

Appendix H

For more information, see
the Table of Specifications.

Grade 12 Prototype Examination

Pre-calculus 30

Course Code 8426

Barcode Number	
Month	Day
Date of Birth	

November 2013

Student Examination Form

**Month and Day
of Your Birth**

JAN	DAY	
	0	0
FEB	1	1
MAR	2	2
JUN	3	3
JUL	4	
AUG	5	
SEP	6	
OCT	7	
NOV	8	
DEC	9	

MULTIPLE CHOICE

- 1 A B C D
2 A B C D
3 A B C D
4 A B C D
5 A B C D
6 A B C D
7 A B C D
8 A B C D

- 9 A B C D
10 A B C D
11 A B C D
12 A B C D
13 A B C D
14 A B C D
15 A B C D
16 A B C D

- 17 A B C D
18 A B C D
19 A B C D
20 A B C D
21 A B C D
22 A B C D
23 A B C D
24 A B C D

- 25 A B C D
26 A B C D
27 A B C D
28 A B C D
29 A B C D
30 A B C D
31 A B C D
32 A B C D

- 33 A B C D
34 A B C D
35 A B C D
36 A B C D
37 A B C D
38 A B C D
39 A B C D
40 A B C D

NUMERICAL RESPONSE

--	--	--	--

--	--	--	--

--	--	--	--

--	--	--	--

--	--	--	--

- 41 0 0 0 0
1 1 1 1
2 2 2 2
3 3 3 3
4 4 4 4
5 5 5 5
6 6 6 6
7 7 7 7
8 8 8 8
9 9 9 9

- 42 0 0 0 0
1 1 1 1
2 2 2 2
3 3 3 3
4 4 4 4
5 5 5 5
6 6 6 6
7 7 7 7
8 8 8 8
9 9 9 9

- 43 0 0 0 0
1 1 1 1
2 2 2 2
3 3 3 3
4 4 4 4
5 5 5 5
6 6 6 6
7 7 7 7
8 8 8 8
9 9 9 9

- 44 0 0 0 0
1 1 1 1
2 2 2 2
3 3 3 3
4 4 4 4
5 5 5 5
6 6 6 6
7 7 7 7
8 8 8 8
9 9 9 9

- 45 0 0 0 0
1 1 1 1
2 2 2 2
3 3 3 3
4 4 4 4
5 5 5 5
6 6 6 6
7 7 7 7
8 8 8 8
9 9 9 9

--	--	--	--

--	--	--	--

--	--	--	--

--	--	--	--

--	--	--	--

- 46 0 0 0 0
1 1 1 1
2 2 2 2
3 3 3 3
4 4 4 4
5 5 5 5
6 6 6 6
7 7 7 7
8 8 8 8
9 9 9 9

- 47 0 0 0 0
1 1 1 1
2 2 2 2
3 3 3 3
4 4 4 4
5 5 5 5
6 6 6 6
7 7 7 7
8 8 8 8
9 9 9 9

- 48 0 0 0 0
1 1 1 1
2 2 2 2
3 3 3 3
4 4 4 4
5 5 5 5
6 6 6 6
7 7 7 7
8 8 8 8
9 9 9 9

- 49 0 0 0 0
1 1 1 1
2 2 2 2
3 3 3 3
4 4 4 4
5 5 5 5
6 6 6 6
7 7 7 7
8 8 8 8
9 9 9 9

- 50 0 0 0 0
1 1 1 1
2 2 2 2
3 3 3 3
4 4 4 4
5 5 5 5
6 6 6 6
7 7 7 7
8 8 8 8
9 9 9 9

Pre-calculus 30

TIME: Two and One-Half Hours

Calculating devices MUST meet the requirements of the Calculator Use Policy. Before an examination begins, devices must be removed from their cases and placed on the students' desks for inspection by a mathematics or science teacher. Students must clear all programmable calculators, both graphing and scientific, of all information that is stored in memory. Internet access and electronic signals are not allowed. Cases must be placed on the floor and left there for the duration of the examination.

Do not spend too much time on any one question. Read each question carefully.

The examination consists of 40 multiple-choice followed by 10 numerical response questions of equal value which will be machine scored. Record your answers on the Student Examination Form which is provided. Each multiple choice question has four suggested answers, one of which is better than the others. Select the **best** answer and record it on the Student Examination Form as shown in the example below:

Student Examination Form:

Multiple Choice

This examination is being written in the subject

- A. Chemistry.
- B. Pre-calculus.
- C. Workplace and Apprenticeship Mathematics.
- D. Foundations of Mathematics.

1.	(A)	<input checked="" type="radio"/>	(C)	(D)
----	-----	----------------------------------	-----	-----

Numerical Response

Record your answer in the numerical response section on the answer sheet.

What is 10% of \$2 000? (Round to the nearest dollar.)

Record 200 on the answer sheet →

2	0	0	
0	1	2	3
1	1	2	3
2	2	2	3
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

What is 10% of \$248.50? (Round to the nearest dollar.)

Record 25 on the answer sheet

2	5		
---	---	--	--

0	0	0	0
1	1	1	1
●	2	2	2
3	3	3	3
4	4	4	4
5	●	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

What is 10% of 24 125? (Round to the nearest whole number.)

Record 2413 on the answer sheet

2	4	1	3
---	---	---	---

0	0	0	0
1	1	●	1
●	2	2	2
3	3	3	●
4	●	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Use an ordinary HB pencil to mark your answers on the Student Examination Form. If you change your mind about an answer, be sure to erase the first mark completely. There should be only one answer marked for each question. Be sure there are no stray pencil marks on your answer sheet. If you need space for rough work, use the space in the examination booklet beside each question.

Do not fold either the Student Examination Form or the examination booklet. Check that all information at the bottom of the Student Examination Form is correct and complete. Make any necessary changes, and fill in any missing information. Be sure to complete the Month and Day of Your Birth section.

Pre-calculus 30

Quadratic Formula

$$\text{For } ax^2 + bx + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Arc Length

$$a = r \theta$$

Trigonometry and Trigonometric Identities

$$\begin{array}{ll} \tan \theta = \frac{\sin \theta}{\cos \theta} & \cot \theta = \frac{\cos \theta}{\sin \theta} \\ \csc \theta = \frac{1}{\sin \theta} & \sec \theta = \frac{1}{\cos \theta} \\ \sin^2 \theta + \cos^2 \theta = 1 & 1 + \tan^2 \theta = \sec^2 \theta \\ & 1 + \cot^2 \theta = \csc^2 \theta \end{array}$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

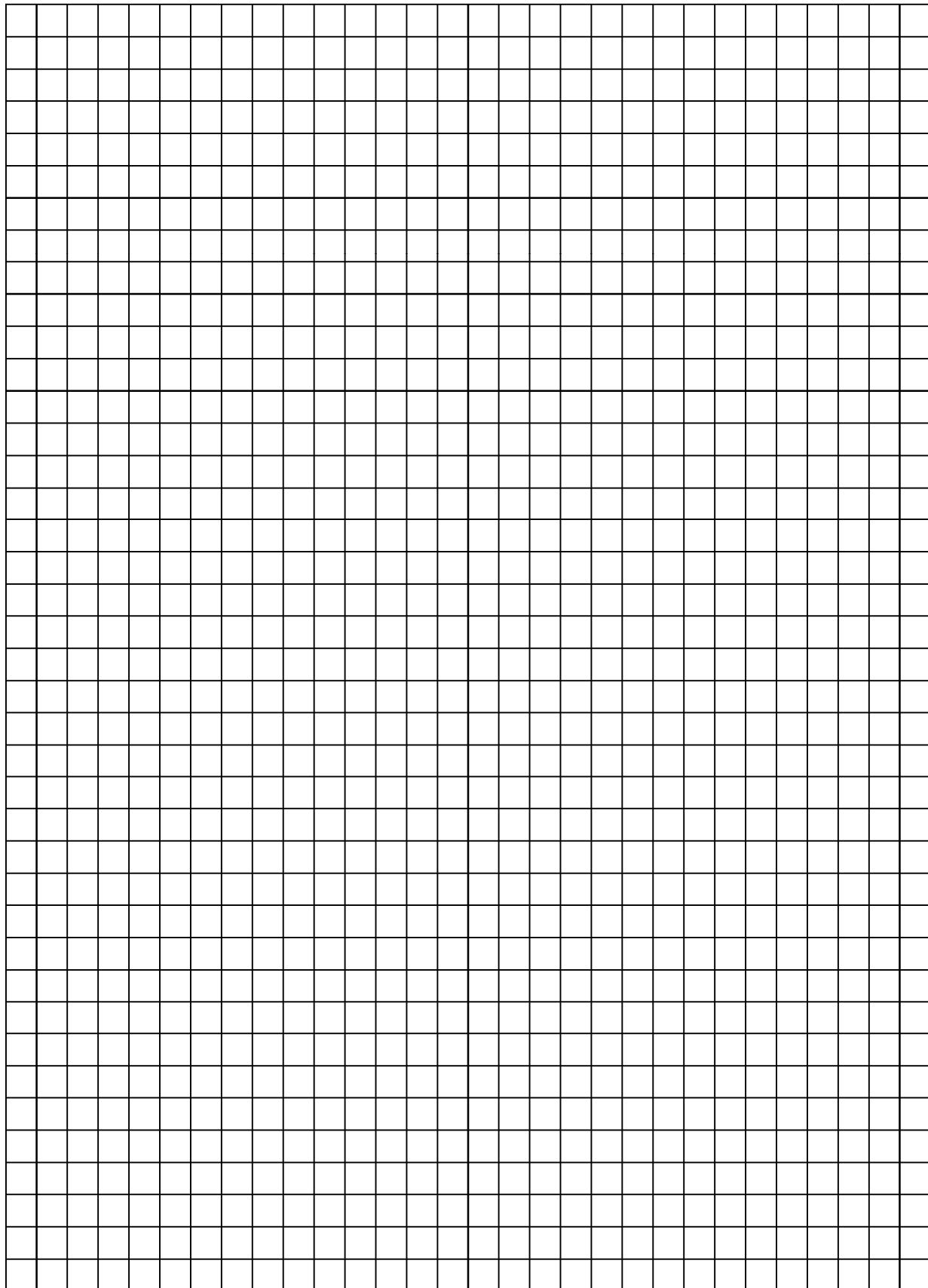
$$\sin 2\theta = 2 \sin \theta \cos \theta \qquad \cos 2\theta = 2 \cos^2 \theta - 1 \qquad \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Permutations, Combinations, and Binomial Theorem

$${_n}P_r = \frac{n!}{(n-r)!} \qquad {_n}C_r = \frac{n!}{(n-r)!r!} \qquad {_n}C_r = \binom{n}{r}$$

$$(a+b)^n = {}_nC_0 a^n b^0 + {}_nC_1 a^{n-1} b^1 + {}_nC_2 a^{n-2} b^2 + \dots + {}_nC_n a^0 b^n$$

