## Chapter 10 Function Operations

## Section 10.1 Sums and Differences of Functions

### Section 10.1 Page 483 Question 1

**a)** For $f(x) = |x + 3|$ and $g(x) = 4$,

$$\begin{aligned} h(x) &= f(x) + g(x) \\ &= |x + 3| + 4 \end{aligned}$$

**b)** For $f(x) = 3x – 5$ and $g(x) = –x + 2$,

$$\begin{aligned} h(x) &= f(x) + g(x) \\ &= 3x - 5 + (-x + 2) \\ &= 3x - 5 - x + 2 \\ &= 2x - 3 \end{aligned}$$

**c)** For $f(x) = x^2 + 2x$ and $g(x) = x^2 + x + 2$,

$$\begin{aligned} h(x) &= f(x) + g(x) \\ &= x^2 + 2x + x^2 + x + 2 \\ &= 2x^2 + 3x + 2 \end{aligned}$$

**d)** For $f(x) = –x – 5$ and $g(x) = (x + 3)^2$,

$$\begin{aligned} h(x) &= f(x) + g(x) \\ &= -x - 5 + (x + 3)^2 \\ &= -x - 5 + x^2 + 6x + 9 \\ &= x^2 + 5x + 4 \end{aligned}$$

### Section 10.1 Page 483 Question 2

**a)** For $f(x) = 6x$ and $g(x) = x – 2$,

$$\begin{aligned} h(x) &= f(x) - g(x) \\ &= 6x - (x - 2) \\ &= 6x - x + 2 \\ &= 5x + 2 \end{aligned}$$

**b)** For $f(x) = –3x + 7$ and $g(x) = 3x^2 + x – 2$,

$$\begin{aligned} h(x) &= f(x) - g(x) \\ &= -3x + 7 - (3x^2 + x - 2) \\ &= -3x + 7 - 3x^2 - x + 2 \\ &= -3x^2 - 4x + 9 \end{aligned}$$

**c)** For $f(x) = 6 – x$ and $g(x) = (x + 1)^2 – 7$,

$$\begin{aligned} h(x) &= f(x) - g(x) \\ &= 6 - x - ((x + 1)^2 - 7) \\ &= 6 - x - (x^2 + 2x + 1 - 7) \\ &= 6 - x - x^2 - 2x + 6 \\ &= -x^2 - 3x + 12 \end{aligned}$$

**d)** For $f(x) = \cos x$ and $g(x) = 4$,
$h(x) = f(x) - g(x)$
$= \cos x - 4$

**Section 10.1 Page 483 Question 3**

For $f(x) = -6x + 1$ and $g(x) = x^2$,

**a)** $h(x) = f(x) + g(x)$
$= -6x + 1 + x^2$
$= x^2 - 6x + 1$
$h(2) = 2^2 - 6(2) + 1$
$= 4 - 12 + 1$
$= -7$

**b)** $m(x) = f(x) - g(x)$
$= -6x + 1 - x^2$
$= -x^2 - 6x + 1$
$m(1) = -1^2 - 6(1) + 1$
$= -1 - 6 + 1$
$= -6$

**c)** $p(x) = g(x) - f(x)$
$= x^2 - (-6x + 1)$
$= x^2 + 6x - 1$
$p(1) = 1^2 + 6(1) - 1$
$= 1 + 6 - 1$
$= 6$

**Section 10.1 Page 483 Question 4**

For $f(x) = 3x^2 + 2$, $g(x) = \sqrt{x+4}$, and $h(x) = 4x - 2$,

**a)** $y = (f + g)(x)$
$= f(x) + g(x)$
$= 3x^2 + 2 + \sqrt{x+4}$

The domain of $f(x) = 3x^2 + 2$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(x) = \sqrt{x+4}$ is $\{x \mid x \geq -4, x \in \mathrm{R}\}$.
The domain of $y = (f + g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \geq -4, x \in \mathrm{R}\}$.

**b)** $y = (h - g)(x)$
$= h(x) - g(x)$
$= 4x - 2 - \sqrt{x+4}$

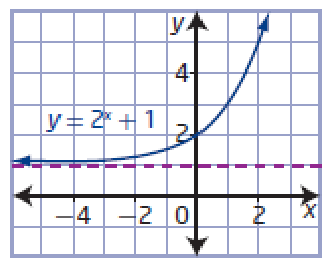
The domain of $h(x) = 4x - 2$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(x) = \sqrt{x+4}$ is $\{x \mid x \geq -4, x \in \mathrm{R}\}$.
The domain of $y = (h - g)(x)$ consists of all values that are in both the domain of $h(x)$ and the domain of $g(x)$: $\{x \mid x \geq -4, x \in \mathrm{R}\}$.

**c)** $y = (g - h)(x)$
$= g(x) - h(x)$
$= \sqrt{x+4} - (4x - 2)$
$= \sqrt{x+4} - 4x + 2$
The domain of $g(x) = \sqrt{x+4}$ is $\{x \mid x \geq -4, x \in \mathrm{R}\}$.
The domain of $h(x) = 4x - 2$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $y = (g - h)(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$: $\{x \mid x \geq -4, x \in \mathrm{R}\}$.

**d)** $y = (f + h)(x)$
$= f(x) + h(x)$
$= 3x^2 + 2 + 4x - 2$
$= 3x^2 + 4x$
The domain of $f(x) = 3x^2 + 2$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $h(x) = 4x - 2$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $y = (f + h)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $h(x)$: $\{x \mid x \in \mathrm{R}\}$.

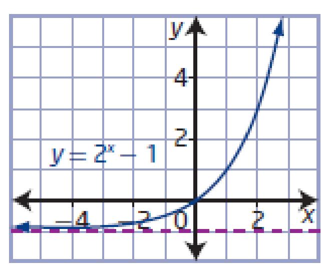
**Section 10.1 Page 483 Question 5**

For $f(x) = 2^x$ and $g(x) = 1$,
**a)** $y = (f + g)(x)$
$= f(x) + g(x)$
$= 2^x + 1$

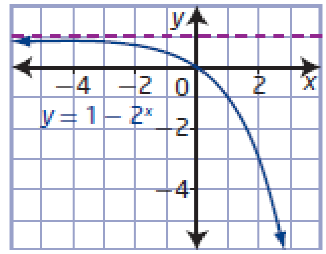



The domain of $f(x) = 2^x$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(x) = 1$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $y = (f + g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (f + g)(x)$ is $\{y \mid y > 1, y \in \mathrm{R}\}$.

**b)** $y = (f - g)(x)$
$= f(x) - g(x)$
$= 2^x - 1$
The domain of $f(x) = 2^x$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(x) = 1$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $y = (f - g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (f - g)(x)$ is $\{y \mid y > -1, y \in \mathrm{R}\}$.



**c)** $y = (g - f)(x)$
$= g(x) - f(x)$
$= 1 - 2^x$




The domain of $g(x) = 1$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $f(x) = 2^x$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $y = (g - f)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (g - f)(x)$ is $\{y \mid y < 1, y \in \mathrm{R}\}$.

**Section 10.1 Page 483 Question 6**

**a)** From the graph, $f(4) = 5$ and $g(4) = 3$ and. So, $(f + g)(4) = 5 + 3 = 8$.

**b)** From the graph, $f(-4) = 5$ and $g(-4) = 1$. So, $(f + g)(-4) = 5 + 1 = 6$.

**c)** From the graph, $f(-5) = 7$ and $g(-5) = 0$. So, $(f + g)(-5) = 7 + 0 = 7$.

**d)** From the graph, $f(-6) = 9$ and $g(-6)$ does not exist. So, $(f + g)(-6)$ is not in the domain.

**Section 10.1 Page 484 Question 7**

First, determine the equations of the functions in the graph as $f(x) = (x - 2)^2 - 5$ and $g(x) = -0.5x^2 + 3$.

**a)** $y = (f + g)(x)$
$= f(x) + g(x)$
$= (x - 2)^2 - 5 + (-0.5x^2 + 3)$
$= x^2 - 4x + 4 - 5 - 0.5x^2 + 3$
$= 0.5x^2 - 4x + 2$

By completing the square, the function in vertex form is $y = 0.5(x - 4)^2 - 6$, where $a = 0.5$, $h = 4$, and $k = -6$. The graph of this parabola has vertex at $(4, -6)$ and has been vertically stretched by a factor of 0.5: graph **B**.

**b)** $y = (f - g)(x)$
$= f(x) + g(x)$
$= (x - 2)^2 - 5 - (-0.5x^2 + 3)$
$= x^2 - 4x + 4 - 5 + 0.5x^2 - 3$
$= 1.5x^2 - 4x - 4$

By completing the square, the function in vertex form is $y = \frac{3}{2}\left(x - \frac{4}{3}\right)^2 - \frac{20}{3}$, where $a = \frac{3}{2}$, $h = \frac{4}{3}$, and $k = -\frac{20}{3}$. The graph of this parabola has vertex at $\left(\frac{4}{3}, -\frac{20}{3}\right)$ and has been vertically stretched by a factor of $\frac{3}{2}$: graph **C**.

**c)** $y = (g - f)(x)$
$= g(x) + f(x)$
$= -0.5x^2 + 3 - ((x - 2)^2 - 5)$
$= -0.5x^2 + 3 - (x^2 - 4x + 4 - 5)$
$= -0.5x^2 + 3 - x^2 + 4x + 1$
$= -1.5x^2 + 4x + 4$

By completing the square, the function in vertex form is $y = -\frac{3}{2}\left(x - \frac{4}{3}\right)^2 + \frac{20}{3}$, where $a = -\frac{3}{2}$, $h = \frac{4}{3}$, and $k = \frac{20}{3}$. The graph of this parabola has vertex at $\left(\frac{4}{3}, \frac{20}{3}\right)$ and has been vertically stretched by a factor of $\frac{3}{2}$ and reflected in the $x$-axis: graph **A**.

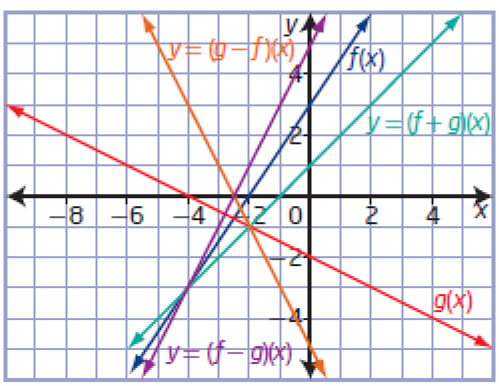
**Section 10.1 Page 484 Question 8**

**a)** First, determine the equations of the functions in the graph as $f(x) = 1.5x + 3$ and $g(x) = -0.5x - 2$.

**i)** $y = (f + g)(x)$
$= f(x) + g(x)$
$= 1.5x + 3 + (-0.5x - 2)$
$= x + 1$

**ii)** $y = (f - g)(x)$
$= f(x) - g(x)$
$= 1.5x + 3 - (-0.5x - 2)$
$= 2x + 5$

**iii)** $y = (g - f)(x)$
$= g(x) - f(x)$
$= -0.5x - 2 - (1.5x + 3)$
$= -2x - 5$

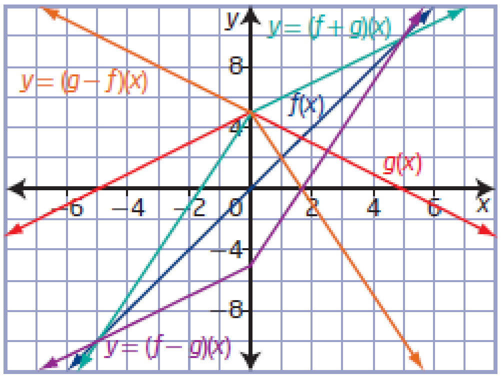



**b)** First, determine the equations of the functions in the graph as $f(x) = 2x$ and $g(x) = -|x| + 5$.

**i)** $y = (f + g)(x)$
$= f(x) + g(x)$
$= 2x + (-|x| + 5)$
$= 2x - |x| + 5$

**ii)** $y = (f - g)(x)$
$= f(x) - g(x)$
$= 2x - (-|x| + 5)$
$= 2x + |x| - 5$

**iii)** $y = (g - f)(x)$
$= g(x) - f(x)$
$= -|x| + 5 - 2x$




**Section 10.1 Page 484 Question 9**

For $f(x) = 3x^2 + 2$, $g(x) = 4x$, and $h(x) = 7x - 1$,

**a)** $y = f(x) + g(x) + h(x)$
$= 3x^2 + 2 + 4x + 7x - 1$
$= 3x^2 + 11x + 1$

**b)** $y = f(x) + g(x) - h(x)$
$= 3x^2 + 2 + 4x - (7x - 1)$
$= 3x^2 - 3x + 3$

**c)** $y = f(x) - g(x) + h(x)$
$= 3x^2 + 2 - 4x + 7x - 1$
$= 3x^2 + 3x + 1$

**d)** $y = f(x) - g(x) - h(x)$
$= 3x^2 + 2 - 4x - (7x - 1)$
$= 3x^2 - 11x + 3$

**Section 10.1 Page 484 Question 10**

$h(x) = (f + g)(x)$
$h(x) = f(x) + g(x)$
$g(x) = h(x) - f(x)$

**a)** Substitute $h(x) = x^2 + 5x + 2$ and $f(x) = 5x + 2$.
$g(x) = h(x) - f(x)$
$= x^2 + 5x + 2 - (5x + 2)$
$= x^2$

**b)** Substitute $h(x) = \sqrt{x+7} + 5x + 2$ and $f(x) = 5x + 2$.
$g(x) = h(x) - f(x)$
$= \sqrt{x+7} + 5x + 2 - (5x + 2)$
$= \sqrt{x+7}$

