-service Account</i>	<i>Full-service Account</i>	<i>Bonus Savings Account</i>
Monthly fee	Waived	Waived	Waived	No fee
Transaction fees	10 free $5 \times \$0.50 = \2.50	25 free	40 free	2 free $13 \times \$1.25 = \16.25
Total cost	\$2.50	Free	Free	\$16.25

4. a) Jeanette's Value Account allows 10 free self-service transactions. Each additional self-service transaction will cost \$0.50 and teller-assisted transactions will cost \$1.00.

<i>Transaction</i>	<i>Description</i>	<i>Withdrawal</i>	<i>Deposit</i>	<i>Balance</i>
				\$4986.54
ATM	Cash	\$250.00		$\$4986.54 - \$250.00 = \$4736.54$
Bank card	Dinner	\$25.32		$\$4736.54 - \$25.32 = \$4711.22$
Bank card	Groceries	\$145.93		$\$4711.22 - \$145.93 = \$4565.29$
Direct deposit	Paycheque		\$524.66	$\$4565.29 + \$524.66 = \$5089.95$
Bank card	Movie	\$12.98		$\$5089.95 - \$12.98 = \$5076.97$
ATM	Cash	\$100.00		$\$5076.97 - \$100.00 = \$4976.97$
Bank card	Gas	\$48.96		$\$4976.97 - \$48.96 = \$4928.01$
Teller	Utilities	$\$123.23 + \1.00 service charge		$\$4928.01 - \$123.23 - \$1.00 = \4803.78
Auto-withdrawal	Rent	\$550.00		$\$4803.78 - \$550.00 = \$4253.78$
Direct deposit	Paycheque		\$524.65	$\$4253.78 - \$524.65 = \$4778.43$
Bank card	Groceries	\$165.24		$\$4778.43 - \$165.24 = \$4613.19$
ATM	Cash	$\$100.00 + \0.50 service charge		$\$4613.19 - \$100.00 - \$0.50 = \4512.69
Teller	Phone	$\$47.25 + \1.00 service charge		$\$4512.69 - \$47.25 - \$1.00 = \4464.44
Bank card	Misc.	$\$12.32 + \0.50 service charge		$\$4464.44 - \$12.32 - \$0.50 = \4451.62
Bank card	Dinner	$\$15.88 + \0.50 service charge		$\$4451.62 - \$15.88 - \$0.50 = \4435.24
ATM	Cash	$\$200.00 + \0.50 service charge		$\$4435.24 - \$200.00 - \$0.50 = \4234.74
Bank card	Prescription	$\$32.54 + \0.50 service charge		$\$4234.74 - \$32.54 - \$0.50 = \4201.70

- b) No, she should switch to a Full-service Account because she keeps enough money in her account for the fee to be waived and she would have more transactions included for free with her account.

5. Use a table to calculate Mariya's balance. Her Value Account allows 10 free self-service transactions. Each additional self-service transaction will cost \$0.50 and teller-assisted transactions will cost \$1.00.

(Note that Mariya likely did not complete the transactions in this order, because they result in a significant negative balance mid-month, but this does not matter for calculating the final balance at the end of the month.)

<i>Transaction</i>	<i>Description</i>	<i>Withdrawal</i>	<i>Deposit</i>	<i>Balance</i>
				\$432.98
ATM	Cash	\$200.00		$\$432.98 - \$200.00 = \$232.98$
ATM	Cash	\$200.00		$\$232.98 - \$200.00 = \$32.98$
ATM	Cash	\$200.00		$\$32.98 - \$200.00 = -\$167.02$
Teller	Phone bill payment	$\$68.21 + \1.00 service charge		$-\$167.02 - \$68.21 - \$1.00 = -\236.23
Bank card	Groceries	\$103.56		$-\$236.23 - \$103.56 = -\$339.79$
Bank card	Movie	\$12.87		$-\$339.79 - \$12.87 = -\$352.66$
Bank card	Meal	\$15.89		$-\$352.66 - \$15.89 = -\$368.55$
Bank card	Meal	\$22.48		$-\$368.55 - \$22.48 = -\$391.03$
Direct deposit	Paycheque		\$658.42	$-\$391.03 + \$658.42 = \$267.39$
Online	Credit card bill payment	\$243.56		$\$267.39 - \$243.56 = \$23.83$
Auto-withdrawal	Account fee	\$3.90		$\$23.83 - \$3.90 = \$19.93$

Mariya did not maintain the minimum monthly balance of \$1000.00, so she does have to pay the \$3.90 account fee. Her closing balance is \$19.93.

6.2

Simple and Compound Interest

BUILD YOUR SKILLS, p. 292

1. a) $I = Prt$

$$I = (\$1000.00)(0.025)(1)$$

$$I = \$25.00$$

b) $I = Prt$

$$I = (\$1000.00)(0.05)(1)$$

$$I = \$50.00$$

c) $I = Prt$

$$I = (\$1000.00)(0.025)(2)$$

$$I = \$50.00$$

d) $I = Prt$

$$I = (\$2000.00)(0.025)(1)$$

$$I = \$50.00$$

e) When the principal and the term stay the same but the rate doubles—for example, from question a) to question b) above—the amount of interest earned doubles.

f) When the principal and the rate stay the same but the term doubles—for example, from question a) to question c) above—the amount of interest earned doubles.

2. $I = Prt$

$$I = (\$600.00)(0.0375)(5)$$

$$I = \$112.50$$

$$A = P + I$$

$$A = \$600.00 + \$112.50$$

$$A = \$712.50$$

3. $I = Prt$

$$I = (\$1000.00)(0.0450)(10)$$

$$I = \$450.00$$

$$A = P + I$$

$$A = \$1000.00 + \$450.00$$

$$A = \$1450.00$$

4. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$A = (\$5000.00)\left(1 + \frac{0.03}{1}\right)^{1 \times 2}$$

$$A = (\$5000.00)(1.03)^2$$

$$A = \$5304.50$$

5. Option 1: \$4000.00 invested at 3.50% per annum, compounded annually, for 3 years

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = (\$4000.00)\left(1 + \frac{0.035}{1}\right)^{1 \times 3}$$

$$A = (\$4000.00)(1.035)^3$$

$$A \approx \$4434.87$$

Option 2: \$4000.00 invested at 3.50% simple interest for 3 years

$$I = Prt$$

$$I = (\$4000.00)(0.0350)(3)$$

$$I = \$420.00$$

$$A = P + I$$

$$A = \$4000.00 + \$420.00$$

$$A = \$4420.00$$

difference = Option 1 – Option 2
 difference = \$4434.87 – \$4420.00
 difference = \$14.87

6. $A = P \left(1 + \frac{r}{n}\right)^{nt}$

$$A = (\$8000.00) \left(1 + \frac{0.025}{1}\right)^{1 \times 5}$$

$$A = (\$8000.00)(1.025)^5$$

$$A \approx \$9051.27$$

$$I = A - P$$

$$I = \$9051.27 - \$8000.00$$

$$I = \$1051.27$$

7. a) $A = P \left(1 + \frac{r}{n}\right)^{nt}$

$$A = (\$4000.00) \left(1 + \frac{0.04}{1}\right)^{1 \times 8}$$

$$A = (\$4000.00)(1.04)^8$$

$$A \approx \$5474.28$$

b) $A = P \left(1 + \frac{r}{n}\right)^{nt}$

$$A = (\$4000.00) \left(1 + \frac{0.04}{2}\right)^{2 \times 8}$$

$$A = (\$4000.00)(1.02)^{16}$$

$$A \approx \$5491.14$$

c) $A = P \left(1 + \frac{r}{n}\right)^{nt}$

$$A = (\$4000.00) \left(1 + \frac{0.04}{4}\right)^{4 \times 8}$$

$$A = (\$4000.00)(1.01)^{32}$$

$$A \approx \$5499.76$$

d) $A = P \left(1 + \frac{r}{n}\right)^{nt}$

$$A = (\$4000.00) \left(1 + \frac{0.04}{12}\right)^{12 \times 8}$$

$$A = (\$4000.00)(1.00333)^{32}$$

$$A \approx \$5505.58$$

8. Compounded annually:

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A = (\$10\,000.00) \left(1 + \frac{0.0375}{1}\right)^{1 \times 1}$$

$$A = (\$10\,000.00)(1.0375)$$

$$A = \$10\,375.00$$

Compounded daily:

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

$$A = (\$10\,000.00) \left(1 + \frac{0.0375}{365}\right)^{365 \times 1}$$

$$A \approx \$10\,382.10$$

difference = daily – annually

$$\text{difference} = \$10\,382.10 - \$10\,375.00$$

$$\text{difference} = \$7.10$$

9. The investment is compounded quarterly, so the term of each interest period is one-quarter of a year, or 0.25.

Interest period	Investment value at beginning of period	Interest earned ($I = Prt$)	Investment value at end of period
1	\$3000.00	$\$3000.00 \times 0.0325 \times 0.25 \approx \24.38	$\$3000.00 + \$24.38 = \$3024.38$
2	\$3024.38	$\$3024.38 \times 0.0325 \times 0.25 \approx \24.57	$\$3024.38 + \$24.57 = \$3048.95$
3	\$3048.95	$\$3048.95 \times 0.0325 \times 0.25 \approx \24.77	$\$3048.95 + \$24.77 = \$3073.72$
4	\$3073.72	$\$3073.72 \times 0.0325 \times 0.25 \approx \24.97	$\$3073.72 + \$24.97 = \$3098.69$
5	\$3098.69	$\$3098.69 \times 0.0325 \times 0.25 \approx \25.18	$\$3098.69 + \$25.18 = \$3123.87$
6	\$3123.87	$\$3123.87 \times 0.0325 \times 0.25 \approx \25.38	$\$3123.87 + \$25.38 = \$3149.25$
7	\$3149.25	$\$3149.25 \times 0.0325 \times 0.25 \approx \25.59	$\$3149.25 + \$25.59 = \$3174.84$
8	\$3174.84	$\$3174.84 \times 0.0325 \times 0.25 \approx \25.80	$\$3174.84 + \$25.80 = \$3200.64$

Students can check their final answer by using the compound interest formula.

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = (\$3000.00) \left(1 + \frac{0.0325}{4} \right)^{4 \times 2}$$

$$A \approx \$3200.64$$

10. a) Years to double investment = $72 \div$ (interest rate as a percent)

$$\text{Years to double investment} = 72 \div 4$$

$$\text{Years to double investment} = 18$$

It will take about 18 years for the investment to double in value.

- b) Years to double investment = $72 \div$ (interest rate as a percent)

$$\text{Years to double investment} = 72 \div 2.45$$

$$\text{Years to double investment} = 29.4$$

It will take about 29.4 years for the investment to double in value.

- c) Years to double investment = $72 \div$ (interest rate as a percent)

$$\text{Years to double investment} = 72 \div 1.95$$

$$\text{Years to double investment} = 36.9$$

It will take about 36.9 years for the investment to double in value.