The r^{th} term of the expansion of $(a+b)^n$ is: $\binom{n}{r-1} a^{n-(r-1)} b^{r-1}$



GRADE 12 DEPARTMENTAL EXAMINATION

PRE-CALCULUS 30, NOVEMBER 2013

**VALUE Answer the following 50 questions on the computer sheet entitled
100 "Student Examination Form."
(50 × 2)**

MULTIPLE CHOICE

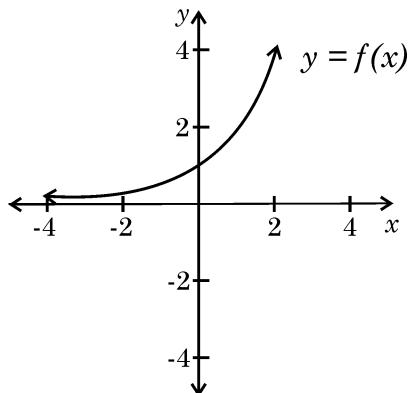
1. If $f(x) = x^2 - 3x + 4$ and $g(x) = 5x + 2$, what is $f(x) - g(x)$?

- A. $x^2 - 2x + 2$
- B. $x^2 - 2x + 6$
- C. $x^2 - 8x + 2$
- D. $x^2 - 8x + 6$

2. What is $y = \log_b x$ rewritten in exponential form?

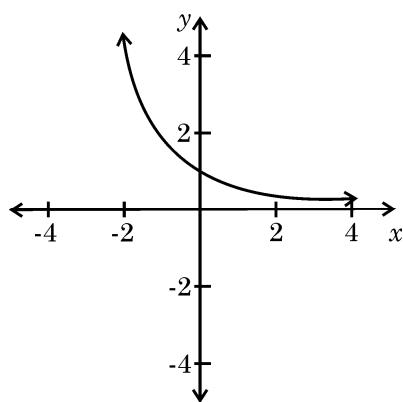
- A. $b^x = y$
- B. $b^y = x$
- C. $x^b = y$
- D. $y^b = x$

3. The graph of $y = f(x)$ is shown below.

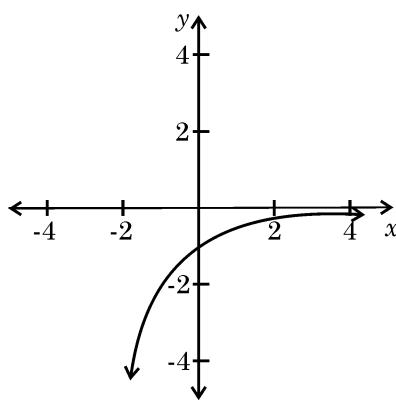


Which of the following graphs represents the inverse of this function?

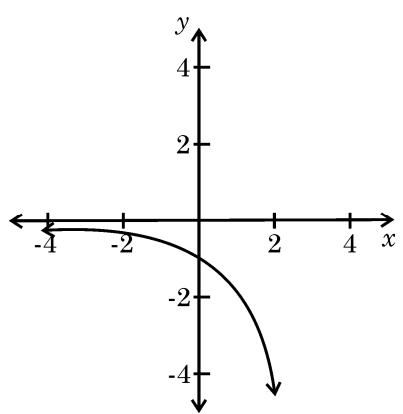
A.



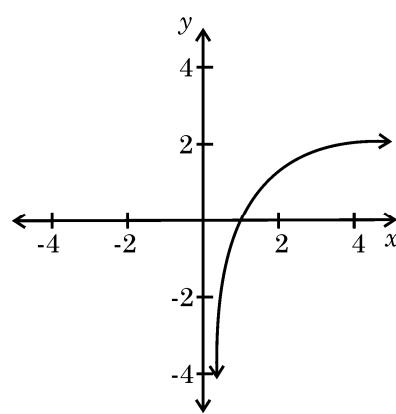
B.



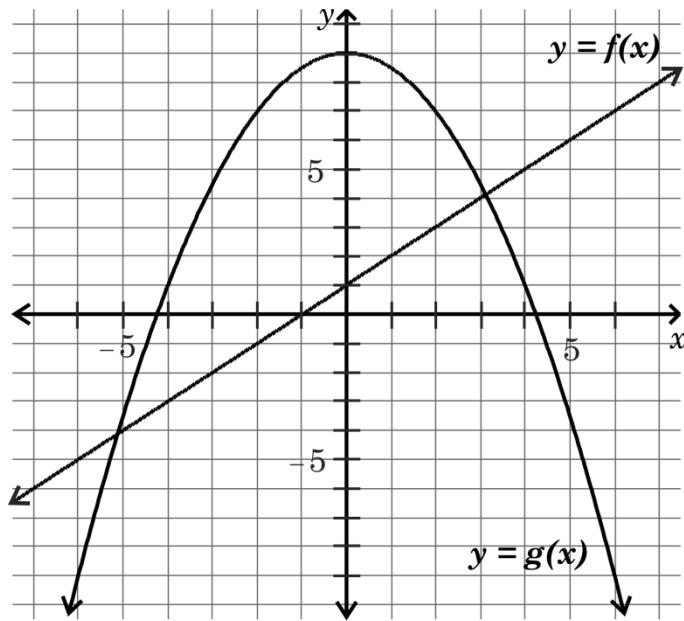
C.



D.



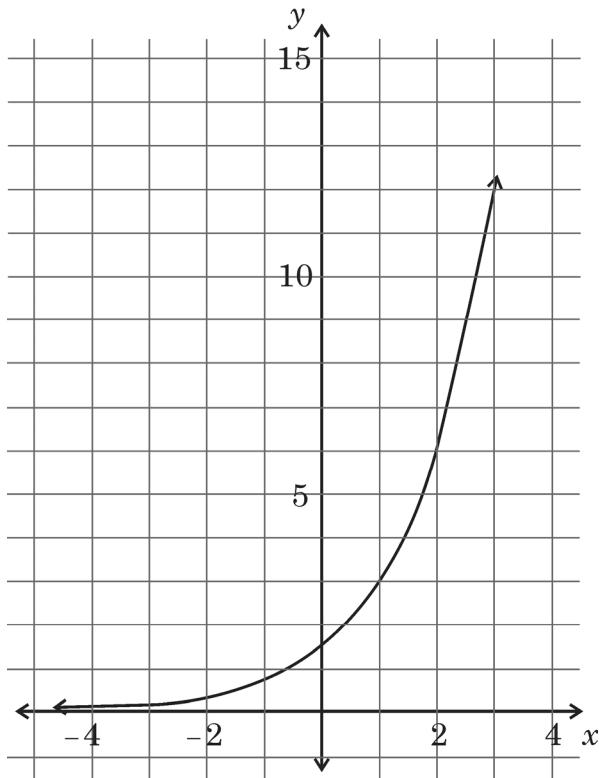
4. Functions $y = f(x)$ and $y = g(x)$ are shown graphed below.



If $h(x) = f(x) + g(x)$, what is the value of $h(-2)$?

- A. 4
- B. 6
- C. 8
- D. 10

5. The graph of an exponential function with a base of 2 is shown below.

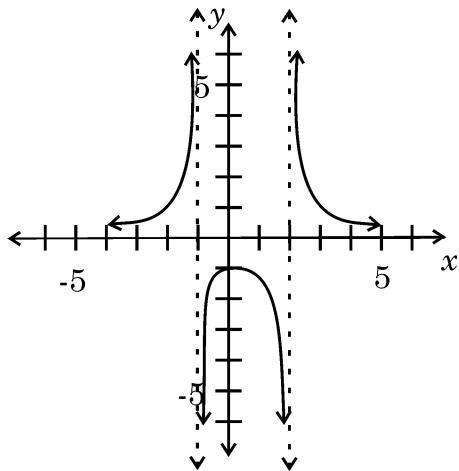


Which equation best describes this function?

- A. $y = (2^x) - (0.5)$
 - B. $y = (2^x) + (0.5)$
 - C. $y = (3)(2^{x-1})$
 - D. $y = (3)(2^{x+1})$
6. The function $f(x) = \log_2(x)$ is transformed into a new function $g(x) = \log_2(x - 2)$. How has the graph of $f(x)$ been translated?
- A. 2 units up
 - B. 2 units down
 - C. 2 units to the left
 - D. 2 units to the right

7. How will the graph of $f(x) = \sqrt{x}$ be transformed if the function is changed to $g(x) = \sqrt{x-3} + 4$?
- A. The new graph will shift 3 units to the left and 4 units up.
 - B. The new graph will shift 3 units to the right and 4 units up.
 - C. The new graph will shift 3 units to the left and 4 units down.
 - D. The new graph will shift 3 units to the right and 4 units down.