**c)** Substitute $h(x) = 2x + 3$ and $f(x) = 5x + 2$.
$g(x) = h(x) - f(x)$
$= 2x + 3 - (5x + 2)$
$= -3x + 1$

**d)** Substitute $h(x) = 3x^2 + 4x - 2$ and $f(x) = 5x + 2$.
$g(x) = h(x) - f(x)$
$= 3x^2 + 4x - 2 - (5x + 2)$
$= 3x^2 - x - 4$

**Section 10.1 Page 484 Question 11**

$h(x) = (f - g)(x)$
$h(x) = f(x) - g(x)$
$g(x) = f(x) - h(x)$

**a)** Substitute $h(x) = -x^2 + 5x + 3$ and $f(x) = 5x + 2$.

$$\begin{aligned} g(x) &= f(x) - h(x) \\ &= 5x + 2 - (-x^2 + 5x + 3) \\ &= x^2 - 1 \end{aligned}$$

**b)** Substitute $h(x) = \sqrt{x-4} + 5x + 2$ and $f(x) = 5x + 2$.

$$\begin{aligned} g(x) &= f(x) - h(x) \\ &= 5x + 2 - (\sqrt{x-4} + 5x + 2) \\ &= -\sqrt{x-4} \end{aligned}$$

**c)** Substitute $h(x) = -3x + 11$ and $f(x) = 5x + 2$.

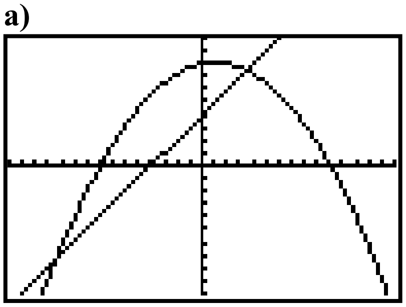
$$\begin{aligned} g(x) &= f(x) - h(x) \\ &= 5x + 2 - (-3x + 11) \\ &= 8x - 9 \end{aligned}$$

**d)** Substitute $h(x) = -2x^2 + 16x + 8$ and $f(x) = 5x + 2$.

$$\begin{aligned} g(x) &= f(x) - h(x) \\ &= 5x + 2 - (-2x^2 + 16x + 8) \\ &= 2x^2 - 11x - 6 \end{aligned}$$

**Section 10.1 Page 485 Question 12**

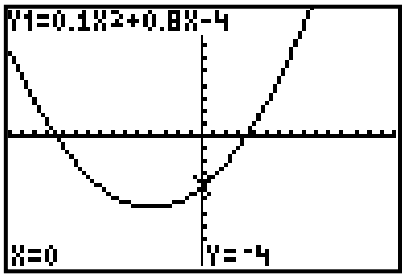
**a)**



The points of intersection represent where the supply equals the demand. The intersection point in quadrant III should not be considered since the price cannot be negative.

**b)** Substitute $S(p) = p + 4$ and $D(p) = -0.1(p + 8)(p - 10)$.

$$\begin{aligned} y &= S(p) - D(p) \\ &= p + 4 - (-0.1(p + 8)(p - 10)) \\ &= p + 4 + 0.1(p^2 - 2p - 80) \\ &= p + 4 + 0.1p^2 - 0.2p - 8 \\ &= 0.1p^2 + 0.8p - 4 \end{aligned}$$

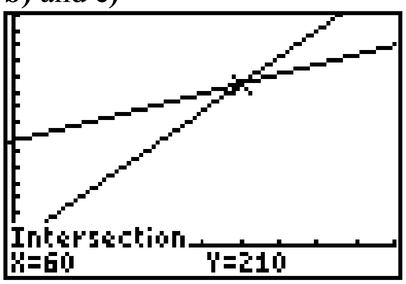



This function models the excess supply as a function of cost.

**Section 10.1 Page 485 Question 13**

**a)** The total cost is represented by $C(n) = 135 + 1.25n$.
The total revenue is represented by $R(n) = 3.5n$.

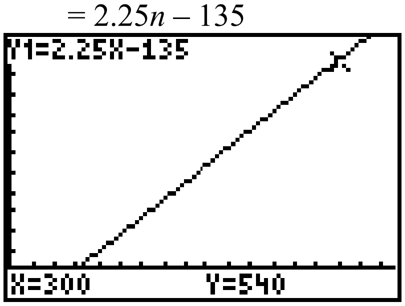
**b)** and **c)**




The break-even point is (60, 210). When 60 hamburgers are sold, the cost equals the revenue, \$210.

**d)** Profit can be represented by a difference function:

$$\begin{aligned}P(n) &= R(n) - C(n)\\ &= 3.5n - (135 + 1.25n)\\ &= 2.25n - 135\end{aligned}$$



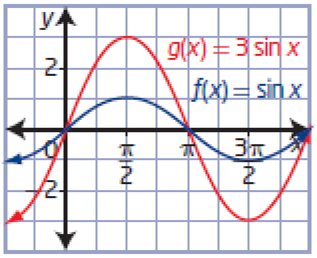

**e)** Substitute $n = 300$, the maximum number of hamburgers that can be sold in a day.

$$\begin{aligned}P(n) &= 2.25n - 135\\ P(300) &= 2.25(300) - 135\\ P(300) &= 675 - 135\\ P(300) &= 540\end{aligned}$$

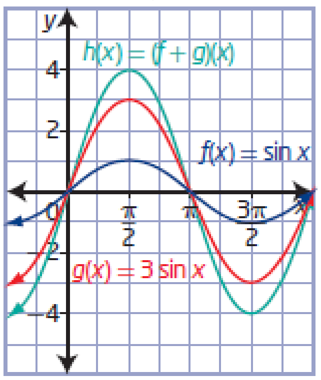
The maximum daily profit the vendor can earn is \$540.

## Section 10.1 Page 485 Question 14

**a)**



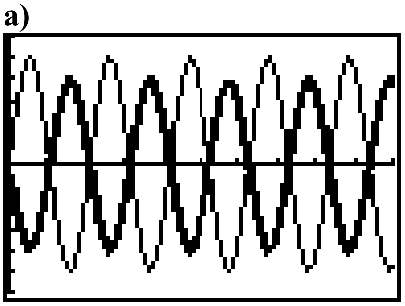

**b)** Add the $y$-values at each $x$-value and draw a smooth curve through the points.




**c)** The maximum height of the resultant wave is 4 cm.

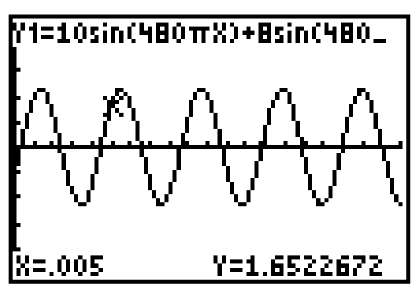
## Section 10.1 Page 485 Question 15

**a)**



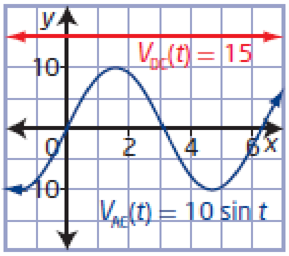
**b)** The maximum values of one function are located at the same $x$-coordinates as the minimum values of the other function, and vice versa. This will result in destructive interference.

**c)** $y = E(t) + R(t)$
$= 10 \sin 480\pi t + 8 \sin 480\pi(t - 0.002)$

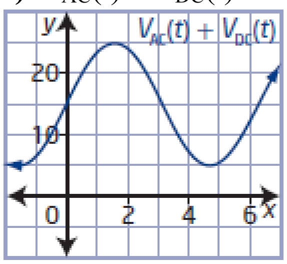


**Section 10.1 Page 486 Question 16**

**a)**




**b)** $V_{AC}(t) + V_{DC}(t) = 10 \sin t + 15$




**c)** The domain of $V_{AC}(t) + V_{DC}(t)$ is $\{t \mid t \in \mathrm{R}\}$ and the range is $\{V \mid 5 \leq V \leq 25, V \in \mathrm{R}\}$.

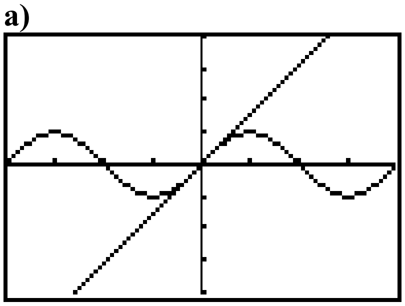
**d) i)** The minimum value of the voltage signal is 5 V.
**ii)** The maximum value of the voltage signal is 25 V.

**Section 10.1 Page 486 Question 17**

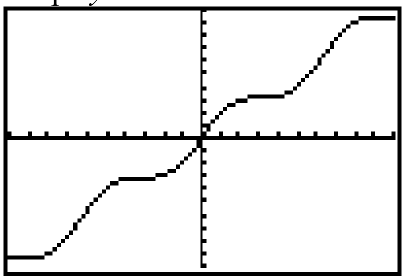
$$\begin{aligned} h(t) &= d_1(t) - d_2(t) \\ &= 10t^2 - 5(t + 2)^2 \\ &= 10t^2 - 5(t^2 + 4t + 4) \\ &= 10t^2 - 5t^2 - 20t - 20 \\ &= 10t^2 - 20t - 20 \end{aligned}$$

**Section 10.1 Page 486 Question 18**

**a)**



**b)** My prediction is that the shape of $h(x) = f(x) + g(x)$ will be a sinusoidal function on a diagonal according to $y = x$.
Graph $y = \sin x + x$:

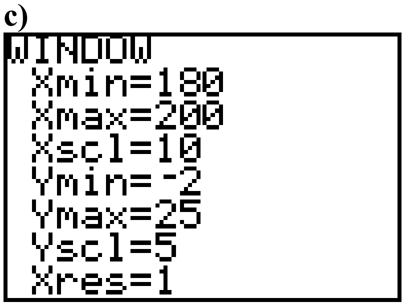


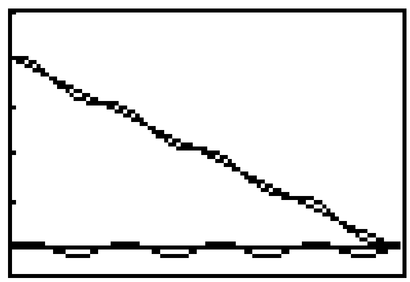
**Section 10.1 Page 486 Question 19**

**a)** A function representing the skier's distance, $d$, from the base of the hill versus time, $t$, in seconds, is $d = 200 - t$.

**b)** If the height, $m$, of the skier through the moguls is $m(t) = 0.75 \sin 1.26t$, then a function that represents the skier's actual path of height versus time is
$h(t) = 200 - t + 0.75 \sin 1.26t$.