PRACTISE YOUR NEW SKILLS, p. 301

1. a) $I = Prt$

$$I = (\$400.00)(0.0125)(8)$$

$$I = \$40.00$$

$$A = P + I$$

$$A = \$400.00 + \$40.00$$

$$A = \$440.00$$

- b) $I = Prt$

$$I = (\$750.00)(0.0275)(5)$$

$$I = \$103.13$$

$$A = P + I$$

$$A = \$750.00 + \$103.13$$

$$A = \$853.13$$

- c) $I = Prt$

$$I = (\$1000.00)(0.0450)(10)$$

$$I = \$450.00$$

$$A = P + I$$

$$A = \$1000.00 + \$450.00$$

$$A = \$1450.00$$

- d) $I = Prt$

$$I = (\$1200.00)(0.0395)(9)$$

$$I = \$426.60$$

$$A = P + I$$

$$A = \$1200.00 + \$426.60$$

$$A = \$1626.60$$

2. a) $A = P \left(1 + \frac{r}{n} \right)^{nt}$

$$A = (\$400.00) \left(1 + \frac{0.0125}{12} \right)^{12 \times 8}$$

$$A \approx \$442.05$$

- b) $A = P \left(1 + \frac{r}{n} \right)^{nt}$

$$A = (\$750.00) \left(1 + \frac{0.0275}{12} \right)^{12 \times 5}$$

$$A \approx \$860.42$$

- c) $A = P \left(1 + \frac{r}{n} \right)^{nt}$

$$A = (\$1000.00) \left(1 + \frac{0.0450}{12} \right)^{12 \times 10}$$

$$A \approx \$1566.99$$

$$d) A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = (\$1200.00) \left(1 + \frac{0.0395}{12} \right)^{12 \times 9}$$

$$A \approx \$1711.27$$

3. The investment is compounded semi-annually, so the term for each interest period is 0.5 years.

Interest period	Investment value at beginning of period	Interest earned ($I = Prt$)	Investment value at end of period
1	\$1000.00	$\$1000.00 \times 0.0385 \times 0.5 = \19.25	$\$1000.00 + \$19.25 = \$1019.25$
2	\$1019.25	$\$1019.25 \times 0.0385 \times 0.5 = \19.62	$\$1019.25 + \$19.62 = \$1038.87$
3	\$1038.87	$\$1038.87 \times 0.0385 \times 0.5 = \20.00	$\$1038.87 + \$20.00 = \$1058.87$
4	\$1058.87	$\$1058.87 \times 0.0385 \times 0.5 = \20.38	$\$1058.87 + \$20.38 = \$1079.25$

Students can check their answer using the compound interest formula.

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = (\$1000.00) \left(1 + \frac{0.0385}{2} \right)^{2 \times 2}$$

$$A \approx \$1079.25$$

$$4. a) A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = (\$4000.00) \left(1 + \frac{0.0300}{365} \right)^{365 \times 3}$$

$$A \approx \$4376.68$$

$$b) A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = (\$4000.00) \left(1 + \frac{0.0300}{365} \right)^{365 \times 10}$$

$$A \approx \$5399.37$$

5. a) Years to double in value = $72 \div$ (interest rate as a percent)

$$\text{Years to double in value} = 72 \div 2.80$$

$$\text{Years to double in value} \approx 26$$

It will take about 26 years for the investment to double in value.

$$b) A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = (\$1000.00) \left(1 + \frac{0.0280}{1} \right)^{1 \times 26}$$

$$A \approx \$2050.32$$

$$6. a) A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = (\$1000.00) \left(1 + \frac{0.0250}{12} \right)^{12 \times 5}$$

$$A \approx \$1133.00$$

- b) Years to double in value = $72 \div$ (interest rate as a percent)

$$\text{Years to double in value} = 72 \div 2.5$$

$$\text{Years to double in value} = 28.8$$

It will take about 28.8 years for the investment to double in value.

7. Choose a principal, such as \$1000.00, and test both investment options.

Option 1: investment at a rate of 1.90% per annum, compounded annually

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = (\$1000.00) \left(1 + \frac{0.0190}{1} \right)^{1 \times 5}$$

$$A \approx \$1098.68$$

Option 2: investment at a rate of 1.75% per annum, compounded monthly

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

$$A = (\$1000.00) \left(1 + \frac{0.0175}{12} \right)^{12 \times 5}$$

$$A \approx \$1091.37$$

The investment at 1.90% per annum compounded annually is a better investment.

8. a) $A = P \left(1 + \frac{r}{n} \right)^{nt}$

$$A = (\$5000.00) \left(1 + \frac{0.0260}{1} \right)^{1 \times 10}$$

$$A = (\$5000.00)(1.0260)^{10}$$

$$A \approx \$6463.14$$

b) $A = P \left(1 + \frac{r}{n} \right)^{nt}$

$$A = (\$5000.00) \left(1 + \frac{0.0260}{4} \right)^{4 \times 10}$$

$$A \approx \$6479.20$$

c) $A = P \left(1 + \frac{r}{n} \right)^{nt}$

$$A = (\$5000.00) \left(1 + \frac{0.0260}{12} \right)^{12 \times 10}$$

$$A \approx \$6482.83$$

d) $A = P \left(1 + \frac{r}{n} \right)^{nt}$

$$A = (\$5000.00) \left(1 + \frac{0.0260}{365} \right)^{365 \times 10}$$

$$A \approx \$6484.59$$

6.3

Credit Cards and Store Promotions

BUILD YOUR SKILLS, p. 306

1. a)
- $I = Prt$

$$I = (\$2076.54)(0.1950)(15 \div 365)$$

$$I = \$16.64$$

- b)
- $I = Prt$

$$I = (\$1007.48)(0.2150)(38 \div 365)$$

$$I = \$22.55$$

- c) The term of this investment is 18 months.
-
- In the interest calculation, divide 18 months by the number of months per year.

$$I = Prt$$

$$I = (\$2019.64)(0.1850)(18 \div 12)$$

$$I = \$560.27$$

2. Calculate the number of days for which Marcia will be charged interest. She will be charged interest for March 10–31 and April 1–2.

March 10–31 = 22 days (including March 10)

April 1–2 = 2 days

Total days = 24

$$I = Prt$$

$$I = (\$568.93)(0.2400)(24 \div 365)$$

$$I = \$8.98$$

$$A = P + I$$

$$A = \$568.93 + \$8.98$$

$$A = \$577.91$$

Marcia will owe \$577.91.

3. a) Add up her total purchases.

$$\begin{aligned} & \$124.32 + \$187.54 + \$32.42 + \$154.21 + \\ & \$54.24 + \$654.32 = \$1207.05 \end{aligned}$$

Harley will owe \$1207.05.

- b) Calculate 5% of Harley's unpaid balance.

$$\$1207.05 \times 0.05 \approx \$60.35$$

This is more than \$10.00, so her minimum payment will be \$60.35.

- c) Calculate her balance after she makes the minimum payment.

$$\$1207.05 - \$60.35 = \$1146.70$$

Calculate how many days she will be charged interest.

November 28–30 = 3 days

December 1–28 = 28 days

Total = 31 days

$$I = Prt$$

$$I = (\$1146.70)(0.1850)(31 \div 365)$$

$$I = \$18.02$$

$$A = P + I$$

$$A = \$1146.70 + \$18.02$$

$$A = \$1164.72$$

4. a) Calculate the number of days.

December 10–21 = 12 days (including December 10)

b) $I = Prt$

$$I = (\$550.00)(0.2490)(12 \div 365)$$

$$I \approx \$4.50$$

$$A = P + I$$

$$A = \$550.00 + \$4.50$$

$$A = \$554.50$$

Javier will owe \$554.50.

c) Calculate the number of days.

December 10–31 = 22 days (including
December 10)

January 1–10 = 10 days

Total = 32 days

$$I = Prt$$

$$I = (\$550.00)(0.2490)(32 \div 365)$$

$$I \approx \$12.01$$

$$A = P + I$$

$$A = \$550.00 + \$12.01$$

$$A = \$562.01$$

5. Calculate her total purchases.

$$\$28.95 + \$45.39 + \$106.15 = \$180.49$$

Calculate 5%.

$$\$180.49 \times 0.05 \approx \$9.02$$

This is less than \$10.00, so her minimum payment is \$10.00.

Calculate her unpaid balance.

$$\$180.49 - \$10.00 = \$170.49$$

Calculate the number of days she will be charged interest on this balance.

October 30–31 = 2 days

November 1–29 = 29 days

Total = 31 days

Calculate her interest charges on the unpaid balance.

$$I = Prt$$

$$I = (\$170.49)(0.2195)(31 \div 365)$$

$$I \approx \$3.18$$

Next, calculate how many days she will be charged interest on the November 12 purchase.

November 12–29 = 18 days (including
November 12)

Calculate the interest on this purchase.

$$I = Prt$$

$$I = (\$119.65)(0.2195)(18 \div 365)$$

$$I \approx \$1.30$$

Add the unpaid balance, the new purchase, and the two interest charges.

$$\$170.49 + \$119.65 + \$3.18 + \$1.30 = \$294.62$$

6. Calculate the total cost of the payment plan.

$$25 \times \$75.00 = \$1800.00$$

Calculate the difference between the cash price and the payment plan price.

$$\$1800.00 - \$1675.89 = \$124.11$$

Calculate the interest rate.

$$I = Prt$$

$$\$124.11 = \$1675.89 \times r \times 2$$

$$\$124.11 = \$3351.78r$$

$$\frac{\$124.11}{\$3351.78} = r$$

$$0.0370 \approx r$$

Multiply by 100 to convert to a percent.

$$0.0370 \times 100 = 3.70\%$$

Sol will pay an annual interest rate of 3.70%.

7. Calculate the cost of each option.

Option 1:

$$10\% \text{ down payment} = 0.10 \times \$689.98$$

$$10\% \text{ down payment} \approx \$69.00$$

Monthly payments:

$$6 \times \$115.00 = \$690.00$$

Total cost:

$$\$69.00 + \$690.00 = \$759.00$$

Option 2:

$$24 \times \$35.00 = \$840.00$$

Option 3:

$$I = Prt$$

$$I = (\$689.98)(0.2095)(20 \div 365)$$

$$I \approx \$7.92$$

$$A = P + I$$

$$A = \$689.98 + \$7.92$$

$$A = \$697.90$$

Option 3 is the best deal.

8. Calculate the total cost of the payment plan.

$$60 \times \$450.00 = \$27\,000.00$$

Calculate the difference between the cash price and the payment plan price.

$$\$27\,000.00 - \$24\,789.00 = \$2211.00$$

Calculate the annual interest rate. The term is 60 months, or 5 years.

$$I = Prt$$

$$\$2211.00 = \$24\,789.00 \times r \times 5$$

$$\$2211.00 = \$123\,945.00r$$

$$\frac{\$2211.00}{\$123\,945.00} = r$$

$$0.0178 \approx r$$

Multiply by 100 to convert to a percent.

$$0.0178 \times 100 = 1.78\%$$

Jacque is paying an annual interest rate of 1.78%.

PRACTISE YOUR NEW SKILLS, p. 313

1. a) $I = Prt$

$$I = (\$2987.69)(0.2150)(45 \div 365)$$

$$I \approx \$79.19$$

- b) $I = Prt$

$$I = (\$1539.99)(0.2095)(6 \div 12)$$

$$I \approx \$161.31$$

2. Calculate Simona's minimum payment.

$$\$1630.45 \times 0.05 \approx \$81.52$$

This is more than \$10.00, so her minimum payment is \$81.52.

Calculate her balance after she makes the minimum payment.

$$\$1630.45 - \$81.52 = \$1548.93$$

Calculate how many days she is charged interest.

June 19–30 = 12 days (including June 19)

July 1–18 = 18 days

Total = 30 days

Calculate the interest due.

$$I = Prt$$

$$I = (\$1548.93)(0.1950)(30 \div 365)$$

$$I \approx \$24.83$$

$$A = P + I$$

$$A = \$1548.93 + \$24.83$$

$$A = \$1573.76$$

3. a) Calculate Vlad's minimum payment.

$$\$398.51 \times 0.05 \approx \$19.93$$

This is more than \$10.00, so his minimum payment is \$19.93.

Calculate his balance after he made the minimum payment.

$$\$398.51 - \$19.93 = \$378.58$$

Calculate the number of days he paid interest on this amount (June 13–July 12).

June 13–30 = 18 days (including June 13)

July 1–12 = 12 days

Total = 30 days

Calculate the interest on the unpaid balance.

$$I = Prt$$

$$I = (\$378.58)(0.1850)(30 \div 365)$$

$$I \approx \$5.76$$

Calculate the number of days he is charged interest on the June 14 purchase.