8. Which equation best represents the graph shown below?

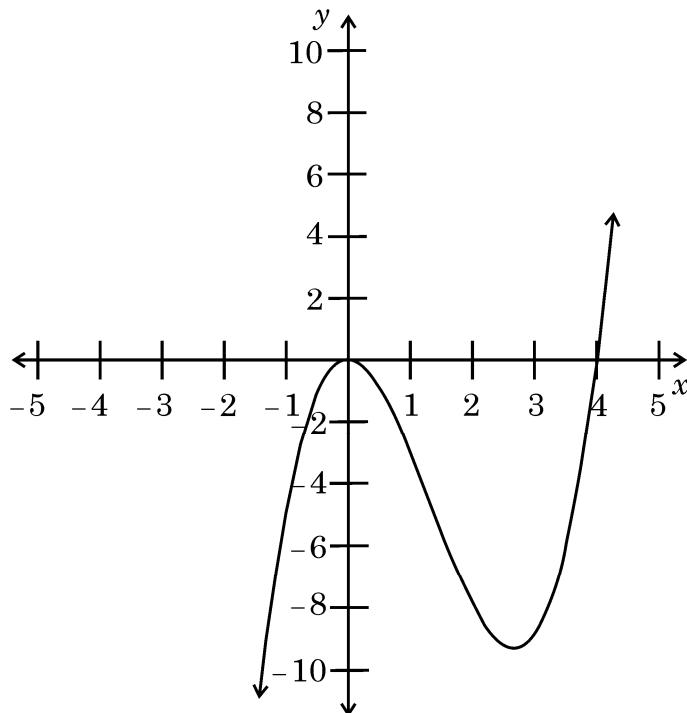


- A. $f(x) = \frac{2}{(x-2)(x+1)}$
- B. $f(x) = \frac{2}{(x+2)(x-1)}$
- C. $f(x) = \frac{(x-2)(x+1)}{2}$
- D. $f(x) = \frac{(x+2)(x-1)}{2}$

9. Which of the following statements is true for the graph of any fourth degree polynomial function?

- A. There will be a minimum of 1 x -intercept.
- B. There will be a maximum of 1 x -intercept.
- C. There will be a minimum of 4 x -intercepts.
- D. There will be a maximum of 4 x -intercepts.

10. A polynomial function is sketched below.



What is the equation of this polynomial function?

- A. $y = x(x-4)^2$
- B. $y = x(x+4)^2$
- C. $y = x^2(x-4)$
- D. $y = x^2(x+4)$

11. What is the value of x in the equation $27^{x-1} = \sqrt{3}$?

- A. $\frac{1}{2}$
- B. $\frac{2}{3}$
- C. $\frac{7}{6}$
- D. $\frac{3}{2}$

12. How many distinct x -intercepts does the graph of $f(x) = \frac{x^2 + 5x + 4}{x^2 - 16}$ have?

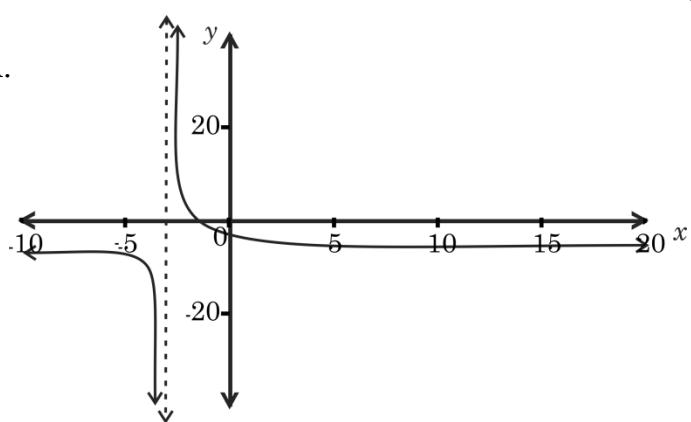
- A. 1
- B. 2
- C. 3
- D. 4

13. What is the equation of the inverse relation of $f(x) = 4x^2 + 10$?

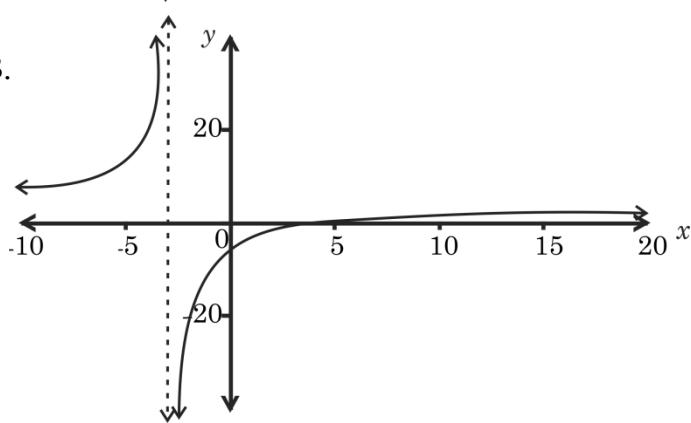
- A. $y = \pm \frac{\sqrt{x-10}}{2}$
- B. $y = \pm \sqrt{\frac{x-10}{2}}$
- C. $y = \pm \sqrt{x-\frac{5}{2}}$
- D. $y = \pm \sqrt{x} - \frac{5}{2}$

14. Which sketch best represents the function $y = \frac{5x-13}{x-3}$?

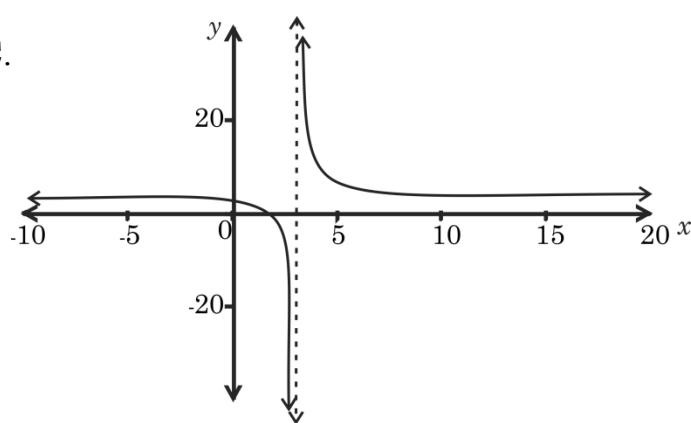
A.



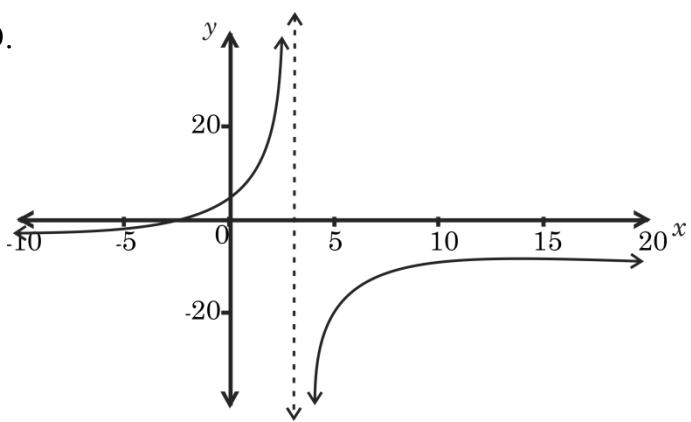
B.



C.



D.



15. Which of the following shows a correct estimation and explanation to approximate the value of x in the equation $\log_3 20 = x$?

- A. $x \approx 2.3$ because x must be somewhere between 2 and 3 and it will be closer to 2.
- B. $x \approx 2.3$ because x must be somewhere between 2 and 3 and it will be closer to 3.
- C. $x \approx 2.7$ because x must be somewhere between 2 and 3 and it will be closer to 2.
- D. $x \approx 2.7$ because x must be somewhere between 2 and 3 and it will be closer to 3.

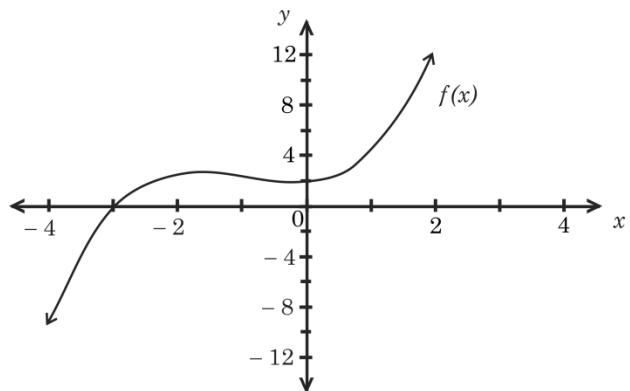
16. What is the value of x in the equation $7^x = 5^{x+3}$?

- A. 7.0
- B. 7.5
- C. 13.5
- D. 14.3

17. A function is defined by $f(x) = \frac{x^2 - 9}{x^2 - 5x + 6}$. On the graph of $f(x)$, where are the vertical asymptote and the point of discontinuity (hole)?