**c)**



**Section 10.1 Page 487 Question 20**

Example: Replace all $x$ with $-x$ and then simplify. If the new function is equal to the original, then it is even. If it is equal to $-1$ times the original, then it is odd. If it is neither equal to the original nor equal to $-1$ times the original, then it is neither.
Let $f(x) = |x + 1|$, $g(x) = x^2 + 2x + 1$, and $h(x) = 2^x - 1$.
Check $k(x) = f(x) + g(x)$.

$$\begin{aligned} k(x) &= f(x) + g(x) \\ &= |x + 1| + x^2 + 2x + 1 \end{aligned}$$

$$\begin{aligned} k(-x) &= |-x + 1| + (-x)^2 + 2(-x) + 1 \\ &= |-x + 1| + x^2 - 2x + 1 \end{aligned}$$

Since $k(-x) \neq k(x)$ and $k(-x) \neq -k(x)$, it is neither even nor odd.
Check $k(x) = f(x) + h(x)$.

$$\begin{aligned} k(x) &= f(x) + h(x) \\ &= |x + 1| + 2^x - 1 \end{aligned}$$

$$k(-x) = |-x + 1| + 2^{-x} - 1$$

Since $k(-x) \neq k(x)$ and $k(-x) \neq -k(x)$, it is neither even nor odd.

Check $k(x) = g(x) + h(x)$.

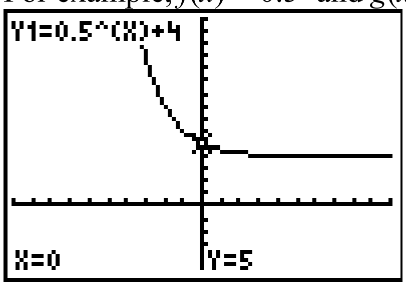
$k(x) = g(x) + h(x)$
$= x^2 + 2x + 1 + 2^x - 1$
$= x^2 + 2x + 2^x$

$k(-x) = (-x)^2 + 2(-x) + 2^{-x}$
$= x^2 - 2x + 2^{-x}$

Since $k(-x) \neq k(x)$ and $k(-x) \neq -k(x)$, it is neither even nor odd.

**Section 10.1 Page 487 Question 21**

The graph shows the sum of an exponential function and a constant function.
For example, $f(x) = 0.5^x$ and $g(x) = 4$.

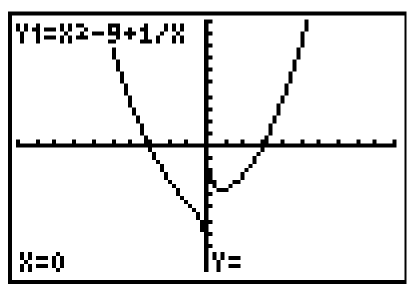



**Section 10.1 Page 487 Question 22**

**a)** The domain of $f(x) = x^2 - 9$ is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -9, y \in \mathrm{R}\}$.
The domain of $g(x) = \dfrac{1}{x}$ is $\{x \mid x \neq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 0, y \in \mathrm{R}\}$.

**b)** $h(x) = f(x) + g(x)$
$= x^2 - 9 + \dfrac{1}{x}$

**c)** The domain of $h(x)$ is $\{x \mid x \neq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \neq 0, y \in \mathrm{R}\}$.
The domain and range of $f(x)$ are different from the domain and range of $h(x)$.
The domain and range of $g(x)$ are the same as that of $h(x)$.




**Section 10.1 Page 487 Question C1**

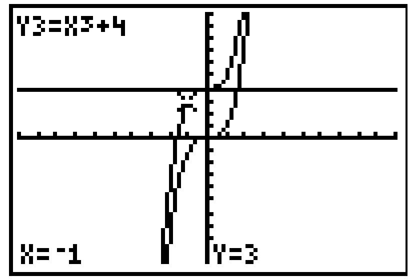
**a)** $f(x) + g(x) = g(x) + f(x)$ is true for all functions because addition is commutative.

**b)** $(f - g)(x) = (g - f)(x)$ is not true for all functions because subtraction is not commutative.

**Section 10.1 Page 487 Question C2**

**a)** For $y_1 = x^3$ and $y_2 = 4$, $y_3$ is the sum of the functions given for $y_1$ and $y_2$.

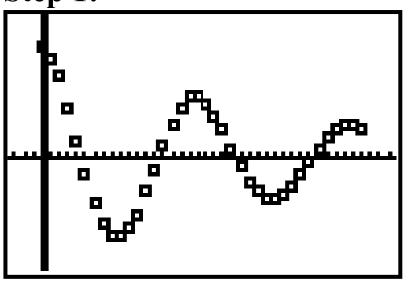
$y_3 = y_1 + y_2$
$\quad = x^3 + 4$




**b)** The domain of $y_3$ is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

**Section 10.1 Page 487 Question C3**

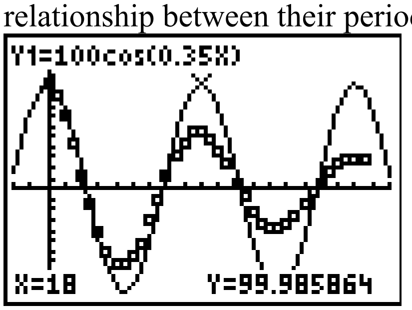
**Step 1:**



The graph exhibits sinusoidal features in its shape and the fact that it is periodic.

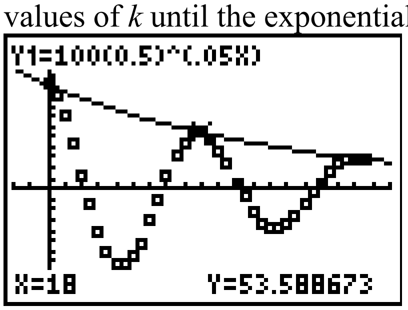
**Step 2:** The graph exhibits exponential features in that it is decreasing and approaching $y = 0$.

**Step 3:** Use technology to graph the function $h = 100 \cos (kt)$, where $k$ is a constant. Test values of $k$ until the crests and troughs of the function occur at the same times as for the scatter plot in step 1. Using $a = 100$ allows you to see both graphs on the screen the relationship between their periods.



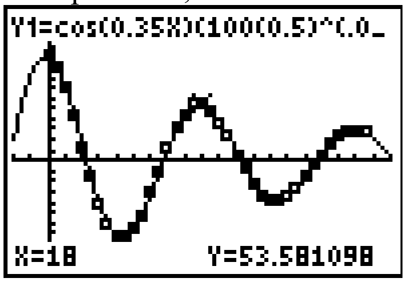

So, a cosine function that has the same period as the scatter plot in step 1 is $h = \cos 0.35t$.

**Step 4:** Use technology to graph the function $h = 100(0.5)^{kt}$, where $k$ is a constant. Test values of $k$ until the exponential curve just touches each crest of the scatter plot in step 1.



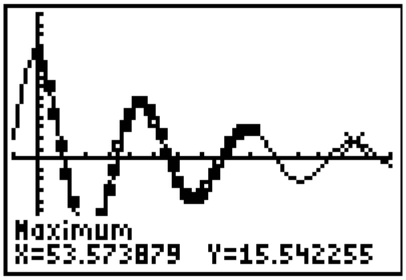

So, an exponential function that models the decay in the amplitude of the scatter plot in step 1 is $h = 100(0.5)^{0.05t}$.

**Step 5:** Try various operations with the two functions: addition, subtraction, multiplication, and division.




So, a combined function that models the height-time relationship of the bungee jumper is $h = (\cos 0.35t)(100(0.5)^{0.05t})$ or $h = (100 \cos 0.35t)((0.5)^{0.05t})$.

**Step 6:** Change the window settings to extend the horizontal axis and determine the maximum value of the fourth crest.




The bungee jumper will be approximately 15.5 m above the rest position at his fourth crest.

**Section 10.2 Products and Quotients of Functions**

**Section 10.2 Page 496 Question 1**

**a)** For $f(x) = x + 7$ and $g(x) = x - 7$,

$$\begin{aligned} h(x) &= f(x)g(x) \\ &= (x + 7)(x - 7) \\ &= x^2 - 49 \end{aligned}$$

$$\begin{aligned} k(x) &= \frac{f(x)}{g(x)} \\ &= \frac{x+7}{x-7}, x \neq 7 \end{aligned}$$

**b)** For $f(x) = 2x - 1$ and $g(x) = 3x + 4$,

$$\begin{aligned} h(x) &= f(x)g(x) \\ &= (2x - 1)(3x + 4) \\ &= 6x^2 + 5x - 4 \end{aligned}$$

$$\begin{aligned} k(x) &= \frac{f(x)}{g(x)} \\ &= \frac{2x-1}{3x+4}, x \neq -\frac{4}{3} \end{aligned}$$

**c)** For $f(x) = \sqrt{x+5}$ and $g(x) = x + 2$,

$$\begin{aligned} h(x) &= f(x)g(x) \\ &= (\sqrt{x+5})(x + 2) \\ &= x\sqrt{x+5} + 2\sqrt{x+5} \end{aligned}$$

$$\begin{aligned} k(x) &= \frac{f(x)}{g(x)} \\ &= \frac{\sqrt{x+5}}{x+2}, x \geq -5, x \neq -2 \end{aligned}$$

**d)** For $f(x) = \sqrt{x-1}$ and $g(x) = \sqrt{6-x}$,

$$
\begin{aligned}
h(x) &= f(x)g(x) \\
&= (\sqrt{x-1})(\sqrt{6-x}) \\
&= \sqrt{x^2+7x-6}
\end{aligned}
$$

$$
\begin{aligned}
k(x) &= \frac{f(x)}{g(x)} \\
&= \frac{\sqrt{x-1}}{\sqrt{6-x}},\ 1 \le x < 6
\end{aligned}
$$

**Section 10.2 Page 496 Question 2**

**a)** From the graph, $f(-2) = 3$ and $g(-2) = -1$. So, $(f \bullet g)(-2) = 3(-1) = -3$.

**b)** From the graph, $f(1) = 0$ and $g(1) = 2$. So, $(f \bullet g)(1) = 0(2) = 0$.

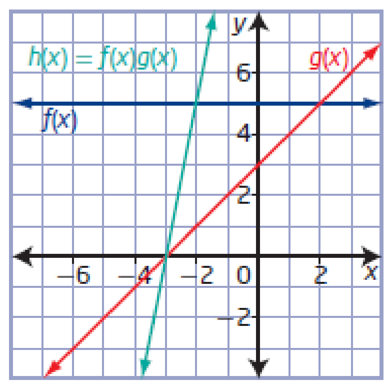
**c)** From the graph, $f(0) = -1$ and $g(0) = 1$. So, $\left(\frac{f}{g}\right)(0) = \frac{-1}{1} = -1$.

**d)** From the graph, $f(1) = 0$ and $g(1) = 2$. So, $\left(\frac{f}{g}\right)(1) = \frac{0}{2} = 0$.

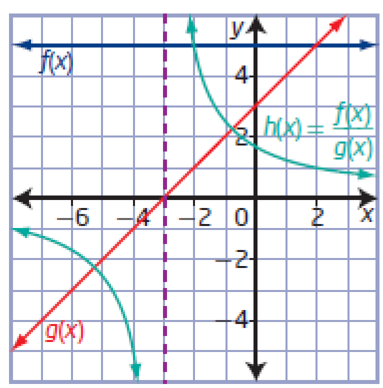
**Section 10.2 Page 496 Question 3**

First, determine the equations of the functions in the graph as $f(x) = 5$ and $g(x) = x + 3$.