June 14–30 = 17 days (including June 14)

July 1–12 = 12 days

Total = 29 days

Calculate the interest on the new purchase.

$$I = Prt$$

$$I = (\$575.54)(0.1850)(29 \div 365)$$

$$I \approx \$8.46$$

Add the unpaid balance, the new purchase, and the two interest charges.

$$\begin{aligned} & \$378.58 + \$575.54 + \$5.76 + \\ & \$8.46 = \$968.34 \end{aligned}$$

Vlad's new balance will be \$968.34.

- b) Calculate 5% of the balance.

$$\$968.34 \times 0.05 \approx \$48.42$$

This is more than \$10.00, so his minimum payment will be \$48.42.

4. a) Calculate the total cost of the living room set on the payment plan.

$$\$435.00 \times 6 = \$2610.00$$

Calculate the difference between the cash price and the payment plan price.

$$\$2610.00 - \$2543.90 = \$66.10$$

He will pay \$66.10 in interest.

- b) Calculate the interest.

$$I = Prt$$

$$I = (\$2543.90)(0.2275)(30 \div 365)$$

$$I \approx \$47.57$$

Calculate the total cost.

$$A = P + I$$

$$A = \$2543.90 + \$47.57$$

$$A = \$2591.47$$

The living room set would cost \$2591.47.

5. Calculate the total cost of each option, and the difference between the payment plan cost and the cash price.

Option 1:

$$\$220.00 \times 4 = \$880.00$$

$$\$880.00 - \$859.40 = \$20.60$$

Option 2:

$$\$150.00 \times 6 = \$900.00$$

$$\$900.00 - \$859.40 = \$40.60$$

Calculate the interest rate of each option.

Option 1:

$$I = Prt$$

$$\$20.60 = \$859.40 \times r \times \left(\frac{4}{12}\right)$$

$$\$20.60 = \$286.47r$$

$$\frac{\$20.60}{\$286.47} = r$$

$$0.0719 \approx r$$

Multiply by 100 to convert the rate to a percent.

$$0.0719 \times 100 = 7.19\%$$

Option 2:

$$I = Prt$$

$$\$40.60 = \$859.40 \times r \times \left(\frac{6}{12}\right)$$

$$\$40.60 = \$429.70r$$

$$\frac{\$40.60}{\$429.70} = r$$

$$0.0945 \approx r$$

Multiply by 100 to convert the rate to a percent.

$$0.0945 \times 100 = 9.45\%$$

Considering interest rate only, Option 1 is a better buy.

6.4

Personal Loans, Lines of Credit, and Overdrafts

BUILD YOUR SKILLS, p. 317

1. Calculate how much interest Barou paid.

$$\$275.00 - \$250.00 = \$25.00$$

Use the simple interest formula.

$$I = Prt$$

$$\$25.00 = \$250.00 \times r \times (15 \div 365)$$

$$\$25.00 = \$10.27r$$

$$\frac{\$25.00}{\$10.27} = r$$

$$2.43 \approx r$$

Multiply by 100 to convert the rate to a percent.

$$2.43 \times 100 = 243\%$$

The annual interest rate is 243%.

2. a) Calculate the amount of interest paid.

$$\$415.00 - \$400.00 = \$15.00$$

Calculate the annual interest rate.

$$I = Prt$$

$$\$15.00 = \$400.00 \times r \times (10 \div 365)$$

$$\$15.00 = \$10.96r$$

$$\frac{\$15.00}{\$10.96} = r$$

$$1.37 \approx r$$

Multiply by 100 to convert the rate to a percent.

$$1.37 \times 100 = 137\%$$

The annual interest rate is 137%.

- b) Calculate the daily interest rate.

$$I = Prt$$

$$\$15.00 = \$400.00 \times r \times 10$$

$$\$15.00 = \$4000.00r$$

$$\frac{\$15.00}{\$4000.00} = r$$

$$0.00375 = r$$

Multiply by 100 to convert the rate to a percent.

$$0.00375 \times 100 = 0.375\%$$

The daily interest rate is 0.375%.

- 3.
- $I = Prt$

$$I = (\$200.00)(3.95)(7 \div 365)$$

$$I \approx \$15.15$$

She paid \$15.15 in interest.

- 4.
- $I = Prt$

$$I = (\$500.00)(0.0112)(25)$$

$$I = \$140.00$$

$$A = P + I$$

$$A = \$500.00 + \$140.00$$

$$A = \$640.00$$

Arleta has to repay \$640.00.

5. Calculate how much interest Helen paid.

$$\$781.50 - \$750.00 = \$31.50$$

$$I = Prt$$

$$\$31.50 = (\$750.00)(.0105)t$$

$$\$31.50 = \$7.875t$$

$$\frac{\$31.50}{\$7.875} = t$$

$$4 = t$$

Helen had the money for 4 days.

6. a) $I = Prt$

$$I = (\$1000.00)(0.0050)(60)$$

$$I = \$300.00$$

$$A = P + I$$

$$A = \$1000.00 + \$300.00$$

$$A = \$1300.00$$

Hans will have to repay \$1300.00.

- b) $I = Prt$

$$\$300.00 = (\$1000.00)(r)(60 \div 365)$$

$$\$300.00 = \$164.38r$$

$$\frac{\$300.00}{\$164.38} = r$$

$$1.825 \approx r$$

Multiply by 100 to convert the rate to a percent.

$$1.825 \times 100 = 182.5 \%$$

The annual interest rate is 182.5%.

7. a) Using the Personal Loan Payment Calculator table, look up 9.00% interest in the left-hand column. In that row, look at the entry under the 2-year term: it is \$45.68.

Divide the amount of the loan (\$3000.00) by \$1000.00 and multiply by \$45.68.

$$(\$3000.00 \div \$1000.00) \times \$45.68 = \$137.04$$

The monthly payment will be \$137.04.

Calculate the total amount paid.

$$2 \text{ years} \times 12 \text{ months/year} \times \$137.04/\text{month} \\ = \$3288.96$$

Calculate the difference between the principal and the total amount paid.

$$\$3288.96 - \$3000.00 = \$288.96$$

The finance charge is \$288.96.

- b) Using the Personal Loan Payment

Calculator table, look up 7.25% interest in the left-hand column. In that row, look at the entry under the 3-year term: it is \$30.99.

Divide the amount of the loan (\$2125.00) by \$1000.00 and multiply by \$30.99.

$$(\$2125.00 \div \$1000.00) \times \$30.99 \approx \$65.85$$

The monthly payment will be \$65.85.

Calculate the total amount paid.

$$3 \text{ years} \times 12 \text{ months/year} \times \$65.85/\text{month} \\ = \$2370.60$$

Calculate the difference between the principal and the total amount paid.

$$\$2370.60 - \$2125.00 = \$245.60$$

The finance charge is \$245.60.

- c) Using the Personal Loan Payment Calculator table, look up 4.75% interest in the left-hand column. In that row, look at the entry under the 4-year term: it is \$22.92.

Divide the amount of the loan (\$11 500.00) by \$1000.00 and multiply by \$22.92.

$$(\$11\,500.00 \div \$1000.00) \times \$22.92 = \$263.58$$

The monthly payment will be \$263.58.

Calculate the total amount paid.

$$4 \text{ years} \times 12 \text{ months/year} \times \$263.58/\text{month} \\ = \$12\,651.84$$

Calculate the difference between the principal and the total amount paid.

$$\$12\,651.84 - \$11\,500.00 = \$1151.84$$

The finance charge is \$1151.84.

8. a) Calculate how much Adele will have to borrow.

$$\$2900.00 - \$1100.00 = \$1800.00$$

- b) Using the Personal Loan Payment Calculator table, look up 6.50% interest in the left-hand column. In that row, look at the entry under the 2-year term: it is \$44.55.

Divide the amount of the loan (\$1800.00) by \$1000.00 and multiply by \$44.55.

$$(\$1800.00 \div \$1000.00) \times \$44.55 = \$80.19$$

The monthly payment will be \$80.19.

- c) Multiply the monthly payment by the amortization period.

$$2 \text{ years} \times 12 \text{ months/year} \times \$80.19/\text{month} \\ = \$1924.56$$

- d) Add the cost of the loan to the amount of her down payment.

$$\$1100.00 + \$1924.56 = \$3024.56$$

The car will cost \$3024.56.

PRACTISE YOUR NEW SKILLS, p. 322

1. a) Calculate how much interest She paid.

$$\$950.00 - \$850.00 = \$100.00$$

The term of the loan is 12 days.

$$I = Prt$$

$$\$100.00 = (\$850.00)(r)(12)$$

$$\$100.00 = \$10\,200.00r$$

$$\frac{\$100.00}{\$10\,200.00} = r$$

$$0.0098 \approx r$$

Multiply by 100 to convert the rate to a percent.

$$0.0098 \times 100 = 0.98\%$$

The daily interest rate is 0.98%.

- b) To calculate the annual interest rate, the term needs to be converted to years. The term will be 12 divided by 365.

$$I = Prt$$

$$\$100.00 = (\$850.00)(r)(12 \div 365)$$

$$\$100.00 = \$27.95r$$

$$\frac{\$100.00}{\$27.95} = r$$

$$3.58 \approx r$$

Multiply by 100 to convert the rate to a percent.

$$3.58 \times 100 = 358\%$$

The annual interest rate is 358%.

2. The interest rate is given as a daily rate, so the term of the investment is also in days.

$$I = Prt$$

$$I = (\$250.00)(0.0117)(18)$$

$$I = \$52.65$$

$$A = P + I$$

$$A = \$250.00 + \$52.65$$

$$A = \$302.65$$

Carmen will have to repay \$302.65.

3. a) Using the Personal Loan Payment Calculator table, look up 8.00% interest in the left-hand column. In that row, look at the entry under the 3-year term: it is \$31.34.

Divide the amount of the loan (\$2500.00) by \$1000.00 and multiply by \$31.34.

$$(\$2500.00 \div \$1000.00) \times \$31.34 = \$78.35$$

The monthly payment will be \$78.35.

Calculate the total amount paid.

$$3 \text{ years} \times 12 \text{ months/year} \times \$78.35/\text{month} \\ = \$2820.60$$

Calculate the difference between the principal and the total amount paid.

$$\$2820.60 - \$2500.00 = \$320.60$$

The finance charge is \$320.60.

- b) Using the Personal Loan Payment Calculator table, look up 6.25% interest in the left-hand column. In that row, look at the entry under the 5-year term: it is \$19.45.

Divide the amount of the loan (\$10 000.00) by \$1000.00 and multiply by \$19.45.

$$(\$10\,000.00 \div \$1000.00) \times \\ \$19.45 = \$194.50$$

The monthly payment will be \$194.50.

Calculate the total amount paid.

$$5 \text{ years} \times 12 \text{ months/year} \times \$194.50/\text{month} \\ = \$11\,670.00$$

Calculate the difference between the principal and the total amount paid.

$$\$11\,670.00 - \$10\,000.00 = \$1670.00$$

The finance charge is \$1670.00.

- c) Using the Personal Loan Payment Calculator table, look up 3.75% interest in the left-hand column. In that row, look at the entry under the 2-year term: it is \$43.31.

Divide the amount of the loan (\$1500.00) by \$1000.00 and multiply by \$43.31.

$$(\$1500.00 \div \$1000.00) \times \$43.31 \approx \$64.97$$

The monthly payment will be \$64.97.

Calculate the total amount paid.

$$2 \text{ years} \times 12 \text{ months/year} \times \$64.97/\text{month} \\ = \$1559.28$$

Calculate the difference between the principal and the total amount paid.

$$\$1559.28 - \$1500.00 = \$59.28$$

The finance charge is \$59.28.