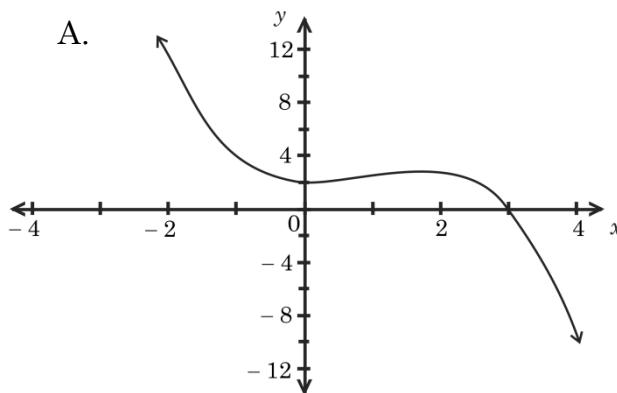
- A. The vertical asymptote is at $x = 2$; the point of discontinuity (hole) is at $(3, 6)$.
- B. The vertical asymptote is at $x = 3$; the point of discontinuity (hole) is at $(2, 5)$.
- C. The vertical asymptote is at $x = -2$; the point of discontinuity (hole) is at $(-3, 0)$.
- D. The vertical asymptote is at $x = -3$; the point of discontinuity (hole) is at $\left(-2, -\frac{1}{4}\right)$.

18. The sketch of the function $y = f(x)$ is shown below.

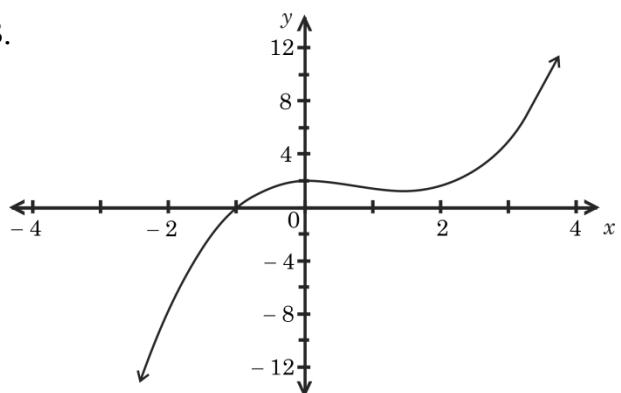


What will be the graph of $y = f(-x) - 2$?

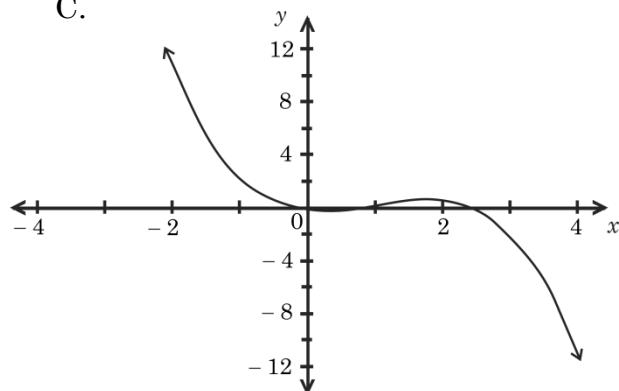
A.



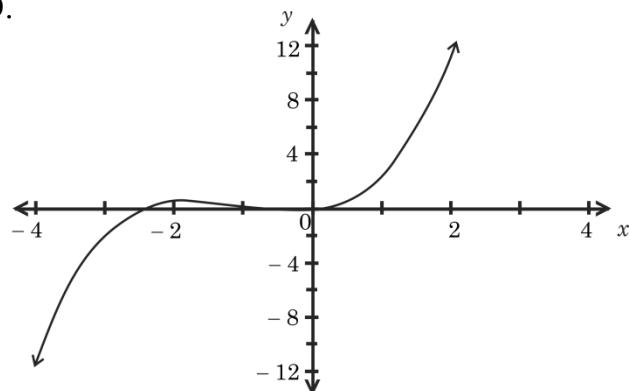
B.



C.



D.



19. What is the solution set for the equation $\sqrt{x+13} = x + 1$?

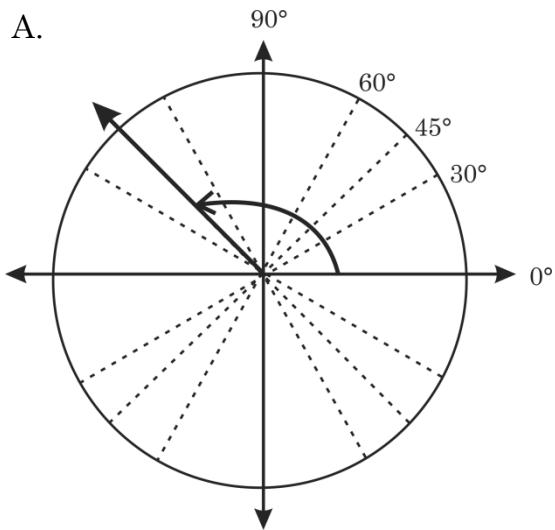
- A. {3}
- B. {4}
- C. {-4, 3}
- D. {-3, 4}

20. What is the solution set for the equation $\log_2(x-1) + \log_2(x+2) = 2$?

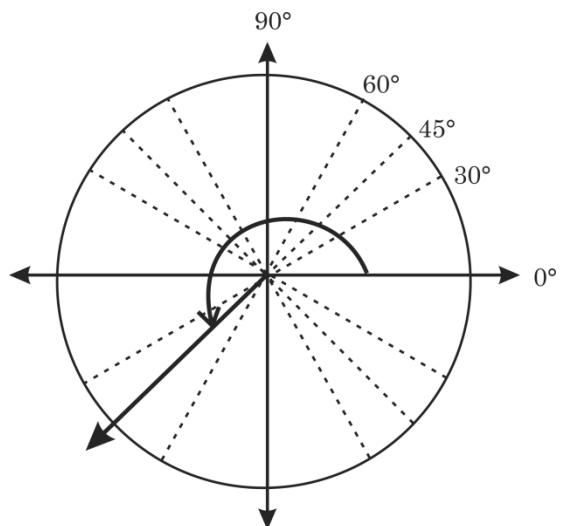
- A. {2}
- B. {-3}
- C. {-3, 2}
- D. {-3, 0, 2}

21. Which of the following represents an angle in standard position measuring $\frac{5\pi}{4}$ radians?

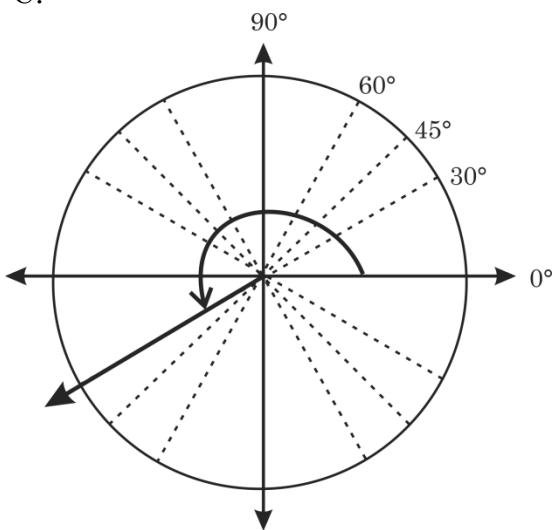
A.



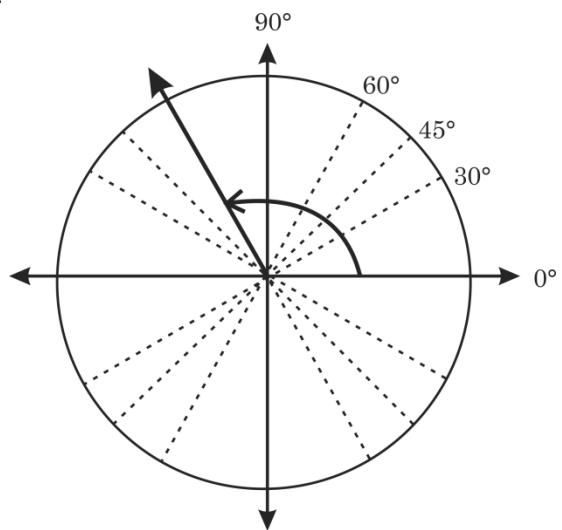
B.



C.



D.



22. Angle θ is in the second quadrant with $\sin \theta = \frac{7}{9}$. What is the exact value of $\cos \theta$?

A. $-\frac{4\sqrt{2}}{7}$

B. $-\frac{4\sqrt{2}}{9}$

C. $\frac{4\sqrt{2}}{9}$

D. $\frac{4\sqrt{2}}{7}$

23. Angle A has a measure of $\frac{4\pi}{3}$ radians. What are the exact values of $\cos A$ and $\sin A$?

A. $\cos A = -\frac{\sqrt{3}}{2}$ and $\sin A = -\frac{1}{2}$

B. $\cos A = -\frac{\sqrt{3}}{2}$ and $\sin A = \frac{1}{2}$

C. $\cos A = -\frac{1}{2}$ and $\sin A = -\frac{\sqrt{3}}{2}$

D. $\cos A = -\frac{1}{2}$ and $\sin A = \frac{\sqrt{3}}{2}$

24. Which of the following pairs of trigonometric ratios have the same value as $\sec 307^\circ$?

A. $\sec (-53^\circ)$ and $\sec (-127^\circ)$

B. $\sec (-53^\circ)$ and $-\sec 127^\circ$

C. $-\sec 53^\circ$ and $\sec (-127^\circ)$

D. $-\sec 53^\circ$ and $-\sec 127^\circ$

25. What are possible solutions for $\sin A = -\frac{\sqrt{2}}{2}$, where $0^\circ \leq A < 360^\circ$?

- A. 45° and 135°
- B. 45° and 315°
- C. 135° and 225°
- D. 225° and 315°

26. What characteristic is the same for the graphs of $y = \sin x$ and $y = \tan x$?

- A. amplitude
- B. asymptotes
- C. period length
- D. x -intercepts

27. Angle θ is in the fourth quadrant with $\tan \theta = -\frac{12}{5}$. What is the exact value of $\sin \theta$?

- A. $-\frac{12}{13}$
- B. $-\frac{5}{13}$
- C. $\frac{5}{13}$
- D. $\frac{12}{13}$

28. The terminal arm of angle A in standard position passes through the point $(-3, 7)$. What is the value of $\cos A$?

A. $-\frac{\sqrt{58}}{3}$

B. $-\frac{3\sqrt{58}}{58}$

C. $\frac{7\sqrt{58}}{58}$

D. $\frac{\sqrt{58}}{7}$

29. What is the exact value of $\frac{\tan 120^\circ + \tan 60^\circ}{1 - \tan 120^\circ \tan 60^\circ}$?

A. 0

B. $\frac{\sqrt{3}}{2}$

C. $\frac{2\sqrt{3}}{3}$

D. $\sqrt{3}$

30. What is $\cos \theta + \sin^2 \theta \sec \theta$ simplified?

A. 1

B. $\sin \theta$

C. $\sec \theta$

D. $1 + \sec \theta$

31. What are the non-permissible values for $\frac{\sin \theta \cdot \cos \theta}{1 - \sin^2 \theta}$, where $0 \leq \theta < 2\pi$?