**a)**
$$
\begin{aligned}
h(x) &= f(x)g(x) \\
&= 5(x+3) \\
&= 5x + 15
\end{aligned}
$$



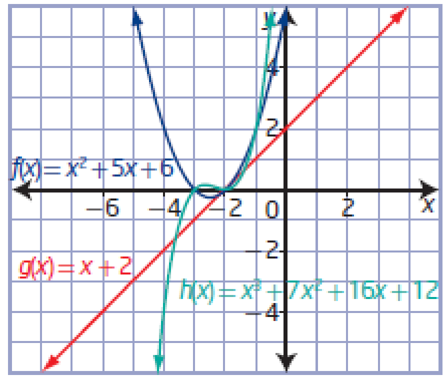

**b)**
$$
\begin{aligned}
k(x) &= \frac{f(x)}{g(x)} \\
&= \frac{5}{x+3},\ x \ne -3
\end{aligned}
$$



**Section 10.2 Page 496 Question 4**

**a)** For $f(x) = x^2 + 5x + 6$ and $g(x) = x + 2$,

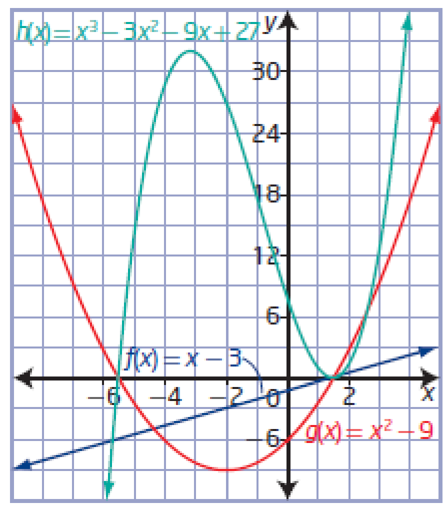
$$\begin{aligned} h(x) &= (f \bullet g)(x) \\ &= f(x)g(x) \\ &= (x^2 + 5x + 6)(x + 2) \\ &= x^3 + 2x^2 + 5x^2 + 10x + 6x + 12 \\ &= x^3 + 7x^2 + 16x + 12 \end{aligned}$$




The function $f(x) = x^2 + 5x + 6$ is quadratic with domain $\{x \mid x \in \mathrm{R}\}$.
The function $g(x) = x + 2$ is linear with domain $\{x \mid x \in \mathrm{R}\}$.
The domain of $h(x) = (f \bullet g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \in \mathrm{R}\}$.

**b)** For $f(x) = x - 3$ and $g(x) = x^2 - 9$,

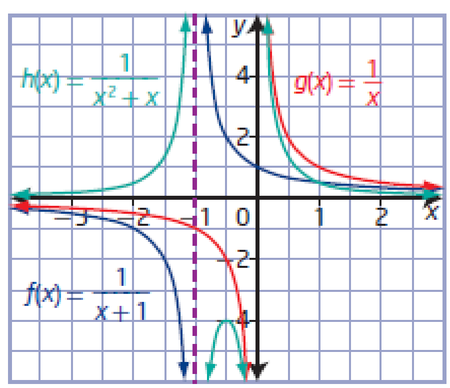
$$\begin{aligned} h(x) &= (f \bullet g)(x) \\ &= f(x)g(x) \\ &= (x - 3)(x^2 - 9) \\ &= x^3 - 3x^2 - 9x + 27 \end{aligned}$$




The function $f(x) = x - 3$ is linear with domain $\{x \mid x \in \mathrm{R}\}$.
The function $g(x) = x^2 - 9$ is quadratic with domain $\{x \mid x \in \mathrm{R}\}$.
The domain of $h(x) = (f \bullet g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \in \mathrm{R}\}$.

**c)** For $f(x) = \dfrac{1}{x+1}$ and $g(x) = \dfrac{1}{x}$,

$$\begin{aligned} h(x) &= (f \bullet g)(x) \\ &= f(x)g(x) \\ &= \frac{1}{x+1}\left(\frac{1}{x}\right) \\ &= \frac{1}{x^2 + x} \end{aligned}$$



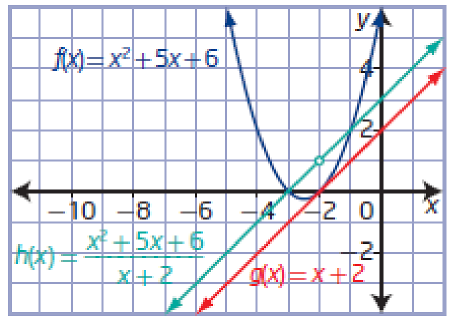

The function $f(x) = \dfrac{1}{x+1}$ has domain $\{x \mid x \neq -1, x \in \mathrm{R}\}$.
The function $g(x) = \dfrac{1}{x}$ has domain $x \mid x \neq 0, x \in \mathrm{R}\}$.

The domain of $h(x) = (f \bullet g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \neq -1, 0, x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \leq -4 \text{ or } y > 0, y \in \mathrm{R}\}$.

**Section 10.2 Page 496 Question 5**

**a)** For $f(x) = x^2 + 5x + 6$ and $g(x) = x + 2$,

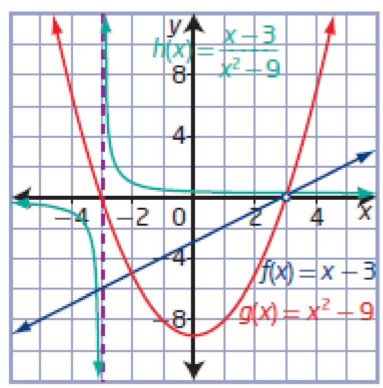
$$\begin{aligned} h(x) &= \left(\frac{f}{g}\right)(x) \\ &= \frac{f(x)}{g(x)} \\ &= \frac{x^2+5x+6}{x+2} \\ &= \frac{(x+2)(x+3)}{x+2} \\ &= x+3, x \neq -2 \end{aligned}$$




The function $f(x) = x^2 + 5x + 6$ is quadratic with domain $\{x \mid x \in \mathrm{R}\}$.
The function $g(x) = x + 2$ is linear with domain $\{x \mid x \in \mathrm{R}\}$.
The domain of $h(x) = \left(\frac{f}{g}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of $x$ where $g(x) = 0$: $\{x \mid x \neq -2, x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \neq 1, y \in \mathrm{R}\}$.

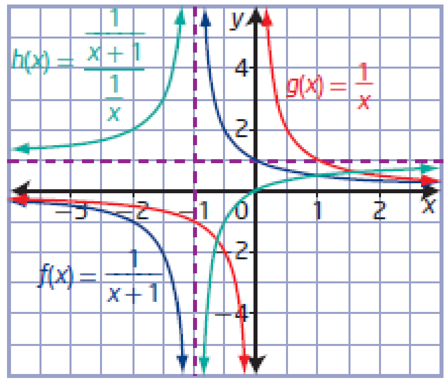
**b)** For $f(x) = x - 3$ and $g(x) = x^2 - 9$,

$$\begin{aligned} h(x) &= \left(\frac{f}{g}\right)(x) \\ &= \frac{f(x)}{g(x)} \\ &= \frac{x-3}{x^2-9} \\ &= \frac{x-3}{(x-3)(x+3)} \\ &= \frac{1}{x+3}, x \neq -3, 3 \end{aligned}$$




The function $f(x) = x - 3$ is linear with domain $\{x \mid x \in \mathrm{R}\}$.
The function $g(x) = x^2 - 9$ is quadratic with domain $\{x \mid x \in \mathrm{R}\}$.
The domain of $h(x) = \left(\frac{f}{g}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of $x$ where $g(x) = 0$: $\{x \mid x \neq -3, 3, x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \neq 0, \frac{1}{6}, y \in \mathrm{R}\}$.

**c)** For $f(x) = \dfrac{1}{x+1}$ and $g(x) = \dfrac{1}{x}$,




$$\begin{aligned} h(x) &= \left(\frac{f}{g}\right)(x) \\ &= \frac{f(x)}{g(x)} \\ &= \frac{\frac{1}{x+1}}{\frac{1}{x}} \\ &= \frac{x}{x+1}, x \neq -1, 0 \end{aligned}$$

The function $f(x) = \dfrac{1}{x+1}$ has domain $\{x \mid x \neq -1, x \in \mathrm{R}\}$.

The function $g(x) = \dfrac{1}{x}$ has domain $\{x \mid x \neq 0, x \in \mathrm{R}\}$.

The domain of $h(x) = \left(\dfrac{f}{g}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of $x$ where $g(x) = 0$: $\{x \mid x \neq -1, 0, x \in \mathrm{R}\}$. The range of $h(x)$ is $\{y \mid y \neq 0, 1, y \in \mathrm{R}\}$.

**Section 10.2 Page 496 Question 6**

For $f(x) = x + 2$, $g(x) = x - 3$, and $h(x) = x + 4$,

**a)**
$$\begin{aligned} y &= f(x)g(x)h(x) \\ &= (x + 2)(x - 3)(x + 4) \\ &= (x^2 - x - 6)(x + 4) \\ &= x^3 + 4x^2 - x^2 - 4x - 6x - 24 \\ &= x^3 + 3x^2 - 10x - 24 \end{aligned}$$

**b)**
$$\begin{aligned} y &= \frac{f(x)g(x)}{h(x)} \\ &= \frac{(x+2)(x-3)}{x+4} \\ &= \frac{x^2-x-6}{x+4}, x \neq -4 \end{aligned}$$

**c)**
$$\begin{aligned} y &= \frac{f(x)+g(x)}{h(x)} \\ &= \frac{x+2+x-3}{x+4} \\ &= \frac{2x-1}{x+4}, x \neq -4 \end{aligned}$$

**d)**
$$\begin{aligned} y &= \frac{f(x)}{h(x)} \times \frac{g(x)}{h(x)} \\ &= \frac{x+2}{x+4}\left(\frac{x-3}{x+4}\right) \\ &= \frac{x^2-x-6}{x^2+8x+16}, x \neq -4 \end{aligned}$$

**Section 10.2 Page 496 Question 7**

$h(x) = f(x)g(x)$

$g(x) = \dfrac{h(x)}{f(x)}$

**a)** Substitute $h(x) = 6x + 15$ and $f(x) = 2x + 5$.

$$\begin{aligned} g(x) &= \frac{h(x)}{f(x)} \\ &= \frac{6x+15}{2x+5} \\ &= \frac{3(2x+5)}{2x+5} \\ &= 3 \end{aligned}$$

**b)** Substitute $h(x) = -2x^2 - 5x$ and $f(x) = 2x + 5$.

$$\begin{aligned} g(x) &= \frac{h(x)}{f(x)} \\ &= \frac{-2x^2-5x}{2x+5} \\ &= \frac{-x(2x+5)}{2x+5} \\ &= -x \end{aligned}$$

**c)** Substitute $h(x) = 2x\sqrt{x} + 5\sqrt{x}$ and $f(x) = 2x + 5$.

$$\begin{aligned} g(x) &= \frac{h(x)}{f(x)} \\ &= \frac{2x\sqrt{x}+5\sqrt{x}}{2x+5} \\ &= \frac{\sqrt{x}(2x+5)}{2x+5} \\ &= \sqrt{x} \end{aligned}$$

**d)** Substitute $h(x) = 10x^2 + 13x - 30$ and $f(x) = 2x + 5$.

$$\begin{aligned} g(x) &= \frac{h(x)}{f(x)} \\ &= \frac{10x^2+13x-30}{2x+5} \\ &= \frac{(5x-6)(2x+5)}{2x+5} \\ &= 5x-6 \end{aligned}$$

**Section 10.2 Page 496 Question 8**

$h(x) = \frac{f(x)}{g(x)}$ and, therefore, $g(x) = \frac{f(x)}{h(x)}$

**a)** Substitute $h(x) = \frac{3x-1}{x+7}$ and $f(x) = 3x - 1$.

$$\begin{aligned} g(x) &= \frac{f(x)}{h(x)} \\ &= \frac{3x-1}{\frac{3x-1}{x+7}} \\ &= x+7 \end{aligned}$$

**b)** Substitute $h(x) = \frac{3x-1}{\sqrt{x+6}}$ and $f(x) = 3x - 1$.

$$\begin{aligned} g(x) &= \frac{f(x)}{h(x)} \\ &= \frac{3x-1}{\frac{3x-1}{\sqrt{x+6}}} \\ &= \sqrt{x+6} \end{aligned}$$

**c)** Substitute $h(x) = 1.5x - 0.5$ and $f(x) = 3x - 1$.

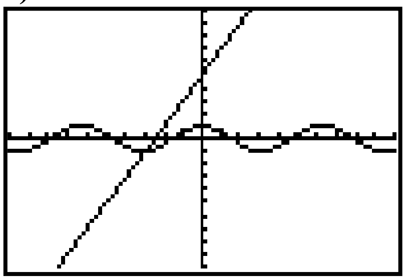
$$\begin{aligned} g(x) &= \frac{f(x)}{h(x)} \\ &= \frac{3x-1}{1.5x-0.5} \\ &= \frac{3x-1}{0.5(3x-1)} \\ &= 2 \end{aligned}$$

**d)** Substitute $h(x) = \frac{1}{x+9}$ and $f(x) = 3x - 1$.

$$\begin{aligned} g(x) &= \frac{f(x)}{h(x)} \\ &= \frac{3x-1}{\frac{1}{x+9}} \\ &= (3x-1)(x+9) \\ &= 3x^2+26x-9 \end{aligned}$$

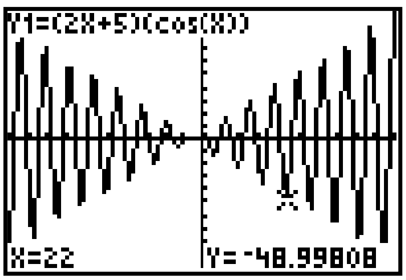
**Section 10.2 Page 496 Question 9**

**a)**



For $f(x) = 2x + 5$, the domain is $\{x \mid x \in \mathrm{R}\}$ and range is $\{y \mid y \in \mathrm{R}\}$.
For $g(x) = \cos x$, the domain is $\{x \mid x \in \mathrm{R}\}$ and range is $\{y \mid -1 \le y \le 1, y \in \mathrm{R}\}$.