4. a) Using the Personal Loan Payment

Calculator table, look up 7.00% interest in the left-hand column. In that row, look at the entry under the 2-year term: it is \$44.77.

Divide the amount of the loan (\$5000.00) by \$1000.00 and multiply by \$44.77.

$$(\$5000.00 \div \$1000.00) \times \$44.77 \approx \$223.85$$

The monthly payment will be \$223.85.

- b) Calculate the total amount paid.

$$2 \text{ years} \times 12 \text{ months/year} \times \$223.85/\text{month} = \$5372.40$$

- c) Calculate the difference between the principal and the total amount paid.

$$\$5372.40 - \$5000.00 = \$372.40$$

The finance charge is \$372.40.

5. a) Calculate how much money Manon would have to borrow from the bank.

$$\$3499.99 - \$1000.00 = \$2499.99$$

Using the Personal Loan Payment

Calculator table, look up 6.50% interest in the left-hand column. In that row, look at the entry under the 2-year term: it is \$44.55.

Divide the amount of the loan (\$2499.99) by \$1000.00 and multiply by \$44.55.

$$(\$2499.99 \div \$1000.00) \times \$44.55 \approx \$111.37$$

The monthly payment will be \$111.37.

- b) Calculate the cost of each option.

Option 1:

$$2 \text{ years} \times 12 \text{ months/year} \times \$111.37/\text{month} = \$2672.88$$

Add the down payment plus the cost of the loan.

$$\$1000.00 + \$2672.88 = \$3672.88$$

Option 1 costs \$3672.88.

Option 2:

$$\$50.00 + (\$325.00 \times 12) = \$3950.00$$

Option 2 costs \$3950.00.

Option 3:

$$I = Prt$$

$$I = (\$2499.99)(0.0112)(30)$$

$$I \approx \$840.00$$

$$A = P + I$$

$$A = \$2499.99 + \$840.00$$

$$A = \$3339.99$$

Add the down payment plus the cost of the loan.

$$\$1000.00 + \$3339.99 = \$4339.99$$

Option 3 costs \$4339.99.

She should choose Option 1.

CHAPTER TEST, p. 325

1. During the month, Salma made 10 self-service withdrawals and 3 deposits, so she will not be charged any transaction fees.

<i>Transaction</i>	<i>Description</i>	<i>Withdrawal</i>	<i>Deposit</i>	<i>Balance</i>
				\$2879.54
ATM	Cash	\$200.00		$\$2879.54 - \$200.00 = \$2679.54$
Direct deposit	Paycheque		\$457.21	$\$2679.54 + \$457.21 = \$3136.75$
Bank card	Groceries	\$172.12		$\$3136.75 - \$172.12 = \$2964.63$
Bank card	Gas	\$42.54		$\$2964.63 - \$42.54 = \$2922.09$
ATM	Cheque—reimbursement		\$175.64	$\$2922.09 + \$175.64 = \$3097.73$
Bank card	Dinner	\$32.42		$\$3097.73 - \$32.42 = \$3065.31$
ATM	Cash	\$100.00		$\$3065.31 - \$100.00 = \$2965.31$
Auto-withdrawal	Hydro	\$112.21		$\$2965.31 - \$112.21 = \$2853.10$
Direct deposit	Paycheque		\$457.21	$\$2853.10 + \$457.21 = \$3310.31$
Auto-withdrawal	Rent	\$645.00		$\$3310.31 - \$645.00 = \$2665.31$
Bank card	Car repairs	\$276.97		$\$2665.31 - \$276.97 = \$2388.34$
Bank card	Movie	\$28.12		$\$2388.34 - \$28.12 = \$2360.22$
ATM	Cash	\$200.00		$\$2360.22 - \$200.00 = \$2160.22$

Salma maintains a minimum balance of \$2000.00, so she does not have to pay the \$6.00 account fee.

2. a) $I = Prt$
 $I = (\$5000.00)(0.0250)(10)$
 $I = \$1250.00$

b) $A = P + I$
 $A = \$5000.00 + \1250.00
 $A = \$6250.00$

3. $I = Prt$
 $\$82.50 = P \times 0.0110 \times 5$
 $\$82.50 = 0.0550P$
 $\frac{\$82.50}{0.0550} = P$
 $\$1500.00 = P$
 The principal was \$1500.00.

4. $A = P \left(1 + \frac{r}{n}\right)^{nt}$
 $A = (\$5000.00) \left(1 + \frac{0.0250}{1}\right)^{1 \times 10}$

$A = (\$5000.00)(1.0250)^{10}$
 $A \approx \$6400.42$

5. $A = P \left(1 + \frac{r}{n}\right)^{nt}$
 $A = (\$1000.00) \left(1 + \frac{0.0300}{12}\right)^{12 \times 10}$

$A \approx \$1349.35$
 $I = A - P$
 $I = \$1349.35 - \1000.00
 $I = \$349.35$

The investment earned \$349.35 in interest.

6. a) $I = Prt$

$$I = (\$1629.53)(0.1950)(21 \div 365)$$

$$I \approx \$18.28$$

b) $I = Prt$

$$I = (\$2639.99)(0.2295)(30 \div 365)$$

$$I \approx \$49.80$$

7. a) Calculate Monique's minimum payment.

$$\$739.65 \times 0.05 \approx \$36.98$$

This is more than \$10.00, so her minimum payment is \$36.98.

Calculate her new unpaid balance.

$$\$739.65 - \$36.98 = \$702.67$$

Calculate how many days she will pay interest on this amount.

$$\text{March } 13\text{--}31 = 19 \text{ days}$$

(including March 13)

$$\text{April } 1\text{--}12 = 12 \text{ days}$$

$$\text{Total} = 31 \text{ days}$$

Calculate the interest on the unpaid balance.

$$I = Prt$$

$$I = (\$702.67)(0.2085)(31 \div 365)$$

$$I \approx \$12.44$$

Calculate the number of days interest is charged on the March 16 purchase.

$$\text{March } 16\text{--}31 = 16 \text{ days}$$

(including March 16)

$$\text{April } 1\text{--}12 = 12 \text{ days}$$

$$\text{Total} = 28 \text{ days}$$

Calculate the interest on the new purchase.

$$I = Prt$$

$$I = (\$179.39)(0.2085)(28 \div 365)$$

$$I \approx \$2.87$$

Add the unpaid balance, the new purchase, and the two interest amounts.

$$\$702.67 + \$179.39 + \$12.44 +$$

$$\$2.87 = \$897.37$$

The new balance on her April 12 statement will be \$897.37.

b) Calculate 5% of the new balance.

$$\$897.37 \times 0.05 \approx \$44.87$$

This is more than \$10.00, so her minimum payment will be \$44.87.

8. a) Calculate the amount of interest paid.

$$\$540.00 - \$500.00 = \$40.00$$

Calculate the daily interest rate, using 7 days as the term.

$$I = Prt$$

$$\$40.00 = (\$500.00)(r)(7)$$

$$\$40.00 = \$3500.00r$$

$$\frac{\$40.00}{\$3500.00} = r$$

$$0.0114 \approx r$$

Multiply by 100 to convert the rate to a percent.

$$0.0114 \times 100 = 1.14\%$$

The daily interest rate is 1.14%.

b) Calculate the annual interest rate, using 7 divided by 365 as the term.

$$I = Prt$$

$$\$40.00 = (\$500.00)(r)(7 \div 365)$$

$$\$40.00 = \$9.589r$$

$$\frac{\$40.00}{\$9.589} = r$$

$$4.17 \approx r$$

The annual interest rate is 417%.

9. a) Calculate how much money Nathaniel needs to borrow.

$$\$2055.99 - \$400.00 = \$1655.99$$

Using the Personal Loan Payment

Calculator table, look up 7.25% interest in the left-hand column. In that row, look at the entry under the 2-year term: it is \$44.89.

Divide the amount of the loan (\$1655.99) by \$1000.00 and multiply by \$44.89.

$$(\$1655.99 \div \$1000.00) \times \$44.89 \approx \$74.34$$

The monthly payment will be \$74.34.

- b) Calculate the cost of the bicycle on the payment plan.

$$12 \times \$180.00 = \$2160.00$$

Calculate the interest paid by subtracting the cash price from the payment plan price.

$$\$2160.00 - \$2055.99 = \$104.01$$

Calculate the annual interest rate.

$$I = Prt$$

$$\$104.01 = (\$2055.99)(r)(1)$$

$$\$104.01 = \$2055.99r$$

$$\frac{\$104.01}{\$2055.99} = r$$

$$0.0506 \approx r$$

Multiply by 100 to convert the rate to a percent.

$$0.0506 \times 100 = 5.06\%$$

The annual interest rate is 5.06%.

- c) Option 1:

Multiply the monthly payment by the term.

$$2 \text{ years} \times 12 \text{ months/year} \times \$74.34/\text{month} \\ = \$1784.16$$

Add the down payment.

$$\$1784.16 + \$400.00 = \$2184.16$$

The total cost of Option 1 is \$2184.16.

The total cost of Option 2 was calculated in b) above. It is \$2160.00.

Nathaniel should choose Option 2 because it is cheaper.

Chapter 7

Personal Budgets

7.1

Preparing to Make a Budget

BUILD YOUR SKILLS, p. 331

1. Answers will vary, particularly for the reason behind the classification. Possible solutions are provided.

<i>Item</i>	<i>Classification</i>	<i>Reason</i>
Semi-monthly paycheque	Regular	This paycheque will happen twice a month.
Birthday gift	Variable	You don't know what you'll get for your birthday and it only happens once a year.
Tips	Variable	You don't know how many tips you'll get in a shift.
Interest from investment	Regular	Your investments will have regular interest payments.
Tax refund	Variable	You don't know how much tax you'll pay or get refunded each year until you do your taxes.

2. Answers will vary, particularly for the reason behind the classification. Possible solutions are provided.

<i>Item</i>	<i>Classification</i>	<i>Reason</i>
Rent	Recurring	You will have to pay the same rent every month.
New shoes	Variable	You don't need to buy new shoes often enough to plan on this.
Loan payments	Recurring	You will have an agreement with whoever issued the loan on a repayment schedule.
Car repairs	Unexpected	You can't plan when your car will break down.
Groceries	Variable	You don't know how much you'll spend on groceries each trip, but you do have to do it regularly.
Meal at restaurant	Variable	You don't know how much you'll spend every time you go for dinner, but you can budget for it.
Replacing broken cell phone	Unexpected	You can't plan when you break your cell phone.
Car insurance	Recurring	You will have an agreed rate of car insurance with your insurance broker.

<i>Item</i>	<i>Income</i>	<i>Expense</i>
Paycheque: \$250.98	Regular	
John's birthday: \$54.25		Variable
Commission: \$75.00	Variable	
Babysitting: \$40.00	Variable	
Paycheque: \$123.42	Regular	
Loan payment: \$125.00		Regular
Donation to earthquake relief fund: \$25.00		Variable
Car insurance: \$115.32		Regular
Savings: \$50.00		Variable
Rent: \$450.00		Regular
Cell phone bill: \$45.00		Regular
Meal at restaurant: \$56.76		Variable

4. Calculate Tonia's total income.

$$4 \times \$450.00 = \$1800.00$$

Calculate her total expenses, including the car repair.

$$\begin{aligned} & \$775.00 + \$225.39 + \$74.00 + \$66.79 + \$47.59 \\ & + \$84.00 + \$250.00 + \$243.25 = \$1766.02 \end{aligned}$$

Calculate the difference between her income and expenses.

$$\$1800.00 - \$1766.02 = \$33.98$$

Tonia will save \$33.98 this month.