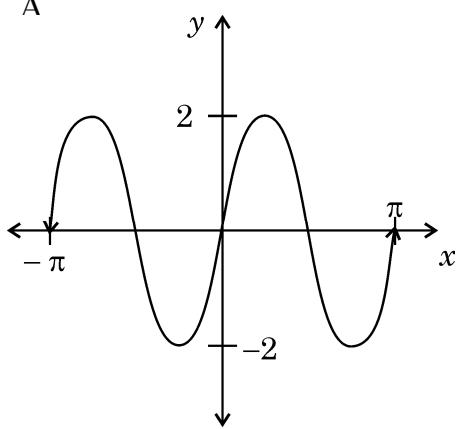
- A. $0, \pi$
- B. $\frac{\pi}{4}, \frac{3\pi}{4}$
- C. $\frac{\pi}{2}, \frac{3\pi}{2}$
- D. $\frac{3\pi}{4}, \frac{3\pi}{2}$

32. Which of the following is a solution for $\tan^2 \theta - 3 \tan \theta + 2 = 0$?

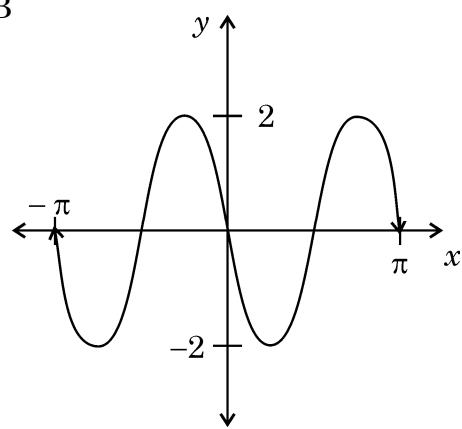
- A. 120°
- B. 135°
- C. 225°
- D. 240°

33. Which of the following represents the graph of $y = 2 \cos 2\left(x + \frac{\pi}{4}\right)$?

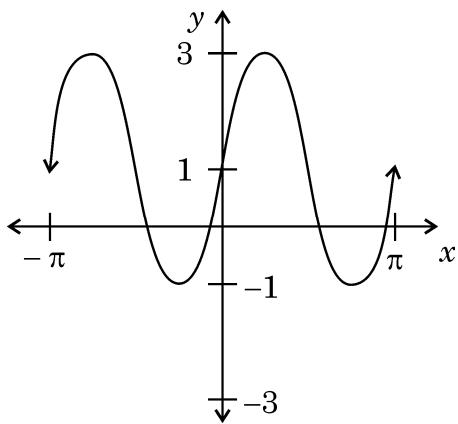
A.



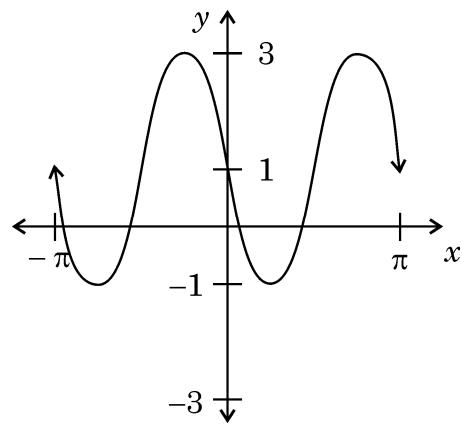
B.



C.



D.



34. Which of the following is a trigonometric identity?
- A. $2 \cos \theta = \cot \theta (\cos \theta \tan \theta + \sin \theta)$
- B. $\cot \theta \sec \theta = \cos \theta + \tan \theta$
- C. $\frac{\cos \theta}{\csc^2 \theta - 1} = \sin^2 \theta$
- D. $\frac{1 - \sin \theta}{\sin \theta} = \cos \theta$
35. What are the exact measures of the angles that satisfy $\csc \theta = -\frac{2\sqrt{3}}{3}$, where $0 \leq \theta < 2\pi$?
- A. $\frac{\pi}{6}, \frac{11\pi}{6}$
- B. $\frac{\pi}{3}, \frac{2\pi}{3}$
- C. $\frac{5\pi}{6}, \frac{7\pi}{6}$
- D. $\frac{4\pi}{3}, \frac{5\pi}{3}$
36. What are the non-permissible values for $\frac{\sin \theta}{\tan \theta} + \frac{\tan \theta}{\csc \theta} = \sec \theta$, where $0^\circ \leq \theta < 360^\circ$?
- A. $0^\circ, 90^\circ, 180^\circ, 270^\circ$
- B. $90^\circ, 180^\circ, 270^\circ$
- C. $90^\circ, 270^\circ$
- D. $0^\circ, 180^\circ$

37. To accessorize her outfit, Jane will choose 1 of 4 handbags, 1 of 5 hats, and 1 of 3 coats. How many different outfits can Jane create by changing these accessories?
- A. 3
B. 12
C. 60
D. 220
38. How many different passwords can be made from all the letters in the word **CALCULUS**?
- A. 2520
B. 5040
C. 6720
D. 40320
39. What is the solution set for r given ${}_7C_r = 21$?
- A. {2}
B. {3}
C. {2, 5}
D. {3, 4}

40. Which of the following represents the 3rd term in the expansion of $(2 - 3x)^7$?

A. $\binom{7}{2}(2)^2(-3x)^5$

B. $\binom{7}{3}(2)^3(-3x)^4$

C. $\binom{7}{3}(2)^4(-3x)^3$

D. $\binom{7}{2}(2)^5(-3x)^2$

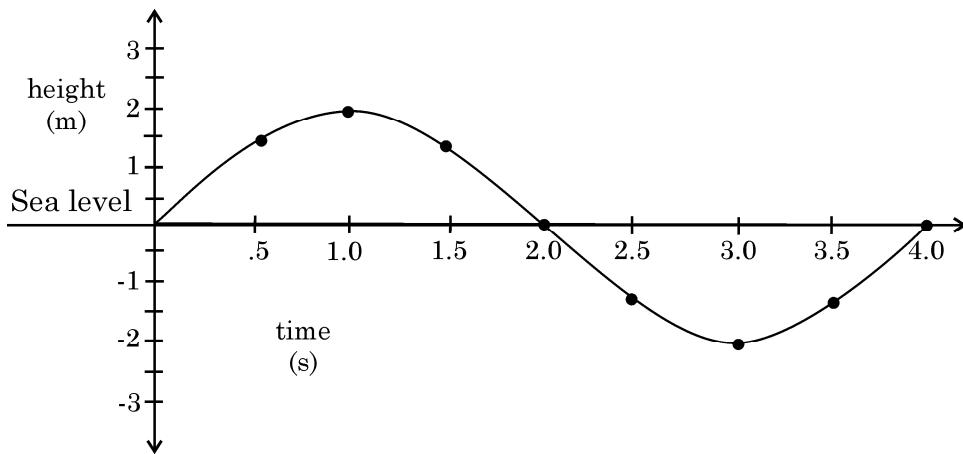
NUMERICAL RESPONSE

Record your answer in the Numerical Response section of the “Student Examination Form.”

41. Functions $f(x)$ and $g(x)$ are defined by $f(x) = 3x - 5$ and $g(x) = (x + 2)^2 - 3$. What is the value of $g(f(4))$?
42. What is the value of a in the equation $\log_5 a = \log_5 6 + 3 \log_5 3 - \frac{1}{2} \log_5 81$?
43. What is the remainder when $2x^3 + x^2 - 8x - 4$ is divided by $x - 3$?
44. Kelsey invested \$1000 at an annual interest rate of 6% compounded monthly. The accumulated value of her investment, A , is given by
$$A = 1000 \left(1 + \frac{0.06}{12}\right)^n$$
 where n is the number of months. What is the fewest number of months required for Kelsey's investment to be worth more than \$1500?
45. What is $\frac{3\pi}{5}$ radians in degrees? (Round to the nearest degree.)

46. How many solutions are there for $4 \sin^2 \theta - 1 = 0$, where $0^\circ \leq \theta < 360^\circ$?

47. The location of a dolphin moving in rhythmic fashion (above and below the surface of the water) is recorded over a time span of 4.0 seconds. The results are shown on the graph below.



What is the length of time it takes the dolphin to complete one cycle?

48. A bicycle tire has a diameter of 724 mm. A point is marked on the outer edge of the tire. After the tire has turned 60° , what is the arc length the point has moved? (Round to the nearest millimetre.)