**b)** $y = f(x)g(x)$
$= (2x + 5)(\cos x)$

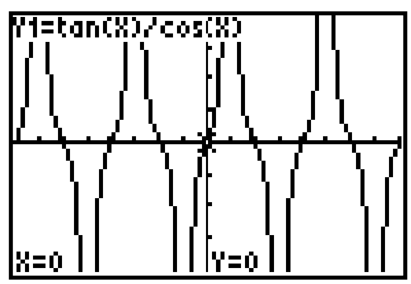



For $y = f(x)g(x)$, the domain is $\{x \mid x \in \mathrm{R}\}$ and range is $\{y \mid y \in \mathrm{R}\}$.

**Section 10.2 Page 496 Question 10**

**a)** Substitute $f(x) = \tan x$ and $g(x) = \cos x$.

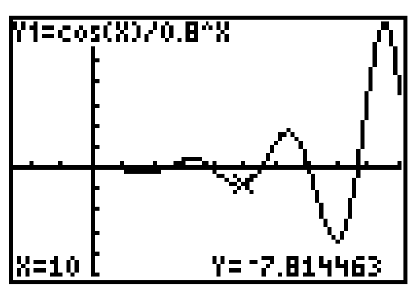
$$y = \left(\frac{f}{g}\right)(x)$$
$$= \frac{\tan x}{\cos x}$$




The domain is $\left\{x \mid x \ne (2n-1)\frac{\pi}{2}, n \in \mathrm{I}, x \in \mathrm{R}\right\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

**b)** Substitute $f(x) = \cos x$ and $g(x) = 0.8^x$.

$$y = \left(\frac{f}{g}\right)(x)$$
$$= \frac{\cos x}{0.8^x}$$




The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

**Section 10.2 Page 497 Question 11**

**a)** $\tan x = \dfrac{\sin x}{\cos x}$

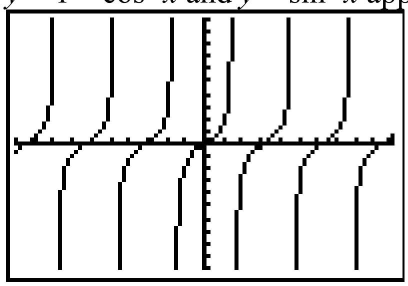
$= \dfrac{f(x)}{g(x)}$

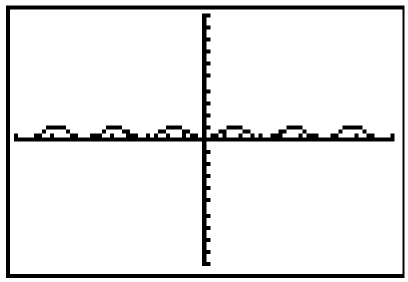
**b)** $1 - \cos^2 x = \sin^2 x$

$= (\sin x)(\sin x)$

$= f(x)f(x)$

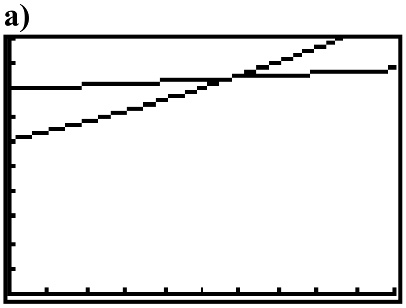
**c)** The graphs of $y = \dfrac{\sin x}{\cos x}$ and $y = \tan x$ appear to be the same. The graphs of $y = 1 - \cos^2 x$ and $y = \sin^2 x$ appear to be the same.





**Section 10.2 Page 497 Question 12**

**a)**

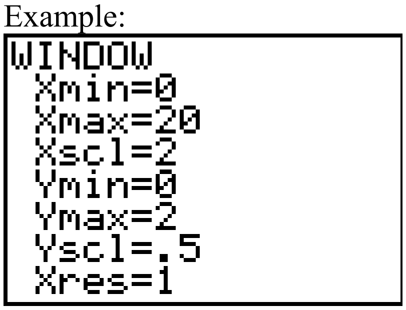


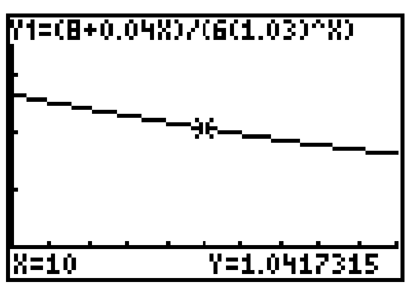
Both graphs are increasing over time. However, the graph of $P(t)$ increases more rapidly and overtakes the graph of $F(t)$.

**b)** $y = \dfrac{F(t)}{P(t)}$

$= \dfrac{8 + 0.04t}{6(1.03)^t}$

Example:



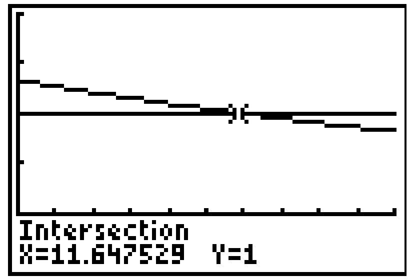




Values of $t$ that are less than 0 should not be considered. Time cannot be negative.

**c)** From the graph, the amount of food per fish is a maximum at $t = 0$.

**d)** The fish farm will no longer be viable when $\dfrac{F(t)}{P(t)} < 1$.

Graph $y = \dfrac{8 + 0.04t}{6(1.03)^t}$ and $y = 1$ and find the point of intersection.

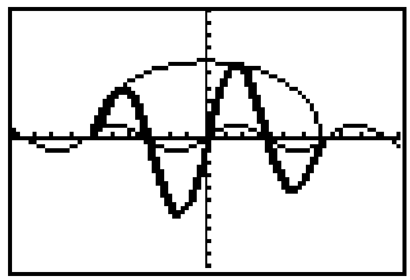



This occurs in about 11.6 years.

**Section 10.2 Page 497 Question 13**

**a)** Substitute $f(x) = \sqrt{36 - x^2}$ and $g(x) = \sin x$.

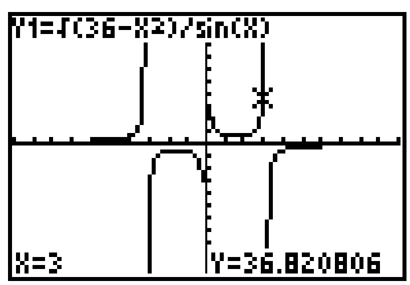
$$\begin{aligned} y &= (f \bullet g)(x) \\ &= f(x)g(x) \\ &= \sqrt{36 - x^2}\,(\sin x) \end{aligned}$$



**b)** The domain of $f(x) = \sqrt{36 - x^2}$ is $\{x \mid -6 \le x \le 6, x \in \mathrm{R}\}$.
The domain for $g(x) = \sin x$ is $\{x \mid x \in \mathrm{R}\}$.
Then, for $y = \sqrt{36 - x^2}$ $(\sin x)$, the domain is $\{x \mid -6 \le x \le 6, x \in \mathrm{R}\}$.
From the graph, the range is about $\{y \mid -5.8 \le y \le 5.8, y \in \mathrm{R}\}$.

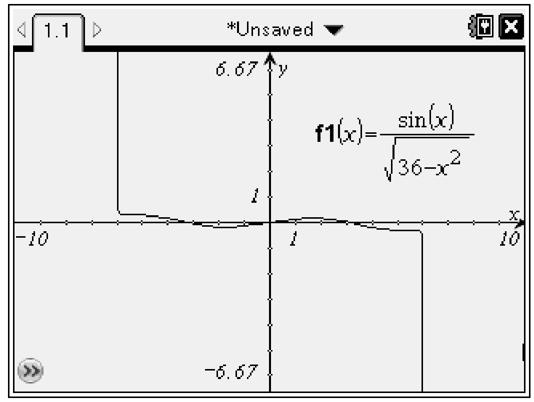
**c)** Substitute $f(x) = \sqrt{36 - x^2}$ and $g(x) = \sin x$.

$$\begin{aligned} y &= \left(\frac{f}{g}\right)(x) \\ &= \frac{f(x)}{g(x)} \\ &= \frac{\sqrt{36 - x^2}}{\sin x} \end{aligned}$$




For $y = \dfrac{\sqrt{36 - x^2}}{\sin x}$, the domain is $\{x \mid -6 \le x \le 6, x \ne n\pi, n \in \mathrm{I}, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

**d)** Substitute $f(x) = \sqrt{36 - x^2}$ and $g(x) = \sin x$.




$$y = \left(\frac{g}{f}\right)(x)$$
$$= \frac{g(x)}{f(x)}$$
$$= \frac{\sin x}{\sqrt{36 - x^2}}$$

For $y = \frac{\sin x}{\sqrt{36 - x^2}}$, the domain is $\{x \mid -6 < x < 6, x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.

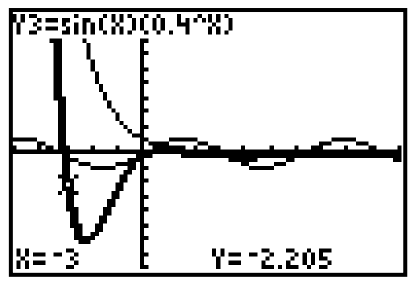
The domain in part d) is restricted to $-6 < x < 6$ but has no non-permissible values. In part c), the domain is restricted to $-6 \le x \le 6$ *with* non-permissible values. The ranges in parts c) and d) are the same.

**Section 10.2 Page 497 Question 14**

**a)** and **b)**

$$d(t) = (A \sin kt) \times 0.4^{ct}$$
$$= f(t)g(t)$$

Then, $f(t) = A \sin kt$ and $g(t) = 0.4^{ct}$.

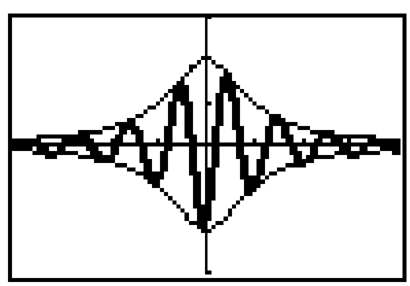



**Section 10.2 Page 498 Question 15**

**a)** Substitute $f(x) = \frac{2}{x^2 + 1}$ and $g(x) = \sin(6x - 1)$.

$$y = f(x)g(x)$$
$$= \frac{2}{x^2 + 1}\sin(6x - 1)$$

Graph $f(x) = \frac{2}{x^2 + 1}$, $-f(x) = -\frac{2}{x^2 + 1}$, and $y = \frac{2}{x^2 + 1}\sin(6x - 1)$.



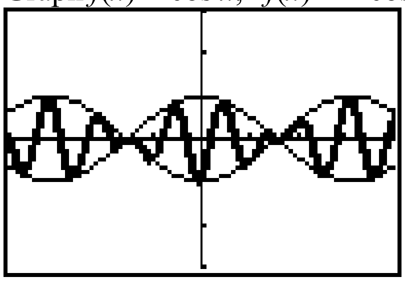
The graph of $y = f(x)g(x)$ oscillates between the graphs of $f(x)$ and $-f(x)$.