5. Calculate Franklin's total expenses.

$$\begin{aligned} & \$1250.46 + \$250.00 + \$135.76 + \$50.00 + \\ & \$245.00 + \$250.00 + \$100.00 = \$2281.22 \end{aligned}$$

Calculate the difference between his income and expenses.

$$\$2456.85 - \$2281.22 = \$175.63$$

Franklin will save \$175.63. Multiply by 12 to find out how much he will save in one year.

$$\$175.63 \times 12 = \$2107.56$$

In one year, he will save \$2107.56.

Calculate what percentage of his income his savings represent.

$$\frac{\$175.63}{\$2456.85} \times 100 \approx 7.1\%$$

Franklin's savings represent about 7.1% of his income.

6. Calculate Marion's total income.

$$\$1037.72 \times 2 = \$2075.44$$

Calculate her total expenses.

$$\begin{aligned} & \$825.00 + \$110.00 + \$100.00 + \$150.00 + \\ & \$25.00 + \$225.00 + \$275.00 + \$100.00 + \\ & \$25.00 + \$50.00 = \$1885.00 \end{aligned}$$

Calculate the difference between her income and his expenses.

$$\$2075.44 - \$1885.00 = \$190.44$$

She saves \$190.44 in one month. Calculate how much she will have saved after 3 months.

$$\$190.44 \times 3 = \$571.32$$

No, she will not be able to pay cash for the \$1399.99 TV. She will have \$571.32 saved.

7. Calculate how much Aidan saves per month.

$$\$1588.25 - \$1275.00 = \$313.25$$

Divide the cost of the computer by the amount he saves per month.

$$\$1798.98 \div \$313.25 \approx 5.7 \text{ months}$$

It will take Aidan about 6 months to save for the new computer.

8. Calculate Karen's monthly expenses.

$$\begin{aligned} & \$775.00 + \$175.00 + \$342.00 + \$123.00 + \\ & \$42.00 + \$225.00 + \$90.00 + \$75.00 + \$50.00 \\ & = \$1897.00 \end{aligned}$$

Calculate how much Karen saves in one month.

$$\$2379.00 - \$1897.00 = \$482.00$$

She saves \$482.00 per month. Calculate how much money Karen will save from January to the beginning of July (or the end of June, so 6 months).

$$\$482.00 \times 6 = \$2892.00$$

Karen will not have enough money saved for her trip.

$$\$3000.00 - \$2892.00 = \$108.00$$

She will be \$108.00 short of her goal.

Calculate her total expenses.

$$\begin{aligned} & \$395.00 + \$85.00 + \$185.00 + \$75.00 \\ & + \$170.00 + \$50.00 + \$25.00 + \$40.00 \\ & = \$1025.00 \end{aligned}$$

Calculate how much she will save each month.

$$\$1240.00 - \$1025.00 = \$215.00$$

Hannah will save \$215.00 per month.

- b) Calculate what percentage of her income her savings represent.

$$\frac{\$215.00}{\$1240.00} \times 100 \approx 17\%$$

Hannah's savings represent about 17% of her income.

- c) Calculate how much she will save in 6 months.

$$\$215.00 \times 6 = \$1290.00$$

Hannah will have enough money saved for the bicycle. She will have \$1290.00 saved.

3. Calculate how much money Perseus has saved in the past 8 months.

$$8 \times \$125.00 = \$1000.00$$

Calculate how much more money he needs to save to be able to afford the trip.

$$\$1900.00 - \$1000.00 = \$900.00$$

Divide the remaining cost of the trip by 6 to find out how much Perseus has to save per month.

$$\$900.00 \div 6 \approx \$150.00$$

PRACTISE YOUR NEW SKILLS, p. 338

1.

<i>Item</i>	<i>Income</i>	<i>Expense</i>
Paycheque	Regular	
Rent		Regular
Car insurance		Regular
Gas		Variable
Clothing		Variable
Cell phone bill		Regular
Mother's Day gift for mom		Variable
Groceries		Variable
Present for Joan		Variable
Entertainment		Variable
Mowing lawns	Variable	
Utilities		Variable

2. a) Calculate Hannah's total income.

$$\begin{aligned} & \$400.00 + \$140.00 + \$150.00 + \$150.00 + \\ & \$400.00 = \$1240.00 \end{aligned}$$

He has been savings \$125.00 a month, so he will need to save an additional \$25.00 a month to be able to afford the trip in 6 months' time.

4. a)

<i>Income</i>		<i>Expenses</i>	
Babysitting	\$40.00	Cell phone	\$40.00
Babysitting	\$40.00	Entertainment	\$105.00
Paycheque	\$375.00	Food	\$195.00
Paycheque	\$375.00	Loan payment	\$65.00
Tutoring	\$50.00	Rent	\$425.00
Tutoring	\$100.00	Renter's insurance	\$25.00
Tutoring	\$50.00	Transportation	\$80.00

b) Calculate Shonda's total income.

$$\$40.00 + \$40.00 + \$375.00 + \$375.00 + \$50.00 + \$100.00 + \$50.00 = \$1030.00$$

Calculate her total expenses.

$$\$40.00 + \$105.00 + \$195.00 + \$65.00 + \$425.00 + \$25.00 + \$80.00 = \$935.00$$

Shona's total income is \$1030.00 and her expenses are \$935.00.

c) Calculate the difference between her income and expenses.

$$\$1030.00 - \$935.00 = \$95.00$$

She can save \$95.00 in one month.

Multiply to find out how much she can save in one year.

$$\$95.00 \times 12 = \$1140.00$$

In one year, Shonda can save \$1140.00, assuming she continues to earn approximately the same income and spend the same amount on her various expenses.

d) Calculate what percentage \$95.00 is of her monthly income (\$1030.00).

$$\frac{\$95.00}{\$1030.00} \times 100 \approx 9\%$$

Shonda's savings represent about 9% of her income.

7.2

The Budgeting Process

BUILD YOUR SKILLS, p. 345

1. a) Answers will vary. A possible budget is provided.

CRAIG AND STÉFANIE'S MONTHLY BUDGET			
<i>Income</i>		<i>Expenses</i>	
Craig's monthly pay	\$2800.00	Mortgage payment	\$1850.00
Stéfanie's monthly pay	\$2770.00	Utilities	\$175.00
		Cat care	\$50.00
		Clothing	\$150.00
		Food	\$400.00
		Gas	\$150.00
		Car insurance	\$112.00
		Gifts	\$150.00
		Entertainment	\$120.00
		Loan payment	\$150.00
		Home insurance	\$122.00
		Trip fund	\$250.00
		Cell phones	\$135.00
		Miscellaneous	\$475.00
		Savings	\$1281.00
<i>Total income</i>	\$5570.00	<i>Total expenses</i>	\$5570.00

- b) Answers will vary, depending on the budget created. In the budget provided above, Craig and Stéfanie save \$1281.00 per month. Multiply by 12 to calculate how much they will save in one year.

$$\$1281.00 \times 12 = \$15\,372.00$$

Based on the budget provided, they would save \$15 372.00 in one year.

2. a) Answers will vary. A possible budget is provided.

MINH'S MONTHLY BUDGET			
<i>Income</i>		<i>Expenses</i>	
Regular	\$1665.00	Rent	\$640.00
Tips	\$300.00	Utilities, phone, cable, internet	\$115.00
		Food	\$300.00
		Transportation	\$110.00
		Entertainment	\$210.00
		Other	\$500.00
		Savings	\$90.00
<i>Total income</i>	\$1965.00	<i>Total expenses</i>	\$1965.00

- b) Answers will vary, depending on the budget created. In the budget above, Minh saves \$90.00 a month. Calculate how much he will save in 6 months.

$$\$90.00 \times 6 = \$540.00$$

Calculate the difference between the cost of the skis and Minh's savings.

$$\$1059.99 - \$540.00 = \$519.99$$

Based on this budget, Minh will not be able to afford the skis. He will be \$519.99 short.

3. a) Answers will vary. A possible budget is provided.

GEORGINA'S MONTHLY BUDGET			
<i>Income</i>		<i>Expenses</i>	
Monthly pay	\$3050.00	Mortgage payment	\$1025.00
		Car insurance	\$122.00
		House repairs	\$70.00
		House taxes	\$350.00
		Gifts	\$45.00
		Charity	\$30.00
		Gas	\$125.00
		Loan payment	\$220.00
		Miscellaneous	\$160.00
		Entertainment	\$160.00
		Home insurance	\$125.00
		Clothing	\$130.00
		Car maintenance	\$50.00
		Food	\$375.00
		Skiing season's pass	\$55.00
		Savings	\$8.00
<i>Total income</i>	\$3050.00	<i>Total expenses</i>	\$3050.00

- b) Answers will vary, depending on the budget created. Based on this budget, Georgina will save only \$8.00 a month. Multiply by 12 to find out how much she will save in one year.

$$\$8.00 \times 12 = \$96.00$$

She will save only \$96.00 in one year.

- c) Answers will vary, depending on the budget created. Based on this budget, Georgina saves \$8.00 per month and earns \$3050.00.

$$\frac{\$8.00}{\$3050.00} \times 100 \approx 0.3\%$$

Georgina saves only 0.3% of her income.

- d) Answers will vary. Options include:

- decreasing her spending on entertainment, clothing, or miscellaneous expenses;
- not buying a season's pass to the ski hill;
- selling her car;
- finding a less expensive place to live.

4. a) Calculate how much money Janae earns in one week in wages.

$$40 \times \$10.75 = \$430.00$$

Calculate how much she would earn in one month, including tips.

$$(\$430.00 \times 4) + \$500.00 = \$2220.00$$

Calculate 80% of her income.

$$\$2220.00 \times 0.8 = \$1776.00$$

Calculate how much she saves per month.

$$\$2220.00 - \$1776.00 = \$444.00$$

- b) Calculate Janae's monthly food expenses.

$$\$75.00 \times 4 = \$300.00$$

Calculate what percentage of her monthly income this represents. Janae's monthly income is \$2220.00.

$$\frac{\$300.00}{\$2220.00} \times 100 \approx 13.5\%$$

Janae spends about 13.5% of her income on food.

5. a) Calculate 12% of \$2700.00.

$$\$2700.00 \times 0.12 = \$324.00$$

Justin will save \$324.00 a month.

- b) Divide \$1500.00 by the amount he saves per month.

$$\$1500.00 \div \$324.00 \approx 4.6$$

It will take Justin about 5 months to save \$1500.00.

6. Calculate how much money Veejay saves per month.

$$\$1725.00 \times 0.55 = \$948.75$$

Calculate how much he needs to save for tuition plus books.

$$\$6000.00 + \$550.00 = \$6550.00$$

Divide \$6550.00 by the amount he saves per month.

$$\$6550.00 \div \$948.75 \approx 6.9$$

It will take Veejay about 7 months to save up for college tuition and books.

PRACTISE YOUR NEW SKILLS, p. 351

1. a) Answers will vary. A possible budget is provided.

PIERRE'S MONTHLY BUDGET			
Income		Expenses	
Semi-monthly pay	\$1475.00	Mortgage payment	\$1450.00
Semi-monthly pay	\$1475.00	Utilities/phone	\$150.00
		Groceries	\$325.00
		Loan repayment	\$250.00
		Charity	\$40.00
		Entertainment	\$150.00
		Clothing	\$150.00
		Transportation	\$220.00
		Miscellaneous	\$80.00
		Savings	\$135.00
<i>Total income</i>	\$2950.00	<i>Total expenses</i>	\$2950.00

- b) Answers will vary, depending on the budget created. Based on the budget provided above, Pierre will save \$135.00 a month. Calculate how much he will save in one year.

$$\$135.00 \times 12 = \$1620.00$$

Pierre will save \$1620.00 in one year.

2. a) Answers will vary. A possible budget is provided.