49. How many different ways could 4 members be selected from a cheerleading squad with 12 members?

50. What is the coefficient of the term containing x^2 in the expansion of $(x + 3)^7$?

GRADE 12 DEPARTMENTAL EXAMINATION
Pre-calculus 30
PROTOTYPE EXAM — Answer Key

(See Explanation of Answers)

1.	C.	11.	C.	21.	B.	31.	C.	41.	78
2.	B.	12.	A.	22.	B.	32.	C.	42.	18
3.	D.	13.	A.	23.	C.	33.	B.	43.	35
4.	B.	14.	C.	24.	B.	34.	A.	44.	82
5.	C.	15.	D.	25.	D.	35.	D.	45.	108
6.	D.	16.	D.	26.	D.	36.	A.	46.	4
7.	B.	17.	A.	27.	A.	37.	C.	47.	4
8.	A.	18.	C.	28.	B.	38.	B.	48.	379
9.	D.	19.	A.	29.	A.	39.	C.	49.	495
10.	C.	20.	A.	30.	C.	40.	D.	50.	5103

Explanation of Answers

1. C.

If $f(x) = x^2 - 3x + 4$ and $g(x) = 5x + 2$, then

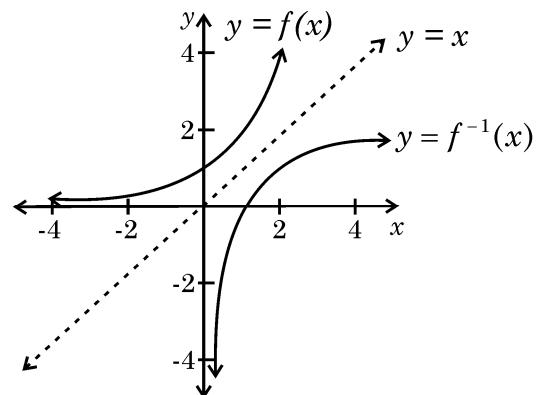
$$\begin{aligned}f(x) - g(x) &= (x^2 - 3x + 4) - (5x + 2) \\&= x^2 - 8x + 2\end{aligned}$$

2. B.

The expression $y = \log_b x$ can be rewritten as $b^y = x$.

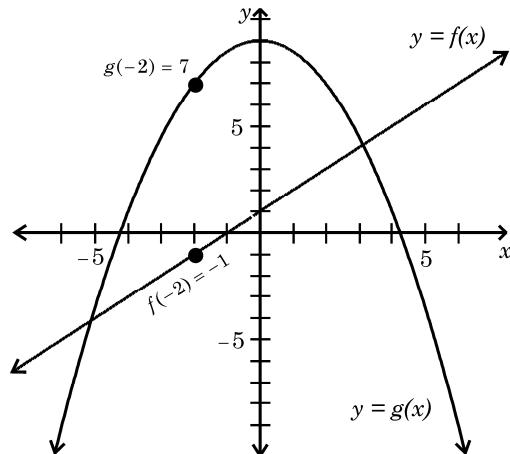
3. D.

The function $y = f(x)$ is an exponential function. The inverse of an exponential function is a logarithmic function. The inverse of any function can be found by reflecting the original function in the line $y = x$.



4. B.

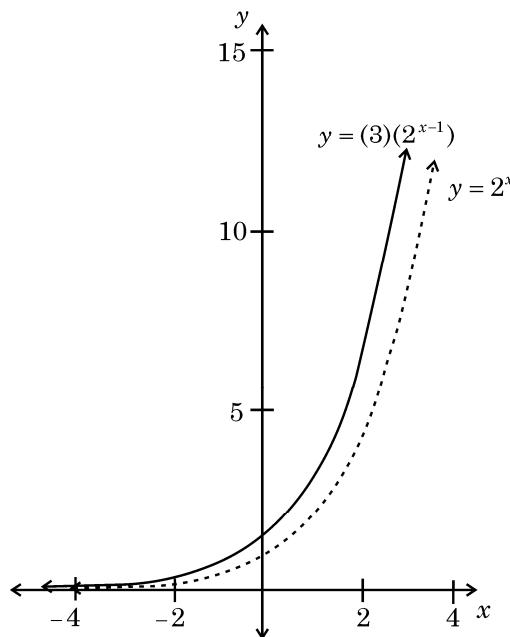
$$\begin{aligned} h(x) &= f(x) + g(x) \\ h(-2) &= f(-2) + g(-2) \\ h(-2) &= -1 + 7 \\ h(-2) &= 6 \end{aligned}$$



5. C.

The graph of $y = 2^x$ is shifted to the right one unit and then stretched vertically by a factor of 3 to get the given graph.

$$y = 3 \cdot 2^{x-1}$$



6. D.

Comparing $f(x) = \log_2 x$ and $g(x) = \log_2(x - 2)$ the graph of $f(x)$ will be translated 2 units horizontally and since it is $x - 2$ the translation is to the right.

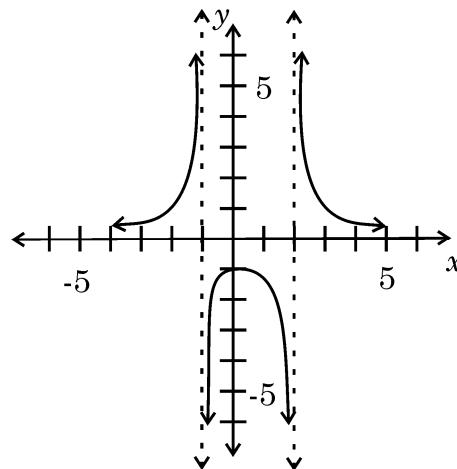
7. B.

The graph of $g(x) = \sqrt{x-3} + 4$ is the graph of $f(x) = \sqrt{x}$ with a horizontal shift of 3 units to the right and a vertical shift of 4 units up.

8. A.

The graph has a horizontal asymptote of $x = 0$, vertical asymptotes of $x = -1$ and $x = 2$ and no zeros.

Therefore $f(x) = \frac{2}{(x-2)(x+1)}$



9. D.

The number of distinct x -intercepts is less than or equal the degree of the function. For a fourth degree polynomial function, the maximum number of x -intercepts is 4.

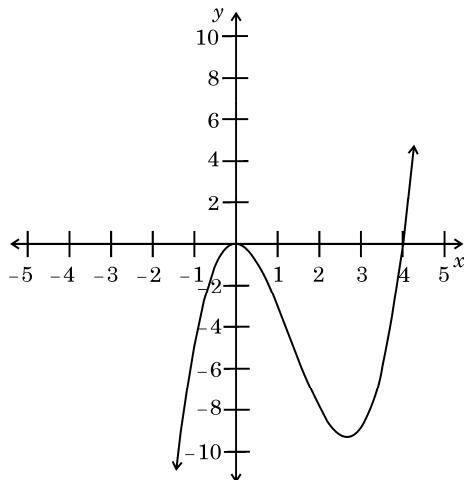
10. C.

The graph has a zero at $x = 0$ with a multiplicity of 2 and a zero at $x = 4$ with a multiplicity of 1.

$$y = ax^2(x - 4)$$

The function begins in quadrant 3 and ends in quadrant 1 therefore a must be positive.

$$y = x^2(x - 4)$$



11. C.

$$27^{x-1} = \sqrt{3}$$

$$(3^3)^{x-1} = 3^{\frac{1}{2}}$$

$$3^{3x-3} = 3^{\frac{1}{2}}$$

$$\therefore 3x - 3 = \frac{1}{2}$$

$$3x = \frac{7}{2}$$

$$x = \frac{7}{6}$$

12. A.

The graph of $f(x) = \frac{x^2 + 5x + 4}{x^2 - 16} = \frac{(x+1)(x+4)}{(x-4)(x+4)} = \frac{(x+1)}{(x-4)}$, $x \neq -4$ will have only

one x -intercept at $(-1, 0)$. The function will have one point of discontinuity

(hole) at $\left(-4, \frac{3}{8}\right)$.

13. A.

$$f(x) = 4x^2 + 10$$

$$y = 4x^2 + 10$$

To find the equation of the inverse relation, switch the x 's and y 's in the equation:

$$x = 4y^2 + 10$$

$$x - 10 = 4y^2$$

$$y^2 = \frac{x - 10}{4}$$

$$\sqrt{y^2} = \pm \sqrt{\frac{x - 10}{4}}$$

$$y = \pm \frac{\sqrt{x - 10}}{2}$$

14. C.

$y = \frac{5x - 13}{x - 3}$ will have a y -intercept at $\left(0, \frac{13}{3}\right)$, an x -intercept at $\left(\frac{13}{5}, 0\right)$, and a vertical asymptote at $x = 3$.