**b)** Substitute $f(x) = \cos x$ and $g(x) = \sin(6x - 1)$.

$y = f(x)g(x)$

$\quad = \cos x \sin(6x - 1)$

Graph $f(x) = \cos x$, $-f(x) = -\cos x$, and $y = \cos x \sin(6x - 1)$.

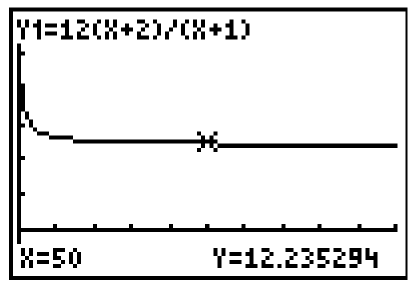


Yes. The graph of $y = f(x)g(x)$ oscillates between the graphs of $f(x)$ and $-f(x)$.

**Section 10.2 Page 498 Question 16**

The price per tonne can be modelled by

$$y = \frac{p(x)}{x} = \frac{12x\left(\frac{x+2}{x+1}\right)}{x} = \frac{12(x+2)}{x+1}$$




The price per tonne decreases as the number of tonnes increases.

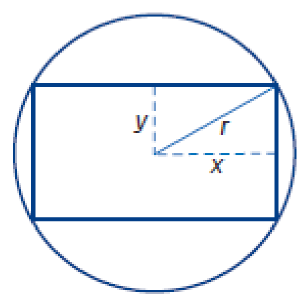
**Section 10.2 Page 498 Question 17**

Given that the length of the rectangle is $2x$, first determine an expression for the width of the rectangle.

Use the Pythagorean theorem to find that $y = \sqrt{r^2 - x^2}$.

Then, the area of the rectangle can be modelled by

$A = 2x(2\sqrt{r^2 - x^2})$ or $A = 4x\sqrt{r^2 - x^2}$.




**Section 10.2 Page 498 Question C1**

Yes; multiplication is commutative. For example, let $f(x) = 6x$ and $g(x) = x - 2$.

$f(x)g(x) = 6x(x - 2)$

$\quad = 6x^2 - 12x$

$g(x)f(x) = (x - 2)6x$

$\quad = 6x^2 - 12x$

$f(x)g(x) = g(x)f(x)$

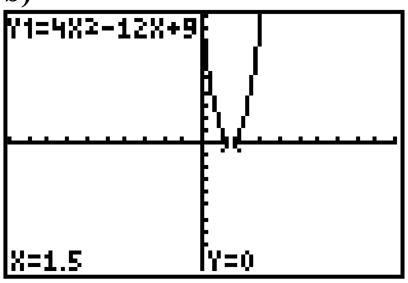
**Section 10.2 Page 498 Question C2**

Example: Multiplication of functions generally increases the domain, although this is not always true. Quotients of functions generally produce asymptotes and points of discontinuity, which restricts the domain, although this is not always true.

**Section 10.2 Page 498 Question C3**

**a)** $A(x) = (2x - 3)(2x - 3)$
$= 4x^2 - 12x + 9$

**b)**



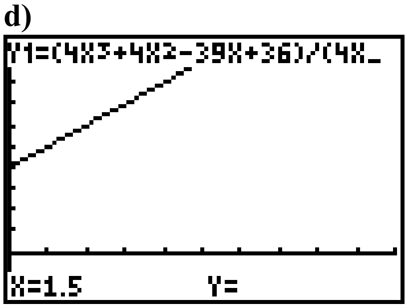

In this context, the domain is $\{x \mid x \geq 1.5, x \in \mathrm{R}\}$ and the range is $\{A \mid A \geq 0, A \in \mathrm{R}\}$.

**c)** Substitute $V(x) = 4x^3 + 4x^2 - 39x + 36$ and $A(x) = 4x^2 - 12x + 9$.

$$\begin{aligned} h(x) &= \frac{V(x)}{A(x)} \\ &= \frac{4x^3 + 4x^2 - 39x + 36}{4x^2 - 12x + 9} \\ &= \frac{(2x-3)^2(x+4)}{(2x-3)^2} \\ &= x + 4,\ x \neq \frac{3}{2} \end{aligned}$$

This represents the height of the box.

**d)**




In this context, the domain is $\{x \mid x > 1.5, x \in \mathrm{R}\}$ and the range is $\{h \mid h > 5.5, h \in \mathrm{R}\}$.

## Section 10.3 Composite Functions

### Section 10.3 Page 507 Question 1

Given: $f(2) = 3$, $f(3) = 4$, $f(5) = 0$, $g(2) = 5$, $g(3) = 2$, and $g(4) = -1$

**a)** Substitute $g(3) = 2$.

$$\begin{aligned} f(g(3)) &= f(2) \\ &= 3 \end{aligned}$$

**b)** Substitute $g(2) = 5$.

$$\begin{aligned} f(g(2)) &= f(5) \\ &= 0 \end{aligned}$$

**c)** Substitute $f(2) = 3$.

$$\begin{aligned} g(f(2)) &= g(3) \\ &= 2 \end{aligned}$$

**d)** Substitute $f(3) = 4$.

$$\begin{aligned} g(f(3)) &= g(4) \\ &= -1 \end{aligned}$$

### Section 10.3 Page 507 Question 2

**a)** From the graph, $g(-4) = 0$ and $f(0) = 2$. So, $f(g(-4)) = 2$.

**b)** From the graph, $g(0) = -4$ and $f(-4) = 2$. So, $f(g(0)) = 2$.

**c)** From the graph, $f(-2) = 0$ and $g(0) = -4$. So, $g(f(-2)) = -4$.

**d)** From the graph, $f(-3) = 1$ and $g(1) = -5$. So, $g(f(-3)) = -5$.

### Section 10.3 Page 507 Question 3

Given: $f(x) = 2x + 8$ and $g(x) = 3x - 2$

**a)** Determine $g(1)$.

$$\begin{aligned} g(x) &= 3x - 2 \\ g(1) &= 3(1) - 2 \\ g(1) &= 1 \end{aligned}$$

Substitute $g(1) = 1$ into $f(x)$.

$$\begin{aligned} f(g(1)) &= f(1) \\ &= 2(1) + 8 \\ &= 10 \end{aligned}$$

**b)** Determine $g(-2)$.

$$\begin{aligned} g(x) &= 3x - 2 \\ g(-2) &= 3(-2) - 2 \\ g(-2) &= -8 \end{aligned}$$

Substitute $g(-2) = -8$ into $f(x)$.

$$\begin{aligned} f(g(-2)) &= f(-8) \\ &= 2(-8) + 8 \\ &= -8 \end{aligned}$$

**c)** Determine $f(-4)$.

$$\begin{aligned} f(x) &= 2x + 8 \\ f(-4) &= 2(-4) + 8 \\ f(-4) &= 0 \end{aligned}$$

Substitute $f(-4) = 0$ into $g(x)$.

$$\begin{aligned} g(f(-4)) &= g(0) \\ &= 3(0) - 2 \\ &= -2 \end{aligned}$$

**d)** Determine $f(1)$.

$$\begin{aligned} f(x) &= 2x + 8 \\ f(1) &= 2(1) + 8 \\ f(1) &= 10 \end{aligned}$$

Substitute $f(1) = 10$ into $g(x)$.

$$\begin{aligned} g(f(1)) &= g(10) \\ &= 3(10) - 2 \\ &= 28 \end{aligned}$$

**Section 10.3 Page 507 Question 4**

Given: $f(x) = 3x + 4$ and $g(x) = x^2 - 1$

**a)** Determine $g(a)$.

$g(x) = x^2 - 1$
$g(a) = a^2 - 1$
Substitute $g(a) = a^2 - 1$ into $f(x)$.
$$\begin{aligned} f(g(a)) &= f(a^2 - 1) \\ &= 3(a^2 - 1) + 4 \\ &= 3a^2 + 1 \end{aligned}$$

**b)** Determine $f(a)$.

$f(x) = 3x + 4$
$f(a) = 3(a) + 4$
$f(a) = 3a + 4$
Substitute $f(a) = 3a + 4$ into $g(x)$.
$$\begin{aligned} g(f(a)) &= g(3a + 4) \\ &= (3a + 4)^2 - 1 \\ &= 9a^2 + 24a + 15 \end{aligned}$$

**c)** $$\begin{aligned} f(g(x)) &= f(x^2 - 1) \\ &= 3(x^2 - 1) + 4 \\ &= 3x^2 + 1 \end{aligned}$$

**d)** $$\begin{aligned} g(f(x)) &= g(3x + 4) \\ &= (3x + 4)^2 - 1 \\ &= 9x^2 + 24x + 15 \end{aligned}$$

**e)** $$\begin{aligned} f(f(x)) &= f(3x + 4) \\ &= 3(3x + 4) + 4 \\ &= 9x + 16 \end{aligned}$$

**f)** $$\begin{aligned} g(g(x)) &= g(x^2 - 1) \\ &= (x^2 - 1)^2 - 1 \\ &= x^4 - 2x^2 \end{aligned}$$

**Section 10.3 Page 507 Question 5**

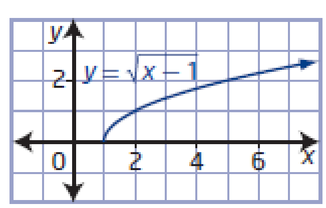
**a)** For $f(x) = x^2 + x$ and $g(x) = x^2 + x$,
$$\begin{aligned} f(g(x)) &= f(x^2 + x) \\ &= (x^2 + x)^2 + x^2 + x \\ &= x^4 + 2x^3 + x^2 + x^2 + x \\ &= x^4 + 2x^3 + 2x^2 + x \end{aligned}$$
$$\begin{aligned} g(f(x)) &= g(x^2 + x) \\ &= (x^2 + x)^2 + x^2 + x \\ &= x^4 + 2x^3 + x^2 + x^2 + x \\ &= x^4 + 2x^3 + 2x^2 + x \end{aligned}$$

**b)** For $f(x) = \sqrt{x^2 + 2}$ and $g(x) = x^2$,
$$\begin{aligned} f(g(x)) &= f(x^2) \\ &= \sqrt{(x^2)^2 + 2} \\ &= \sqrt{x^4 + 2} \end{aligned}$$
$$\begin{aligned} g(f(x)) &= g(\sqrt{x^2 + 2}) \\ &= (\sqrt{x^2 + 2})^2 \\ &= x^2 + 2 \end{aligned}$$

**c)** For $f(x) = |x|$ and $g(x) = x^2$,
$$\begin{aligned} f(g(x)) &= f(x^2) \\ &= |x^2| \\ &= x^2 \end{aligned}$$
$$\begin{aligned} g(f(x)) &= g(|x|) \\ &= |x|^2 \\ &= x^2 \end{aligned}$$

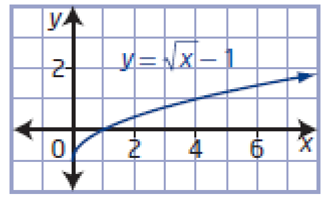
**Section 10.3 Page 507 Question 6**

**a)** For $f(x) = \sqrt{x}$ and $g(x) = x - 1$,
$$\begin{aligned} y &= f(g(x)) \\ &= f(x - 1) \\ &= \sqrt{x - 1} \end{aligned}$$



The domain is $\{x \mid x \geq 1, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

**b)** For $f(x) = \sqrt{x}$ and $g(x) = x - 1$,




$$\begin{aligned} y &= g(f(x)) \\ &= g(\sqrt{x}) \\ &= \sqrt{x} - 1 \end{aligned}$$

The domain is $\{x \mid x \geq 0, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq -1, y \in \mathrm{R}\}$.

**Section 10.3 Page 507 Question 7**

$$\begin{aligned} h(x) &= (f \circ g)(x) \\ &= f(g(x)) \end{aligned}$$

**a)** For $h(x) = (2x - 5)^2$ and $f(x) = x^2$,

$$\begin{aligned} h(x) &= f(g(x)) \\ &= g(x)^2 \\ &= (2x - 5)^2 \end{aligned}$$

Therefore, $g(x) = 2x - 5$.

**b)** For $h(x) = (5x + 1)^2 - (5x + 1)$ and $f(x) = x^2 - x$,

$$\begin{aligned} h(x) &= f(g(x)) \\ &= g(x)^2 - g(x) \\ &= (5x + 1)^2 - (5x + 1) \end{aligned}$$

Therefore, $g(x) = 5x + 1$.