CHANTAL'S MONTHLY BUDGET			
Income		Expenses	
Semi-monthly pay	\$1110.00	Rent	\$675.00
Semi-monthly pay	\$110.00	Utilities, phone, cable, internet	\$150.00
Tips	\$135.00	Food	\$400.00
		Transportation	\$300.00
		Entertainment	\$150.00
		Other	\$375.00
		Savings	\$305.00
<i>Total income</i>	\$2355.00	<i>Total expenses</i>	\$2355.00

- b) Answers will vary, depending on the budget created. Based on this budget, Chantal saves \$305.00 per month. Calculate what percentage of her income this represents.

$$\frac{\$305.00}{\$2355.00} \times 100 \approx 13\%$$

Based on this budget, Chantal saves 13% of her income.

- c) Answers will vary, depending on the budget created. Based on this budget, Chantal saves \$305.00 per month. Divide the cost of the computer by this amount.

$$\$1000.00 \div \$305.00 \approx 3.3$$

It will take Chantal about 3.3 months to save for her computer. Because she will not have enough money saved at the beginning of the third month, you should round the answer up. It will take her 4 months to save for the computer.

3. a) Calculate Marcel's monthly income from regular pay, assuming there are 4 weeks in a month.

$$(40 \text{ h/wk} \times \$14.50/\text{h} \times 4 \text{ wk}) = \$2320.00$$

Calculate his monthly income from overtime pay.

$$6 \text{ h} \times (1.5 \times \$14.50/\text{h}) = \$130.50$$

Calculate his total monthly income.

$$\$2320.00 + \$130.50 = \$2450.50$$

Calculate his total expenses, excluding rent.

$$\$125.00 + \$250.00 + \$165.00 + \$300.00 + \$350.00 + \$400.00 = \$1590.00$$

Calculate how much he can spend on rent.

$$\$2450.50 - \$1590.00 = \$860.50$$

Marcel can spend a maximum of \$860.50 per month on rent.

- b) Divide the cost of the trip by the amount he saves per month.

$$\$2700.00 \div \$300.00 = 9 \text{ months}$$

It will take Marcel 9 months to save up for the trip to Ecuador.

7.3

Analyzing a Budget

BUILD YOUR SKILLS, p. 355

1. a) Calculate how much money Sukh saves per month.

$$\$2865.00 \times 0.15 = \$429.75$$

Divide the amount Sukh needs to save (\$6000.00) by the amount he saves per month.

$$\$6000.00 \div \$429.75 \approx 14$$

It will take Sukh 14 months to save enough money for the college course.

- b) Calculate how much money Sukh will have to save per month by dividing the amount needed (\$6000.00) by 12 months.

$$\$6000.00 \div 12 = \$500.00$$

Calculate what percentage \$500.00 is of his income (\$2865.00).

$$\frac{\$500.00}{\$2865.00} \times 100 \approx 17.5\%$$

Sukh will need to save about 17.5% of his income in order to be able to afford the course in 12 months.

2. a) Calculate Juliet's income.

$$\frac{\$275.00}{x} \times 100 = 12\%$$

$$\frac{\$275.00}{x} = \frac{12}{100}$$

$$\frac{\$275.00}{x} = 0.12$$

$$\$275.00 = 0.12x$$

$$\frac{\$275.00}{0.12} = x$$

$$\$2291.67 \approx x$$

Calculate 8% of her income.

$$\frac{x}{\$2291.67} \times 100 = 8\%$$

$$\frac{x}{\$2291.67} = \frac{8}{100}$$

$$\frac{x}{\$2291.67} = 0.08$$

$$x = \$2291.67 \times 0.08$$

$$x \approx \$183.33$$

Juliet's new monthly entertainment budget is \$183.33.

- b) $\frac{\$275.00}{x} \times 100 = 12\%$

$$\frac{\$275.00}{x} = \frac{12}{100}$$

$$\frac{\$275.00}{x} = 0.12$$

$$\$275.00 = 0.12x$$

$$\frac{\$275.00}{0.12} = x$$

$$\$2291.67 \approx x$$

Juliet's total monthly income is \$2291.67.

3. a) Answers will vary. For example, Petra could decrease her spending by a total of \$100.00 on a combination of expenses, such as food, miscellaneous, and clothing. She could decrease her food expenses to \$275.00, miscellaneous to \$75.00, and clothing to \$75.00. These expenses are all variable, and Petra can choose to buy less expensive items within these categories.
- b) Answers will vary, depending on the suggestions made in a). Based on the suggestions above, the new budget would be as follows.

PETRA'S MONTHLY BUDGET			
Income		Expenses	
Regular	\$1400.00	Housing	\$550.00
		Utilities, phone, cable, internet	\$175.00
		Food	\$275.00
		Transportation	\$75.00
		Entertainment	\$50.00
		Clothing	\$75.00
		Miscellaneous	\$75.00
		Charitable donations	\$25.00
		Savings	\$100.00
Total income	\$1400.00	Total expenses	\$1400.00

To construct a circle graph of the expenses, first calculate what percentage of Petra's income each category represents.

$$\text{Housing: } (\$550.00 \div \$1400.00) \times 100 \approx 39\%$$

$$\text{Utilities, etc.: } (\$175.00 \div \$1400.00) \times 100 \approx 13\%$$

$$\text{Food: } (\$275.00 \div \$1400.00) \times 100 \approx 20\%$$

$$\text{Transportation: } (\$75.00 \div \$1400.00) \times 100 \approx 5\%$$

$$\text{Entertainment: } (\$50.00 \div \$1400.00) \times 100 \approx 4\%$$

$$\text{Clothing: } (\$75.00 \div \$1400.00) \times 100 \approx 5\%$$

$$\text{Miscellaneous: } (\$75.00 \div \$1400.00) \times 100 \approx 5\%$$

$$\text{Charitable donations: } (\$25.00 \div \$1400.00) \times 100 \approx 2\%$$

$$\text{Savings: } (\$100.00 \div \$1400.00) \times 100 \approx 7\%$$

Next, calculate how many degrees each percentage of expenses represents.

$$\text{Housing: } 0.39 \times 360^\circ \approx 140^\circ$$

$$\text{Utilities, etc.: } 0.13 \times 360^\circ \approx 47^\circ$$

$$\text{Food: } 0.20 \times 360^\circ = 72^\circ$$

$$\text{Transportation: } 0.05 \times 360^\circ = 18^\circ$$

$$\text{Entertainment: } 0.04 \times 360^\circ \approx 14^\circ$$

$$\text{Clothing: } 0.05 \times 360^\circ = 18^\circ$$

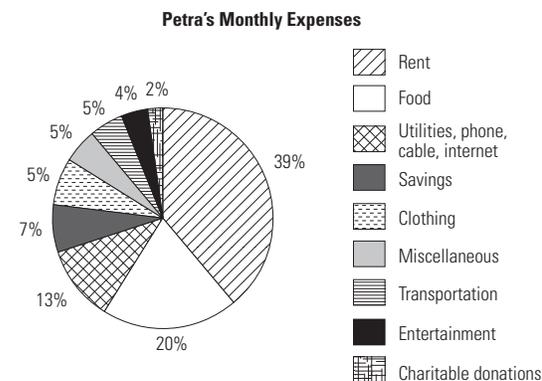
$$\text{Miscellaneous: } 0.05 \times 360^\circ = 18^\circ$$

$$\text{Charitable donations: } 0.02 \times 360^\circ \approx 7^\circ$$

$$\text{Savings: } 0.07 \times 360^\circ \approx 25^\circ$$

(Note that the sections add up to 359° rather than 360° ; this is due to rounding. To construct the graph, you can increase the size of one of the sections by 1° .)

Draw a circle and use a protractor to measure the degrees for each category of expense.



4. a) Calculate what percentage of his income Sammy spends on each category.

$$\text{Housing: } (\$400.00 \div \$1500.00) \times 100 \approx 27\%$$

$$\text{Food: } (\$225.00 \div \$1500.00) \times 100 = 15\%$$

$$\begin{aligned} \text{Transportation: } & (\$350.00 \div \$1500.00) \\ & \times 100 \approx 23\% \end{aligned}$$

$$\begin{aligned} \text{Entertainment: } & (\$250.00 \div \$1500.00) \\ & \times 100 \approx 17\% \end{aligned}$$

$$\begin{aligned} \text{Charitable donations: } & (\$25.00 \div \$1500.00) \\ & \times 100 \approx 2\% \end{aligned}$$

$$\text{Savings: } (\$50.00 \div \$1500.00) \times 100 \approx 3\%$$

$$\text{Other: } (\$200.00 \div \$1500.00) \times 100 \approx 13\%$$

Sammy's spending falls within the guidelines for housing, food, and charitable donations (but for this last category, just barely). He is currently spending too much on transportation, entertainment, and other. He is currently not putting enough money into savings.

- b) Multiply Sammy's monthly savings by 12 months.

$$\$50.00 \times 12 = \$600.00$$

Sammy will save \$600.00 in one year.

- c) Answers will vary. A possible budget is provided.

SAMMY'S MONTHLY BUDGET			
Income		Expenses	
Pay	\$1500.00	Housing	\$400.00
		Food	\$275.00
		Transportation	\$200.00
		Entertainment	\$150.00
		Charitable donations	\$125.00
		Savings	\$150.00
		Other	\$200.00
<i>Total income</i>	\$1500.00	<i>Total expenses</i>	\$1500.00

To construct a circle graph of the expenses, first calculate what percentage of Sammy's income each category represents.

$$\text{Housing: } (\$400.00 \div \$1500.00) \times 100 \approx 27\%$$

$$\text{Food: } (\$275.00 \div \$1500.00) \times 100 \approx 18\%$$

$$\begin{aligned} \text{Transportation: } & (\$200.00 \div \$1500.00) \\ & \times 100 \approx 13\% \end{aligned}$$

$$\begin{aligned} \text{Entertainment: } & (\$150.00 \div \$1500.00) \\ & \times 100 = 10\% \end{aligned}$$

$$\begin{aligned} \text{Charitable donations: } & (\$125.00 \div \$1500.00) \\ & \times 100 \approx 8\% \end{aligned}$$

$$\text{Savings: } (\$150.00 \div \$1500.00) \times 100 = 10\%$$

$$\text{Other: } (\$200.00 \div \$1500.00) \times 100 \approx 13\%$$

Note that the percentages add up to 99% rather than 100%; this is due to rounding. To construct the circle graph, you can increase the size of one of the sections by 1% (for example, housing to 28%).

Next, calculate how many degrees each percentage of expenses represents.

$$\text{Housing: } 0.28 \times 360^\circ \approx 101^\circ$$

$$\text{Food: } 0.18 \times 360^\circ \approx 65^\circ$$

Transportation: $0.13 \times 360^\circ \approx 47^\circ$

Entertainment: $0.10 \times 360^\circ = 36^\circ$

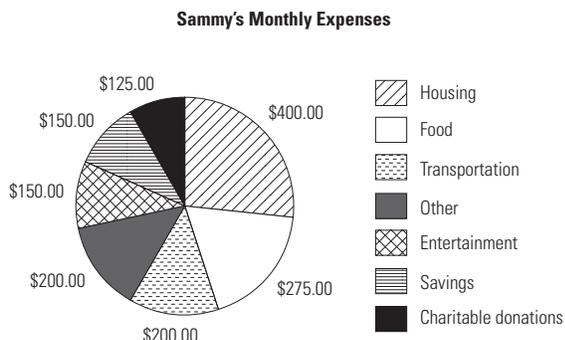
Charitable donations: $0.08 \times 360^\circ \approx 29^\circ$

Savings: $0.10 \times 360^\circ = 36^\circ$

Other: $0.13 \times 360^\circ \approx 47^\circ$

Note that the sections add up to 361° rather than 360° ; this is due to rounding. To construct the graph, you can decrease the size of one of the sections by 1° (for example, housing to 100° , since you rounded the percentage up in the previous section).

Draw a circle and use a protractor to measure the degrees for each category of expense.