15. D.

To solve $\log_3 20 = x$, rewrite the equation into exponential form ($3^x = 20$) and use whole number exponents to approximate the solution.

$$3^1 = 3$$

$$3^2 = 9 \quad 20 \text{ lies in this range and is closer to } 27 \text{ than it is to } 9$$

$$3^3 = 27$$

$$3^4 = 81$$

16. D.

Since the bases are not powers of one another ($5 \neq 7$), you need to take the log of both sides.

$$\log 7^x = \log 5^{x+3}$$

Use the power law of logs

$$x \log 7 = (x + 3) \log 5$$

$$x \log 7 = x \log 5 + 3 \log 5$$

$$x \log 7 - x \log 5 = 3 \log 5$$

$$x(\log 7 - \log 5) = 3 \log 5$$

$$x = \frac{3 \log 5}{\log 7 - \log 5}$$

$$x = 14.34981318$$

or

$$x \log 7 = (x + 3) \log 5$$

$$x(0.845098) = (x + 3)(0.698970)$$

$$0.845098x = 0.698970x + 2.09691$$

$$0.146128x = 2.09691$$

$$x = 14.34981$$

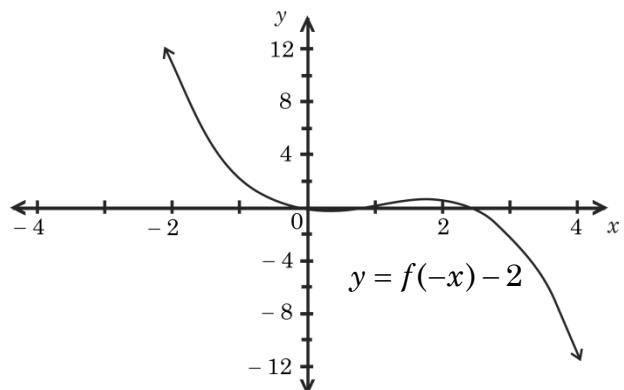
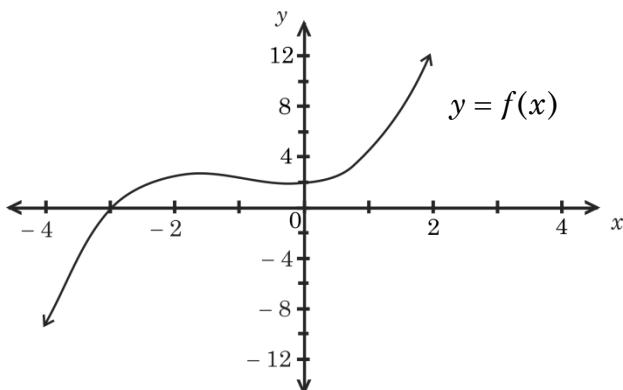
17. A.

The graph of $f(x) = \frac{x^2 - 9}{x^2 - 5x + 6} = \frac{(x+3)(x-3)}{(x-2)(x-3)} = \frac{(x+3)}{(x-2)}$, $x \neq 3$ will have a

vertical asymptote at $x = 2$. The function will have one point of discontinuity (hole) at $(3, 6)$.

18. C.

To obtain $y = f(-x) - 2$, from $y = f(x)$, the graph will be reflected in the y -axis and translated 2 units down.



19. A.

$$\begin{aligned}\sqrt{x+13} &= x+1 \\x+13 &= (x+1)^2 \\x+13 &= x^2 + 2x + 1 \\x^2 + x - 12 &= 0 \\(x+4)(x-3) &= 0 \\x = -4 &\quad x = 3\end{aligned}$$

Verification:

$$\begin{array}{lll}\sqrt{-4+13} = -4+1 & \sqrt{3+13} = 3+1 \\ \sqrt{9} = -3 & \sqrt{16} = 4 \\ 3 = -3 & 4 = 4 \quad \checkmark \\ \text{False, } x = -4 \text{ is extraneous} & \text{True for } x = 3\end{array}$$

∴ The solution is {3}

20. A.

$$\begin{aligned}\log_2(x-1) + \log_2(x+2) &= 2 \\ \log_2(x-1) + \log_2(x+2) &= 2 \\ \log_2(x-1)(x+2) &= 2 \\ x^2 + x - 2 &= 2^2 \\ x^2 + x - 6 &= 0 \\ (x+3)(x-2) &= 0 \\ x+3 = 0 &\quad \text{or} \quad x-2 = 0 \\ x = -3 &\quad \quad \quad x = 2\end{aligned}$$

Verification:

$$\begin{array}{lll}\log_2(-3-1) + \log_2(-3+2) = 2 & \log_2(2-1) + \log_2(2+2) = 2 \\ \log_2(-4) + \log_2(-1) = 2 & \log_2(1) + \log_2(4) = 2 \\ & 0+2=2 \\ \text{Undefined, } x = -3 & 2=2 \quad \checkmark \\ \text{is extraneous} & \end{array}$$

∴ The solution set is {2}

21. B.

$$\begin{aligned}&\frac{5\pi}{4} \\&= \frac{5(180^\circ)}{4} \\&= 225^\circ \\&= (180 + 45)^\circ\end{aligned}$$

22. B.

$$\sin \theta = \frac{7}{9} = \frac{y}{r}$$

$$x^2 + y^2 = r^2$$

$$x^2 = r^2 - y^2$$

$$x^2 = (9)^2 - (7)^2$$

$$x^2 = 81 - 49$$

$$x^2 = 32$$

$$x = \pm\sqrt{32}$$

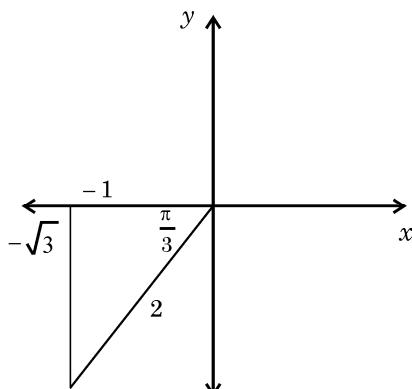
$$x = \pm 4\sqrt{2}$$

Because the terminal arm is in quadrant II: $x = -4\sqrt{2}$; $y = 7$; $r = 9$

$$\cos \theta = \frac{x}{r} = -\frac{4\sqrt{2}}{9}$$

23. C.

$$A = \frac{4\pi}{3}$$



$$\cos A = -\frac{1}{2} \text{ and } \sin A = -\frac{\sqrt{3}}{2}$$

24. B.

$$\sec(-53^\circ) = 1.66$$

$$-\sec 53^\circ = -1.66$$

$$\sec(-127^\circ) = -1.66$$

$$-\sec 127^\circ = 1.66$$

307° is in quadrant 4, so $\sec 307^\circ$ must be positive. The 2 ratios that equal $\sec 307^\circ$ are $\sec(-53^\circ)$ and $-\sec 127^\circ$.

25. D.

Since $\sin A$ is a negative value the two possible solutions are in Quadrant 3 and Quadrant 4. The reference angle for $\sin A = -\frac{\sqrt{2}}{2}$ is 45° .

26. D.

	$y = \sin x$	$y = \tan x$
Amplitude	1	undefined
Asymptotes	None	Yes (example $\frac{\pi}{2}$)
Period	2π	π
x -intercepts	$0, \pi, 2\pi, \dots$	$0, \pi, 2\pi, \dots$

\therefore The x -intercepts are the only characteristic which is the same for the graphs of $y = \sin x$ and $y = \tan x$.

27. A.

$\tan \theta = -\frac{12}{5}$, and θ is in the fourth quadrant, then $x = 5$ and $y = -12$

$$\begin{aligned}x^2 + y^2 &= r^2 \\5^2 + (-12)^2 &= r^2 & \sin \theta &= \frac{y}{r} \\25 + 144 &= r^2 \\ \sqrt{169} &= r & \sin \theta &= -\frac{12}{13} \\13 &= r\end{aligned}$$

28. B.

If the point is $(-3, 7)$, then $x = -3$ and $y = 7$.

$$\begin{aligned}x^2 + y^2 &= r^2 & \cos A &= \frac{x}{r} \\(-3)^2 + 7^2 &= r^2 & \cos A &= -\frac{3}{\sqrt{58}} \\58 &= r^2 \\ \sqrt{58} &= r & \cos A &= -\frac{3\sqrt{58}}{58}\end{aligned}$$

29. A.

$$\frac{\tan 120^\circ + \tan 60^\circ}{1 - \tan 120^\circ \tan 60^\circ} = \tan(120^\circ + 60^\circ) = \tan(180^\circ) = 0$$

30. C.

$$\begin{aligned}\cos \theta + \sin^2 \theta \sec \theta &= \cos \theta + \frac{\sin^2 \theta}{\cos \theta} \\&= \cos \theta \left(\frac{\cos \theta}{\cos \theta} \right) + \frac{\sin^2 \theta}{\cos \theta} \\&= \frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} \\&= \frac{1}{\cos} \\&= \sec \theta\end{aligned}$$

31. C.

The non-permissible values of θ occur when the denominator of $\frac{\sin \theta \cos \theta}{1 - \sin^2 \theta}$ is equal to zero.

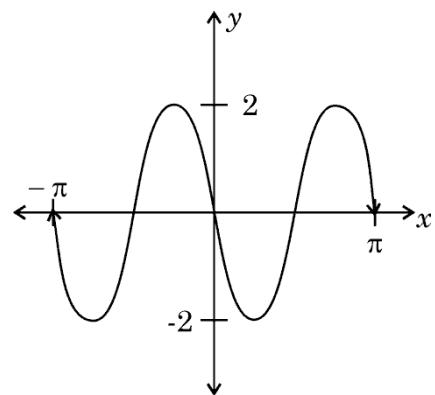
$$\begin{aligned}1 - \sin^2 \theta &= 0 \\ \sin^2 \theta &= 1 \\ \sin \theta &= \pm 1 \\ \sin \theta = 1 &\quad \sin \theta = -1 \\ \theta = \frac{\pi}{2} &\quad \theta = \frac{3\pi}{2}\end{aligned}$$

32. C.

$$\begin{aligned}\tan^2 \theta - 3 \tan \theta + 2 &= 0 \\ (\tan \theta - 2)(\tan \theta - 1) &= 0 \\ \tan \theta = 2 &\quad \tan \theta = 1 \\ \theta = 63.4^\circ, 243.4^\circ &\quad \theta = 45^\circ, 225^\circ\end{aligned}$$

33. B.

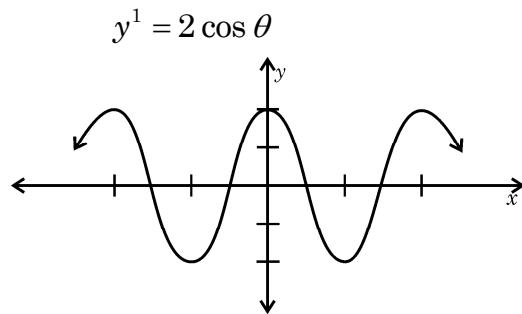
The graph of $y = 2 \cos 2\left(x + \frac{\pi}{4}\right)$ will have
amplitude = 2, period length = π ,
phase shift = $-\frac{\pi}{4}$ (shift left), and no vertical shift.