**Section 10.3 Page 507 Question 8**

Christine's work is correct. Ron forgot to substitute $x^2 + 2$ for all $x$s in the first step.

**Section 10.3 Page 507 Question 9**

For $j(x) = x^2$ and $k(x) = x^3$,

$$\begin{aligned} k(j(x)) &= k(x^3) \\ &= (x^3)^2 \\ &= x^6 \end{aligned}$$

$$\begin{aligned} j(k(x)) &= k(x^2) \\ &= (x^2)^3 \\ &= x^6 \end{aligned}$$

So, $k(j(x)) = j(k(x))$ for all values of $x$.

**Section 10.3 Page 507 Question 10**

For $s(x) = x^2 + 1$ and $t(x) = x - 3$,

$$\begin{aligned} s(t(x)) &= s(x - 3) \\ &= (x - 3)^2 + 1 \\ &= x^2 - 6x + 10 \end{aligned}$$

$$\begin{aligned} t(s(x)) &= t(x^2 + 1) \\ &= x^2 + 1 - 3 \\ &= x^2 - 2 \end{aligned}$$

So, $s(t(x)) \neq t(s(x))$ for all values of $x$.

**Section 10.3 Page 507 Question 11**

**a)** Substitute $C(t) = 100 + 35t$.

$$W(C) = 3\sqrt{C}$$
$$W(C(t)) = 3\sqrt{100+35t}$$

**b)** In this context, the domain is $\{t \mid t \geq 0, t \in \mathrm{R}\}$ and the range is $\{W \mid W \geq 30, W \in \mathrm{W}\}$.

**Section 10.3 Page 508 Question 12**

**a)** The function, $s(p)$, that relates the regular price, $p$, to the sale price, $s$, is $s(p) = 0.75p$.

**b)** The function, $t(s)$, that relates the sale price, $s$, to the total cost including taxes, $t$, is $t(s) = 1.05s$.

**c)** A composite function that expresses the total cost in terms of the regular price is

$$\begin{aligned} t(s(p)) &= t(0.75p) \\ &= 1.05(0.75p) \\ &= 0.7875p \end{aligned}$$

Substitute $p = 89.99$.

$$t(s(p)) = 0.7875p$$
$$t(s(89.99)) = 0.7875(89.99)$$
$$t(s(89.99)) = 70.867\ldots$$

Tobias paid \$70.87 for the jacket.

**Section 10.3 Page 508 Question 13**

**a)** The function, $g(d)$, that relates the distance, $d$, in kilometres, driven to the quantity, $g$, in litres, of gasoline used is $g(d) = 0.06d$.

**b)** The function, $c(g)$, that relates the quantity, $g$, in litres, of gasoline used to the average cost, $c$, in dollars, of a litre of gasoline is $c(g) = 1.23g$.

**c)** The composite function that expresses the cost of gasoline in terms of the distance driven is

$$\begin{aligned} c(g(d)) &= c(0.06d) \\ &= 1.23(0.06d) \\ &= 0.0738d \end{aligned}$$

Substitute $d = 200$.

$$c(g(d)) = 0.0738d$$
$$c(g(200)) = 0.0738(200)$$
$$c(g(200)) = 14.76$$

It would cost Jordan \$14.76 to drive her car 200 km.

**d)** From part a), $d(g) = \dfrac{g}{0.06}$. From part b), $g(c) = \dfrac{c}{1.23}$.

The composite function that expresses the distance driven in terms of the cost of gasoline is

$$\begin{aligned} d(g(c)) &= d\left(\frac{c}{1.23}\right) \\ &= \frac{\frac{c}{1.23}}{0.06} \\ &= \frac{c}{0.0738} \approx 13.55c \end{aligned}$$

Substitute $c = 40$.

$$\begin{aligned} d(g(c)) &= \frac{c}{0.0738} \\ d(g(40)) &= \frac{40}{0.0738} \\ d(g(40)) &= 542.005\ldots \end{aligned}$$

Jordan could drive about 542 km on \$40.

**Section 10.3 Page 508 Question 14**

For $f(x) = 3x$, $g(x) = x - 7$, and $h(x) = x^2$,

**a)** $(f \circ g \circ h)(x) = (f \circ (g \circ h))(x)$

First, determine $(g \circ h)(x)$.

$$\begin{aligned} (g \circ h)(x) &= g(x^2) \\ &= x^2 - 7 \end{aligned}$$

Then,

$$\begin{aligned} (f \circ g \circ h)(x) &= (f \circ (g \circ h))(x) \\ &= f(x^2 - 7) \\ &= 3(x^2 - 7) \\ &= 3x^2 - 21 \end{aligned}$$

**b)** For $g(f(h(x)))$, first, determine $f(h(x))$.

$$\begin{aligned} f(h(x)) &= f(x^2) \\ &= 3x^2 \end{aligned}$$

Then,

$$\begin{aligned} g(f(h(x))) &= g(3x^2) \\ &= 3x^2 - 7 \end{aligned}$$

**c)** For $f(h(g(x)))$, first, determine $h(g(x))$.

$$\begin{aligned} h(g(x)) &= h(x - 7) \\ &= (x - 7)^2 \\ &= x^2 - 14x + 49 \end{aligned}$$

Then,

$$f(h(g(x))) = f(x^2 - 14x + 49)$$

$= 3(x^2 - 14x + 49)$
$= 3x^2 - 42x + 147$

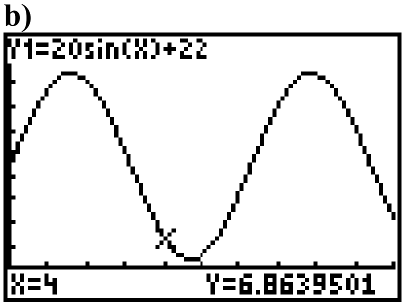
**d)** $(h \circ g \circ f)(x) = (h \circ (g \circ f))(x)$
First, determine $(g \circ f)(x)$.
$(g \circ f)(x) = g()$
$= 3x - 7$
Then,
$(h \circ g \circ f)(x) = (h \circ (g \circ f))(x)$
$= h(3x - 7)$
$= (3x - 7)^2$
$= 9x^2 - 42x + 49$

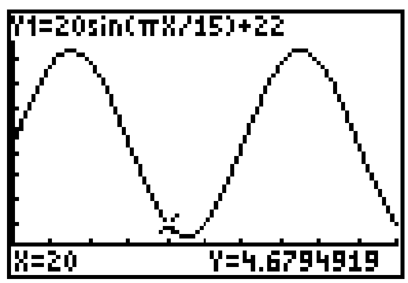
**Section 10.3 Page 508 Question 15**

**a)** The equation of the rider's height in terms of time is

$$h(\theta(t)) = h\left(\frac{\pi t}{15}\right)$$
$$= 20 \sin \frac{\pi t}{15} + 22$$

**b)**




For $h(\theta) = 20 \sin \theta + 22$, $b = 1$. So, the period is $2\pi$.
For $h(\theta(t)) = 20 \sin \frac{\pi t}{15} + 22$, $b = \frac{\pi}{15}$. So, the period is $\frac{2\pi}{\frac{\pi}{15}}$, or 30.
The period of the composite function is much greater.

**Section 10.3 Page 508 Question 16**

**a)** The equation of the concentration of pollutant as a function of time is

$$C(P(t)) = C\left(12.5(2)^{\frac{t}{10}}\right)$$
$$= 1.15\left(12.5(2)^{\frac{t}{10}}\right) + 53.12$$
$$= 14.375\,(2)^{\frac{t}{10}} + 53.12$$

**b)** Substitute $C = 100$.

$$C(P(t)) = 14.375\,(2)^{\frac{t}{10}} + 53.12$$

$$100 = 14.375\,(2)^{\frac{t}{10}} + 53.12$$

$$46.88 = 14.375\,(2)^{\frac{t}{10}}$$

$$\frac{46.88}{14.375} = (2)^{\frac{t}{10}}$$

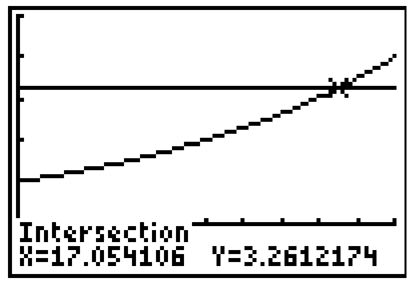
This can be solved algebraically or graphically.

$$\log\frac{46.88}{14.375} = \log 2^{\frac{t}{10}}$$

$$\log\frac{46.88}{14.375} = \frac{t}{10}\log 2$$

$$\frac{10}{\log 2}\log\frac{46.88}{14.375} = t$$

$$t = 17.054...$$




It will take approximately 17.1 years for the concentration to be over 100 ppm.

**Section 10.3 Page 509 Question 17**

**a)** Example: Let $g(x) = x^2$. Then, work backward.

$$h(x) = 2x^2 - 1$$
$$f(g(x)) = 2g(x) - 1$$
$$f(x) = 2x - 1$$

**b)** Example: Let $g(x) = x^2$. Then, work backward.

$$h(x) = \frac{2}{3 - x^2}$$
$$f(g(x)) = \frac{2}{3 - g(x)}$$
$$f(x) = \frac{2}{3 - x}$$

**c)** Let $g(x) = x^2 - 4x + 5$. Then, work backward.

$$h(x) = |x^2 - 4x + 5|$$
$$f(g(x)) = |g(x)|$$
$$f(x) = |x|$$

**Section 10.3 Page 509 Question 18**

For $f(x) = 1 - x$ and $g(x) = \dfrac{x}{1-x}$, $x \neq 1$,

**a)** 
$$\begin{aligned} g(f(x)) &= g(1 - x) \\ &= \frac{1-x}{1-(1-x)} \\ &= \frac{1-x}{x} \\ &= \frac{1}{g(x)} \end{aligned}$$

**b)** 
$$\begin{aligned} f(g(x)) &= f\left(\frac{x}{1-x}\right) \\ &= 1 - \frac{x}{1-x} \\ &= \frac{1-2x}{1-x} \\ &\neq \frac{1}{f(x)} \end{aligned}$$

**Section 10.3 Page 509 Question 19**

**a)** 
$$\begin{aligned} m(v(t)) &= m(t^3) \\ &= \frac{m_0}{\sqrt{1-\dfrac{(t^3)^2}{c^2}}} \\ &= \frac{m_0}{\sqrt{1-\dfrac{t^6}{c^2}}} \end{aligned}$$

**b)** Substitute $t = \sqrt[3]{\dfrac{c}{2}}$.

$$m(v(t)) = \frac{m_0}{\sqrt{1-\dfrac{t^6}{c^2}}}$$

$$m\left(\sqrt[3]{\frac{c}{2}}\right) = \frac{m_0}{\sqrt{1-\dfrac{\left(\sqrt[3]{\dfrac{c}{2}}\right)^6}{c^2}}}$$

$$m\left(\sqrt[3]{\frac{c}{2}}\right) = \frac{m_0}{\sqrt{1-\dfrac{\frac{c^2}{4}}{c^2}}}$$

$$m\left(\sqrt[3]{\frac{c}{2}}\right) = \frac{m_0}{\sqrt{1-\dfrac{1}{4}}}$$

$$m\left(\sqrt[3]{\frac{c}{2}}\right) = \frac{m_0}{\sqrt{\dfrac{3}{4}}}$$

$$m\left(\sqrt[3]{\frac{c}{2}}\right) = \frac{2m_0}{\sqrt{3}}$$

The particle's mass is $\dfrac{2m_0}{\sqrt{3}}$.

**Section 10.3 Page 509 Question 20**

**a)** For $f(x) = 5x + 10$ and $g(x) = \frac{1}{5}x - 2$,

$$\begin{aligned} f(g(x)) &= f\left(\frac{1}{5}x - 2\right) \\ &= 5\left(\frac{1}{5}x - 2\right) + 10 \\ &= x - 10 + 10 \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(5x + 10) \\ &= \frac{1}{5}(5x + 10) - 2 \\ &= x + 2 - 2 \\ &= x \end{aligned}$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, the functions $f(x)$ and $g(x)$ are inverses of each other.