5. a) Answers will vary. A possible budget is provided.

WILLA'S MONTHLY BUDGET			
<i>Income</i>		<i>Expenses</i>	
Regular	\$3530.00	Housing	\$1250.00
		Utilities	\$235.00
		Transportation	\$285.00
		Debt repayment	\$420.00
		Savings	\$375.00
		Food	\$450.00
		Recreation	\$215.00
		Health and personal care	\$300.00
<i>Total income</i>	\$3530.00	<i>Total expenses</i>	\$3530.00

- b) Answers will vary, depending on the budget created in a). The following graph is provided for the budget given above.

To construct a circle graph of the expenses, first calculate what percentage of Willa's income each category represents.

Housing: $(\$1250.00 \div \$3530.00) \times 100 \approx 35\%$

Utilities: $(\$235.00 \div \$3530.00) \times 100 \approx 7\%$

Transportation: $(\$285.00 \div \$3530.00) \times 100 \approx 8\%$

Debt repayment: $(\$420.00 \div \$3530.00) \times 100 \approx 12\%$

Savings: $(\$375.00 \div \$3530.00) \times 100 \approx 11\%$

Food: $(\$450.00 \div \$3530.00) \times 100 \approx 13\%$

Recreation: $(\$215.00 \div \$3530.00) \times 100 \approx 6\%$

Health and personal care: $(\$300.00 \div \$3530.00) \times 100 \approx 8\%$

Next, calculate how many degrees each percentage of expenses represents.

Housing: $0.35 \times 360^\circ = 126^\circ$

Utilities: $0.07 \times 360^\circ \approx 25^\circ$

Transportation: $0.08 \times 360^\circ \approx 29^\circ$

Debt repayment: $0.12 \times 360^\circ \approx 43^\circ$

Savings: $0.11 \times 360^\circ \approx 40^\circ$

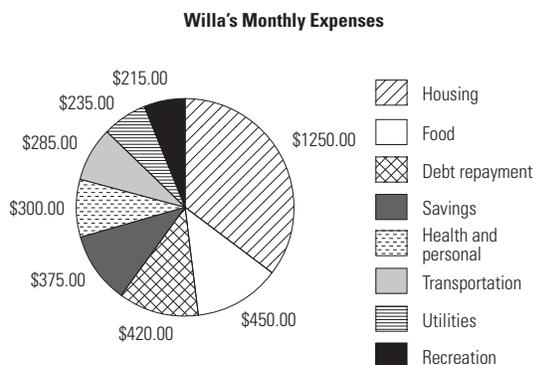
Food: $0.13 \times 360^\circ \approx 47^\circ$

Recreation: $0.06 \times 360^\circ \approx 22^\circ$

Health and personal care: $0.08 \times 360^\circ \approx 29^\circ$

Note that the sections add up to 361° rather than 360° ; this is due to rounding. To construct the graph, you can decrease the size of one of the sections by 1° (for example, recreation to 21°).

Draw a circle and use a protractor to measure the degrees for each category of expense.



PRACTISE YOUR NEW SKILLS, p. 363

1. a) Calculate how much money Manjeet saves in one week.

$$\$550.00 \times 0.14 = \$77.00$$

Multiply by 52 weeks.

$$\$77.00 \times 52 = \$4004.00$$

Manjeet will save \$4004.00 in one year.

- b) Calculate a 5% increase to Manjeet's weekly salary.

$$\$550.00 \times 0.05 = \$27.50$$

Calculate his new weekly salary.

$$\$550.00 + \$27.50 = \$577.50$$

Calculate 14% of his new salary.

$$\$577.50 \times 0.14 = \$80.85$$

Multiply by 52 weeks.

$$\$80.85 \times 52 = \$4204.20$$

In one year, Manjeet will save \$4204.20.

2. a) Calculate Lena's income.

$$\frac{\$3150.00}{x} \times 100 = 7\%$$

$$\frac{\$3150.00}{x} = \frac{7}{100}$$

$$\frac{\$3150.00}{x} = 0.07$$

$$\$3150.00 = 0.07x$$

$$\frac{\$3150.00}{0.07} = x$$

$$\$45\,000.00 = x$$

Lena's annual income is \$45 000.00.

Calculate 10%.

$$\$45\,000.00 \times 0.10 = \$4500.00$$

In one year, Lena will be able to save \$4500.00.

$$\frac{\$3150.00}{x} \times 100 = 7\%$$

$$\frac{\$3150.00}{x} = \frac{7}{100}$$

$$\frac{\$3150.00}{x} = 0.07$$

$$\$3150.00 = 0.07x$$

$$\frac{\$3150.00}{0.07} = x$$

$$\$45\,000.00 = x$$

Lena's annual income is \$45 000.00.

3. a) Add Sean's income.

$$\begin{aligned} & \$1300.00 + \$1300.00 + \$150.00 \\ & = \$2750.00 \end{aligned}$$

Add his expenses.

$$\begin{aligned} & \$1200.00 + \$175.00 + \$150.00 + \$325.00 + \\ & \$150.00 + \$125.00 + \$175.00 + \$100.00 + \\ & \$50.00 + \$100.00 + \$200.00 = \$2750.00 \end{aligned}$$

Sean's income is equal to his expenses, so his budget is balanced.

- b) To construct a circle graph of the expenses, first calculate what percentage of Sean's income each category represents.

$$\text{Housing: } (\$1200.00 \div \$2750.00) \times 100 \approx 44\%$$

$$\text{Utilities: } (\$175.00 \div \$2750.00) \times 100 \approx 6\%$$

$$\begin{aligned} \text{Phone, cable, internet: } & (\$150.00 \div \$2750.00) \\ & \times 100 \approx 5\% \end{aligned}$$

$$\text{Food: } (\$325.00 \div \$2750.00) \times 100 \approx 12\%$$

$$\begin{aligned} \text{Transportation: } & (\$150.00 \div \$2750.00) \\ & \times 100 \approx 5\% \end{aligned}$$

$$\begin{aligned} \text{Entertainment: } & (\$125.00 \div \$2750.00) \\ & \times 100 \approx 5\% \end{aligned}$$

$$\text{Clothing: } (\$175.00 \div \$2750.00) \times 100 \approx 6\%$$

$$\begin{aligned} \text{Miscellaneous: } & (\$100.00 \div \$2750.00) \\ & \times 100 \approx 4\% \end{aligned}$$

$$\begin{aligned} \text{Charitable donations: } & (\$50.00 \div \$2750.00) \\ & \times 100 \approx 2\% \end{aligned}$$

$$\begin{aligned} \text{Medical/health and other emergencies:} \\ & (\$100.00 \div \$2750.00) \times 100 \approx 4\% \end{aligned}$$

$$\text{Savings: } (\$200.00 \div \$2750.00) \times 100 \approx 7\%$$

Next, calculate how many degrees each percentage of expenses represents.

$$\text{Housing: } 0.44 \times 360^\circ \approx 158^\circ$$

$$\text{Utilities: } 0.06 \times 360^\circ \approx 22^\circ$$

$$\text{Phone, cable, internet: } 0.05 \times 360^\circ = 18^\circ$$

$$\text{Food: } 0.12 \times 360^\circ \approx 43^\circ$$

$$\text{Transportation: } 0.05 \times 360^\circ = 18^\circ$$

$$\text{Entertainment: } 0.05 \times 360^\circ = 18^\circ$$

$$\text{Clothing: } 0.06 \times 360^\circ \approx 22^\circ$$

$$\text{Miscellaneous: } 0.04 \times 360^\circ \approx 14^\circ$$

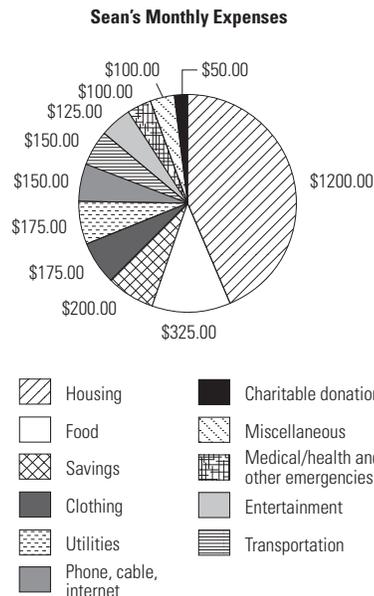
$$\text{Charitable donations: } 0.02 \times 360^\circ \approx 7^\circ$$

$$\begin{aligned} \text{Medical/health and other emergencies:} \\ & 0.04 \times 360^\circ \approx 14^\circ \end{aligned}$$

$$\text{Savings: } 0.07 \times 360^\circ \approx 25^\circ$$

Note that the sections add up to 359° rather than 360° ; this is due to rounding. To construct the graph, you can increase the size of one of the sections by 1° (for example, housing to 159°).

Draw a circle and use a protractor to measure the degrees for each category of expense.



- c) From the calculations in b), Sean is spending about 44% of his income on spending. This is more than suggested by the guidelines.

Calculate 35% of Sean's income.

$$\$2750.00 \times 0.35 = \$962.50$$

He should be spending no more than \$962.50 on housing.

- d) Answers will vary. Calculate how much Sean will need to save per month.

$$\$2750.00 \times 0.15 = \$412.50$$

$$\$412.50 - \$200.00 = \$212.50$$

Sean needs to adjust his budget to save \$212.50 more than he is currently saving per month.

Possible suggestions include:

- decreasing his spending on entertainment, clothing, miscellaneous, and charitable donations. He could decrease his expenses to the following amounts: entertainment, \$75.00; clothing, \$75.00, miscellaneous, \$75.00; charitable donations, \$12.50.
- finding a less expensive mode of transportation; or
- getting a less expensive phone/cable/internet package.

4. a) To construct a circle graph of the expenses, first calculate what percentage of Gabriella's income each category represents.

$$\text{Housing: } (\$840.00 \div \$1990.00) \times 100 \approx 42\%$$

$$\text{Food: } (\$175.00 \div \$1990.00) \times 100 \approx 9\%$$

$$\text{Transportation: } (\$125.00 \div \$1990.00) \times 100 \approx 6\%$$

$$\text{Entertainment: } (\$175.00 \div \$1990.00) \times 100 \approx 9\%$$

$$\text{Clothing: } (\$175.00 \div \$1990.00) \times 100 \approx 9\%$$

$$\text{Miscellaneous: } (\$150.00 \div \$1990.00) \times 100 \approx 8\%$$

$$\text{Loan payments: } (\$275.00 \div \$1990.00) \times 100 \approx 14\%$$

$$\text{Savings: } (\$75.00 \div \$1990.00) \times 100 \approx 4\%$$

Note that the percentages add up to 101%; this is due to rounding. For the next step, you can decrease one of the categories by 1% (for example, housing to 41%).

Next, calculate how many degrees each percentage of expenses represents.

$$\text{Housing: } 0.41 \times 360^\circ \approx 148^\circ$$

$$\text{Food: } 0.09 \times 360^\circ \approx 32^\circ$$

$$\text{Transportation: } 0.06 \times 360^\circ \approx 22^\circ$$

$$\text{Entertainment: } 0.09 \times 360^\circ \approx 32^\circ$$

$$\text{Clothing: } 0.09 \times 360^\circ \approx 32^\circ$$

$$\text{Miscellaneous: } 0.08 \times 360^\circ \approx 29^\circ$$

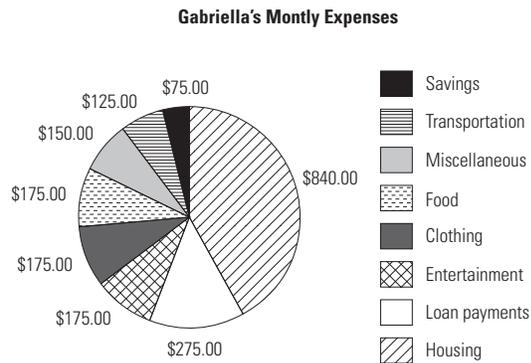
$$\text{Loan payments: } 0.14 \times 360^\circ \approx 50^\circ$$

$$\text{Savings: } 0.04 \times 360^\circ \approx 14^\circ$$

Note that the sections add up to 359° rather than 360°; this is due to rounding. To construct

the graph, you can increase the size of one of the sections by 1° (for example, housing to 149°).