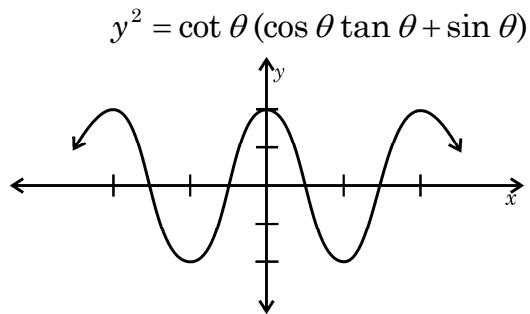
34. A.

Using a graphing calculator the expression that is an identity

$$2 \cos \theta = \cot \theta (\cos \theta \tan \theta + \sin \theta)$$



$$x = 1.0471976 \quad y = 1$$



$$x = 1.0471976 \quad y = 1$$

Verify numerically,

Answer A:

$$2 \cos 45^\circ = \cot 45^\circ (\cos 45^\circ \cdot \tan 45^\circ + \sin 45^\circ)$$

$$2 \left(\frac{\sqrt{2}}{2} \right) \stackrel{?}{=} (1) \left(\frac{\sqrt{2}}{2} (1) + \frac{\sqrt{2}}{2} \right)$$

$$\sqrt{2} \stackrel{?}{=} \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

$$\sqrt{2} = \sqrt{2} \checkmark$$

True \therefore Trigonometric identity

Answer B:

$$\cot 45^\circ \sec 45^\circ = \cos 45^\circ + \tan 45^\circ$$

$$(1) \left(\frac{2}{\sqrt{2}} \right) \stackrel{?}{=} \frac{\sqrt{2}}{2} + 1$$

$$\sqrt{2} \neq \frac{2+\sqrt{2}}{2}$$

False \therefore Not a trigonometric identity

Answer C:

$$\frac{\cos 45^\circ}{\csc^2 45^\circ - 1} \stackrel{?}{=} \sin^2 45^\circ$$

$$\frac{\frac{\sqrt{2}}{2}}{\left(\frac{2}{\sqrt{2}} \right)^2 - 1} \stackrel{?}{=} \left(\frac{\sqrt{2}}{2} \right)^2$$

$$\frac{\frac{\sqrt{2}}{2}}{\frac{4}{2} - 1} \stackrel{?}{=} \frac{2}{4}$$

$$\frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} \stackrel{?}{=} \frac{1}{2}$$

$$\frac{\sqrt{2}}{2} \neq \frac{1}{2}$$

False \therefore Not a trigonometric identity

Answer D:

$$\frac{1 - \sin 30^\circ}{\sin 30} \stackrel{?}{=} \cos 30^\circ$$

$$\frac{1 - \frac{1}{2}}{\frac{1}{2}} \stackrel{?}{=} \frac{\sqrt{3}}{2}$$

$$\frac{\frac{1}{2}}{\frac{1}{2}} \stackrel{?}{=} \frac{\sqrt{3}}{2}$$

$$1 \neq \frac{\sqrt{3}}{2}$$

False \therefore Not a trigonometric identity

35. D.

$\csc \theta = -\frac{2\sqrt{3}}{3}$ is the reciprocal of $\sin \theta = -\frac{\sqrt{3}}{2}$.

\therefore The angles will be $\frac{4\pi}{3}$ and $\frac{5\pi}{3}$.

36. A.

To determine non-permissible values, look at each trigonometric function and assess which terms have non-permissible values.

First term, left side:

$$\frac{\sin \theta}{\tan \theta}$$

$\sin \theta$: no non-permissible values

$\frac{1}{\tan \theta}$: will have non-permissible values where $\tan \theta$ is undefined and also

where $\tan \theta = 0$.

$\tan \theta$ is undefined at 90° and 270°

$\tan \theta = 0$ when $\theta = 0^\circ$ and 180°

Second term, left side:

$$\frac{\tan \theta}{\csc \theta}$$

$\tan \theta$: will have non-permissible values where $\tan \theta$ is undefined
 $\tan \theta$ is undefined at 90° and 270°

$\frac{1}{\csc \theta}$: will have non-permissible values where $\csc \theta = 0$ (which will never happen) and also where $\csc \theta$ is undefined
 $\csc \theta$ is undefined at 0 and 180°

Right side:

$\sec \theta$: will have a non-permissible value where $\sec \theta = 0$ (which will never happen) and also where $\sec \theta$ is undefined
 $\sec \theta$ is undefined at 90° and 270°

All Non-Permissible Values = $0^\circ, 90^\circ, 180^\circ, 270^\circ$

37. C.

$$\text{Number of handbags} \times \text{number of hats} \times \text{number of coats} = 4 \times 5 \times 3 = 60$$
$$4 \times 5 \times 3 = 60$$

38. B.

$$\frac{n!}{r! s! t! \dots} = \frac{8!}{2! 2! 2!} = 5040$$

39. C

$${}_7C_0 = 1$$

$${}_7C_3 = 35$$

$${}_7C_1 = 7$$

$${}_7C_4 = 35$$

$${}_7C_2 = 21$$

$${}_7C_5 \text{ will also equal } 21$$

40. D.

The 3rd term in the expansion of $(2 - 3x)^7$ is found using $\binom{7}{2}(2)^5(-3x)^2$

41. Numerical Response: 78

$$f(x) = 3x - 5 \text{ and } g(x) = (x + 2)^2 - 3$$

$$f(4) = 3(4) - 5 = 12 - 5 = 7$$

$$g(f(4)) = g(7) = (7 + 2)^2 - 3 = 81 - 3 = 78$$

42. Numerical Response: 18

$$\log_5 6 + 3 \log_5 3 - \frac{1}{2} \log_5 81$$

$$= \log_5 6 + \log_5 3^3 - \log_5 81^{\frac{1}{2}}$$

$$= \log_5 6 + \log_5 27 - \log_5 9$$

$$= \log_5 \frac{6 \cdot 27}{9}$$

$$= \log_5 18$$

$$\therefore a = 18$$

43. Numerical Response: 35

$$\begin{array}{r} 2x^3 + x^2 - 8x - 4 \\ \hline x - 3 \end{array}$$

$$2(3)^3 + (3)^2 - 8(3) - 4$$

$$54 + 9 - 24 - 4 = 35$$

or

$$\begin{array}{r} 2 \quad 1 - 8 - 4 \\ 3 \left| \begin{array}{r} \downarrow & 6 \quad 21 \quad 39 \\ 2 \quad 7 \quad 13 \quad 35 \end{array} \right. \end{array}$$

44. Numeric Response: 82

$$A = 1000(1 + 0.005)^n$$

$$1500 = 1000(1.005)^n$$

$$1.5 = (1.005)^n$$

$$\log(1.5) = \log(1.005)^n$$

$$\log(1.5) = n \cdot \log(1.005)$$

$$\frac{\log(1.5)}{\log(1.005)} = n$$

$$81.3 = n$$

∴ need at least 82 months

45. Numerical Response: 108

$$\frac{3\pi}{5} \cdot \frac{180^\circ}{\pi} = 108^\circ$$

46. Numerical Response: 4

$$4 \sin^2 \theta - 1 = 0$$

$$4 \sin^2 \theta = 1$$

$$\sin^2 \theta = \frac{1}{4}$$

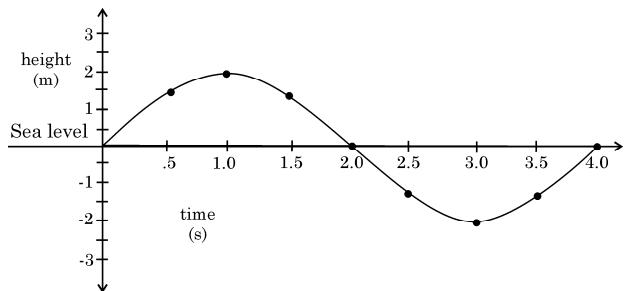
$$\sin \theta = \pm \frac{1}{2}$$

$\sin \theta = \frac{1}{2}$ will occur in quadrants 1 and 2.

$\sin \theta = -\frac{1}{2}$ will occur in quadrants 3 and 4.

47. Numerical Response: 4

One complete wavelength (cycle) is completed in 4 seconds.



48. Numerical Response: 379

Solution 1:

Convert the measure of the central angle to radians

$$\frac{\pi}{180^\circ} = \frac{x}{60^\circ}$$

$$x = \frac{\pi}{3}$$

$$a = \theta r$$

$$a = \left(\frac{\pi}{3}\right)(362)$$

$$a = 379.0855135$$

$$a = 379 \text{ mm}$$

Solution 2: Use proportion

$$\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{full rotation}}$$

$$\frac{a}{2\pi(362)} = \frac{60^\circ}{360^\circ}$$

$$a = 379.0855135$$

$$a = 379 \text{ mm}$$

49. Numerical Response: 495

$${}_{12}C_4 = 495$$

50. Numerical Response: 5103

Given $(x+3)^7$ the term containing x^2 will be given by

$$_7C_5 \cdot (x)^2 \cdot (3)^5 = 21 \cdot x^2 \cdot 243 = 5103 x^2$$