Draw a circle and use a protractor to measure the degrees for each category of expense.



b) Compare Gabriella's spending to the guidelines.

Item	Gabriella's spending (percentage)	Guidelines (percentage)
Housing	42%	30–35%
Food	9%	8–15%
Transportation	6%	8–12%
Debt repayment	14%	0–12%
Entertainment	9%	8–12%
Personal	9% (clothing) + 8% (misc.) = 17%	8–12%
Savings	4%	at least 10%

Gabriella is spending too much on housing, debt repayment, and personal expenses.

She is spending below the guidelines on transportation, and is not putting enough money into savings.

She is within the guidelines for food and entertainment.

Calculate how much money Gabriella should be spending in each category.

Housing:

$$\$1990.00 \times 0.35 = \$696.50$$

Compare to her current spending.

$$\$840.00 - \$696.50 = \$143.50$$

She is spending about \$145.00 too much on housing.

Transportation:

$$\$1990.00 \times 0.12 = \$238.80$$

$$\$238.80 - \$125.00 = \$113.80$$

Gabriella could spend an additional \$113.80 on transportation and still fall within the guidelines.

Debt repayment:

$$\$1990.00 \times 0.12 = \$238.80$$

$$\$275.00 - \$238.80 = \$36.20$$

Gabriella is spending about \$36.00 more than recommended to repay her loan. However, this expense is likely not negotiable so she will have to continue paying this same amount.

Personal expenses (clothing and miscellaneous):

$$\$1990.00 \times 0.12 = \$238.80$$

$$(\$175.00 + \$150.00) - \$238.80 = \$86.20$$

Gabriella is spending about \$90.00 too much on personal expenses. She could decrease her spending on clothing to \$100.00 and on miscellaneous to \$135.00.

Savings:

$$\$1990.00 \times 0.10 = \$199.00$$

$$\$199.00 - \$75.00 = \$124.00$$

Gabriella is saving about \$125.00 less per month than recommended.

- c) Divide the total savings goal by Gabriella's current savings per month (\$75.00).

$$\$1000.00 \div \$75.00 \approx 13.3 \text{ months}$$

It will take Gabriella about 13.3 months to save \$1000.00. Because she will not have enough money saved at the beginning of the 13th month, you should round the answer up. It will take her 14 months to save \$1000.00.

Calculate how much she would have to save per month to save \$1000.00 in 6 months.

$$\$1000.00 \div 6 \approx \$166.67$$

Calculate the difference from what she is currently saving.

$$\$166.67 - \$75.00 = \$91.67$$

If she wanted to save \$1000.00 in 6 months, she would need to save \$91.67 more per month. Her savings would still be lower than the guidelines suggest.

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1.

<i>Item</i>	<i>Income</i>	<i>Expense</i>
Biweekly paycheque: \$735.00	Regular	
Monthly bus pass: \$75.00		Recurring
Gas: \$42.95		Variable
Cell phone bill: \$39.96		Recurring
New jacket: \$69.95		Variable
Tips: \$52.00	Variable	
Debt repayment: \$75.00		Recurring
Emergency vet bill for cat: \$250.00		Unexpected
Movie ticket and snack: \$20.00		Variable
Dinner at restaurant: \$27.00		Variable
Charitable donation: \$25.00		Variable
Commission: \$100.00	Variable	

2. a) Answers will vary. A possible budget is provided.

TAMARA'S MONTHLY BUDGET

<i>Income</i>		<i>Expenses</i>	
Semi-monthly	\$975.00	Rent	\$575.00
Semi-monthly	\$975.00	Utilities, phone, cable, internet	\$155.00
		Groceries	\$275.00
		Debt repayment	\$100.00
		Transportation	\$325.00
		Personal	\$465.00
		Savings	\$55.00
<i>Total income</i>	\$1950.00	<i>Total expenses</i>	\$1950.00

- b) Answers will vary, depending on the budget created. Based on the budget above, Tamara saves \$55.00 per month.

$$\$55.00/\text{month} \times 12 \text{ months} = \$660.00$$

If she follows this budget, Tamara will save \$660.00 in one year.

- c) Answers will vary, depending on the budget created. Calculate what percentage \$55.00 represents of \$1950.00.

$$\frac{\$55.00}{\$1950.00} \times 100 \approx 3\%$$

Based on this budget, Tamara saves about 3% of her income.

3. a) Calculate how much Masaru earns per week.

$$(35 \text{ h} \times \$11.75/\text{h}) + \$125.00 = \$536.25$$

Calculate what percentage of his income Masaru puts into savings.

$$100 - 85 = 15\%$$

Calculate how much money Masaru puts into savings in one week.

$$\$536.25 \times 0.15 \approx \$80.44$$

Multiply to find how much he saves in one year.

$$\$80.44/\text{week} \times 52 \text{ weeks} = \$4182.88$$

In one year, Masaru saves about \$4182.88.

- b) Calculate Masaru's annual income.

$$\$536.25/\text{week} \times 52 \text{ weeks} = \$27\,885.00$$

Calculate what percentage \$7200.00 represents of his income.

$$\frac{\$7200.00}{\$27\,885.00} \times 100 \approx 25.8\%$$

Masaru spends about 26% of his income on rent.

4. a)

MADELEINE'S MONTHLY INCOME & EXPENSES			
<i>Income</i>		<i>Expenses</i>	
Semi-monthly	\$885.00	Rent	\$595.00
Semi-monthly	\$885.00	Utilities, phone, cable, and internet	\$275.00
Tips	\$250.00	Food	\$295.00
		Transportation	\$100.00
		Entertainment	\$225.00
		Other	\$350.00
		Savings	\$180.00
<i>Total income</i>	\$2020.00	<i>Total expenses</i>	\$2020.00

- b) Calculate what percentage of her income each category represents.

$$\text{Rent: } (\$595.00 \div \$2020.00) \times 100 \approx 29.5\%$$

$$\text{Utilities, phone, cable, and internet: } (\$275.00 \div \$2020.00) \times 100 \approx 13.6\%$$

$$\text{Food: } (\$295.00 \div \$2020.00) \times 100 \approx 14.6\%$$

$$\text{Transportation: } (\$100.00 \div \$2020.00) \times 100 \approx 5.0\%$$

$$\text{Entertainment: } (\$225.00 \div \$2020.00) \times 100 \approx 11.1\%$$

$$\text{Other: } (\$350.00 \div \$2020.00) \times 100 \approx 17.3\%$$

$$\text{Savings: } (\$180.00 \div \$2020.00) \times 100 \approx 8.9\%$$

- c) Calculate a 5% increase to Madeleine's income.

$$\$2020.00 \times 0.05 = \$101.00$$

$$\$2020.00 + \$101.00 = \$2121.00$$

Calculate 8.9% of \$2121.00.

$$\$2121.00 \times 0.089 \approx \$188.77$$

Madeleine will save \$188.77 per month.

5. a) Calculate what percentage of his income Noah spends on each category.

$$\text{Housing: } (\$795.00 \div \$2400.00) \times 100 \approx 33\%$$

$$\text{Utilities: } (\$150.00 \div \$2400.00) \times 100 \approx 6\%$$

$$\text{Food: } (\$500.00 \div \$2400.00) \times 100 \approx 21\%$$

$$\text{Transportation: } (\$375.00 \div \$2400.00) \times 100 \approx 16\%$$

$$\text{Entertainment: } (\$150.00 \div \$2400.00) \times 100 \approx 6\%$$

$$\text{Clothing: } (\$100.00 \div \$2400.00) \times 100 \approx 4\%$$

$$\text{Savings: } (\$30.00 \div \$2400.00) \times 100 \approx 1\%$$

$$\text{Other: } (\$300.00 \div \$2400.00) \times 100 \approx 13\%$$

Compare these percentages to the guidelines:

- Housing: within the guidelines
- Utilities: within the guidelines
- Food: higher than recommended
- Transportation: higher than recommended
- Entertainment: within the guidelines
- Clothing: within the guidelines
- Savings: lower than recommended
- Other: higher than recommended

For food, transportation, savings, and other, calculate how much Noah should be spending.

Food:

$$\$2400.00 \times 0.15 = \$360.00$$

$$\$500.00 - \$360.00 = \$140.00$$

To stay within the guidelines, Noah would need to spend \$140.00 less on food.

Transportation:

$$\$2400.00 \times 0.15 = \$360.00$$

$$\$375.00 - \$360.00 = \$15.00$$

He would need to spend \$15.00 less on transportation.

Other:

$$\$2400.00 \times 0.10 = \$240.00$$

$$\$300.00 - \$240.00 = \$60.00$$

To stay within the guidelines, Noah would have to spend \$60.00 less on savings.

Savings:

$$\$2400.00 \times 0.05 = \$120.00$$

$$\$120.00 - \$30.00 = \$90.00$$

Noah would need to save at least \$90.00 more per month.

b) $\$2400.00 \times 0.05 = \120.00

He should put at least \$120.00 into savings per month.

- c) Answers will vary. A possible budget is provided.

NOAH'S MONTHLY BUDGET			
Income		Expenses	
Semi-monthly	\$1200.00	Rent	\$795.00
Semi-monthly	\$1200.00	Utilities	\$150.00
		Food	\$360.00
		Transportation	\$360.00
		Entertainment	\$150.00
		Clothing	\$100.00
		Savings	\$245.00
		Other	\$240.00
<i>Total income</i>	\$2400.00	<i>Total expenses</i>	\$2400.00

To construct a circle graph of the expenses, first calculate what percentage of Noah's income each category represents.

$$\text{Housing: } (\$795.00 \div \$2400.00) \times 100 \approx 33\%$$

$$\text{Utilities: } (\$150.00 \div \$2400.00) \times 100 \approx 6\%$$

$$\text{Food: } (\$360.00 \div \$2400.00) \times 100 = 15\%$$

$$\begin{aligned} \text{Transportation: } & (\$360.00 \div \$2400.00) \\ & \times 100 = 15\% \end{aligned}$$

$$\begin{aligned} \text{Entertainment: } & (\$150.00 \div \$2400.00) \\ & \times 100 \approx 6\% \end{aligned}$$

$$\text{Clothing: } (\$100.00 \div \$2400.00) \times 100 \approx 4\%$$

$$\text{Savings: } (\$245.00 \div \$2400.00) \times 100 \approx 10\%$$

$$\text{Other: } (\$240.00 \div \$2400.00) \times 100 = 10\%$$

Note that the percentages add up to 99%; this is due to rounding. For the next step, you can increase one of the categories by 1% (for example, housing to 34%).

Next, calculate how many degrees each percentage of expenses represents.

$$\text{Housing: } 0.34 \times 360^\circ \approx 122^\circ$$

$$\text{Utilities: } 0.06 \times 360^\circ \approx 22^\circ$$

$$\text{Food: } 0.15 \times 360^\circ = 54^\circ$$

$$\text{Transportation: } 0.15 \times 360^\circ = 54^\circ$$

$$\text{Entertainment: } 0.06 \times 360^\circ \approx 22^\circ$$

$$\text{Clothing: } 0.04 \times 360^\circ \approx 14^\circ$$

$$\text{Savings: } 0.10 \times 360^\circ = 36^\circ$$

$$\text{Other: } 0.10 \times 360^\circ = 36^\circ$$

Draw a circle and use a protractor to measure the degrees for each category of expense.