**b)** For $f(x) = \frac{x-1}{2}$ and $g(x) = 2x + 1$,

$$\begin{aligned} f(g(x)) &= f(2x + 1) \\ &= \frac{2x+1-1}{2} \\ &= \frac{2x}{2} \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g\left(\frac{x-1}{2}\right) \\ &= 2\left(\frac{x-1}{2}\right) + 1 \\ &= x - 1 + 1 \\ &= x \end{aligned}$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, the functions $f(x)$ and $g(x)$ are inverses of each other.

**c)** For $f(x) = \sqrt[3]{x+1}$ and $g(x) = x^3 - 1$,

$$\begin{aligned} f(g(x)) &= f(x^3 - 1) \\ &= \sqrt[3]{x^3 - 1 + 1} \\ &= \sqrt[3]{x^3} \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(\sqrt[3]{x+1}) \\ &= (\sqrt[3]{x+1})^3 - 1 \\ &= x + 1 - 1 \\ &= x \end{aligned}$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, the functions $f(x)$ and $g(x)$ are inverses of each other.

**d)** For $f(x) = 5^x$ and $g(x) = \log_5 x$,

$$\begin{aligned} f(g(x)) &= f(\log_5 x) \\ &= 5^{\log_5 x} \\ &= x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(5^x) \\ &= \log_5 5^x \\ &= x \log_5 5 \\ &= x \end{aligned}$$

Since $f(g(x)) = x$ and $g(f(x)) = x$, the functions $f(x)$ and $g(x)$ are inverses of each other.

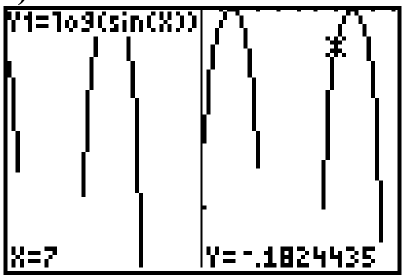
**Section 10.3 Page 509 Question 21**

**a)** The domain of $f(x) = \log x$ is $\{x \mid x > 0, x \in \mathrm{R}\}$.

**b)** For $f(x) = \log x$ and $g(x) = \sin x$,

$$\begin{aligned} f(g(x)) &= f(\sin x) \\ &= \log(\sin x) \end{aligned}$$

**c)**




**d)** For $f(g(x)) = \log(\sin x)$, the domain is $\{x \mid 2\pi n < x < (2n + 1)\pi n, n \in \mathrm{I}, x \in \mathrm{R}\}$, since $\sin x$ is positive in quadrants I and II of each rotation on a unit circle. The range is $\{y \mid y \le 0, y \in \mathrm{R}\}$.

**Section 10.3 Page 509 Question 22**

For $f(x) = \dfrac{1}{1+x}$ and $g(x) = \dfrac{1}{2+x}$,

$$\begin{aligned}
f(g(x)) &= f\left(\frac{1}{2+x}\right)\\
&= \frac{1}{1+\frac{1}{2+x}}\\
&= \frac{1}{\frac{x+3}{2+x}}\\
&= \frac{x+2}{x+3}, x \neq -1, -2, -3
\end{aligned}$$

**Section 10.3 Page 509 Question 23**

**a)** For $f_1(x) = x$, $f_2(x) = \dfrac{1}{x}$, $f_3(x) = 1 - x$, $f_4(x) = \dfrac{x}{x-1}$, $f_5(x) = \dfrac{1}{1-x}$, and $f_6(x) = \dfrac{x-1}{x}$,

**i)**
$$\begin{aligned}
f_2(f_3(x)) &= f_2(1 - x)\\
&= \frac{1}{1-x}, x \neq 1
\end{aligned}$$

**ii)**
$$\begin{aligned}
(f_3 \circ f_5)(x) &= f_3\left(\frac{1}{1-x}\right)\\
&= 1 - \frac{1}{1-x}\\
&= \frac{1-x-1}{1-x}\\
&= \frac{-x}{1-x}, x \neq 1
\end{aligned}$$

**iii)** $f_1(f_2(x)) = f_1\left(\frac{1}{x}\right)$

$= \frac{1}{x}, x \neq 0$

**iv)** $f_2(f_1(x)) = f_2(x)$

$= \frac{1}{x}, x \neq 0$

**b)** $f_6(x) = \frac{x-1}{x}$

$$
\begin{aligned}
y &= \frac{x-1}{x} \\
x &= \frac{y-1}{y} \\
xy &= y - 1 \\
xy - y &= -1 \\
y(x-1) &= -1 \\
y &= \frac{-1}{x-1} \\
y &= \frac{1}{1-x}
\end{aligned}
$$

This is the same as $f_2(f_3(x))$.

**Section 10.3 Page 509 Question C1**

No, $f(g(x))$ does not mean the same as $(f \bullet g)(x)$. One is a composite function, $f(g(x))$, and the other is the product of functions, $(f \bullet g)(x)$. For example, let $f(x) = x + 4$ and $g(x) = x^2$.

$$
\begin{aligned}
f(g(x)) &= f(x^2) \\
&= x^2 + 4
\end{aligned}
$$

$$
\begin{aligned}
(f \bullet g)(x) &= (x + 4)(x^2) \\
&= x^3 + 4x^2
\end{aligned}
$$

**Section 10.3 Page 509 Question C2**

**a)** From the list, $f(1) = 5$ and $g(5) = 10$.
So,

$$
\begin{aligned}
g(f(1)) &= g(5) \\
&= 10
\end{aligned}
$$

**b)** From the list, $f(3) = 7$ and $g(7) = 0$.
So,

$$
\begin{aligned}
g(f(3)) &= g(7) \\
&= 0
\end{aligned}
$$

**Section 10.3 Page 509 Question C3**

For $f(x) = 4 - 3x$ and $g(x) = \frac{4-x}{3}$,

$$
\begin{aligned}
g(f(x)) &= g(4 - 3x) \\
&= \frac{4-(4-3x)}{3} \\
&= \frac{3x}{3} \\
&= x
\end{aligned}
$$

$$
\begin{aligned}
f(g(x)) &= f\left(\frac{4-x}{3}\right) \\
&= 4 - 3\left(\frac{4-x}{3}\right) \\
&= 4 - 4 + x \\
&= x
\end{aligned}
$$

Yes, $g(f(x)) = f(g(x))$ for all $x$. The pair of functions are inverses of each other.

**Section 10.3 Page 509 Question C4**

**Step 1:**

**a)** $f(x) = 2x + 3$
$f(x + h) = 2(x + h) + 3$
$f(x + h) = 2x + 2h + 3$

**b)** $$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{2x+2h+3-(2x+3)}{h}\\ &= \frac{2x+2h+3-2x-3}{h}\\ &= \frac{2h}{h}\\ &= 2\end{aligned}$$

**Step 2:**

**a)** $f(x) = -3x - 5$
$f(x + h) = -3(x + h) - 5$
$f(x + h) = -3x - 3h - 5$

**b)** $$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{-3x-3h-5-(-3x-5)}{h}\\ &= \frac{-3x-3h-5+3x+5}{h}\\ &= \frac{-3h}{h}\\ &= -3\end{aligned}$$

**Step 3:** I predict that $\frac{f(x+h)-f(x)}{h}$ for $f(x) = \frac{3}{4}x - 5$ will equal $\frac{3}{4}$.
Each value is the slope of the related linear function.

## Chapter 10 Review

**Chapter 10 Review Page 510 Question 1**

For $f(x) = 3x - 1$ and $g(x) = 2x + 7$,

**a)** $$\begin{aligned}(f + g)(x) &= f(x) + g(x)\\ &= 3x - 1 + 2x + 7\\ &= 5x + 6\end{aligned}$$
$$\begin{aligned}(f + g)(4) &= 5(4) + 6\\ &= 26\end{aligned}$$

**b)** From part a), $(f + g)(x) = 5x + 6$.
$$\begin{aligned}(f + g)(-1) &= 5(-1) + 6\\ &= 1\end{aligned}$$

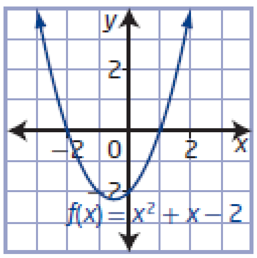
**c)** $(f - g)(x) = f(x) - g(x)$
$= 3x - 1 - (2x + 7)$
$= x - 8$
$(f - g)(3) = 3 - 8$
$= -5$

**d)** $(g - f)(x) = g(x) - f(x)$
$= 2x + 7 - (3x - 1)$
$= -x + 8$
$(g - f)(-5) = -(-5) + 8$
$= 13$

**Chapter 10 Review   Page 510   Question 2**

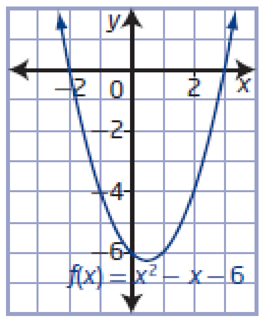
**a)** For $g(x) = x + 2$ and $h(x) = x^2 - 4$,

**i)** $f(x) = g(x) + h(x)$
$= x + 2 + x^2 - 4$
$= x^2 + x - 2$



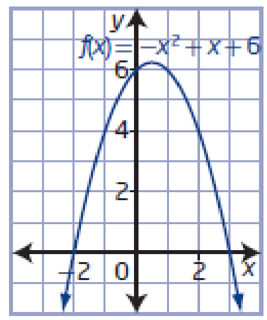

domain: $\{x \mid x \in \mathrm{R}\}$
range:
$\{y \mid y \geq -2.25, y \in \mathrm{R}\}$

**ii)** $f(x) = h(x) - g(x)$
$= x^2 - 4 - (x + 2)$
$= x^2 - x - 6$




domain: $\{x \mid x \in \mathrm{R}\}$
range:
$\{y \mid y \geq -6.25, y \in \mathrm{R}\}$

**iii)** $f(x) = g(x) - h(x)$
$= x + 2 - (x^2 - 4)$
$= -x^2 + x + 6$




domain: $\{x \mid x \in \mathrm{R}\}$
range: $\{y \mid y \leq 6.25, y \in \mathrm{R}\}$

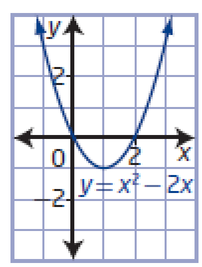
**b)**

**i)** $f(x) = x^2 + x - 2$
$f(2) = 2^2 + 2 - 2$
$f(2) = 4$

**ii)** $f(x) = x^2 - x - 6$
$f(2) = 2^2 - 2 - 6$
$f(2) = -4$

**iii)** $f(x) = -x^2 + x + 6$
$f(2) = -2^2 + 2 + 6$
$f(2) = 4$

**Chapter 10 Review   Page 510   Question 3**

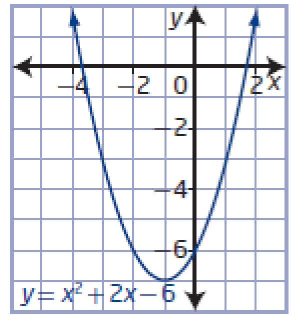



**a)** For $f(x) = x^2 - 3$ and $g(x) = -2x + 3$,
$y = (f + g)(x)$
$= f(x) + g(x)$
$= x^2 - 3 + (-2x + 3)$
$= x^2 - 2x$

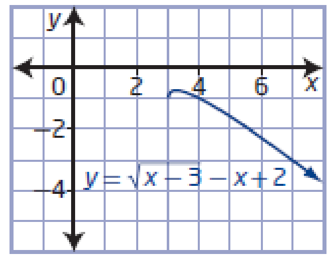
The domain of $y = (f + g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (f + g)(x)$ is $\{y \mid y \geq -1, y \in \mathrm{R}\}$.

$y = (f - g)(x)$
$= f(x) - g(x)$
$= x^2 - 3 - (-2x + 3)$
$= x^2 + 2x - 6$

The domain of $y = (f - g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (f - g)(x)$ is $\{y \mid y \geq -7, y \in \mathrm{R}\}$.



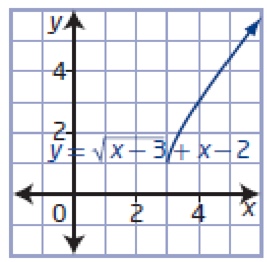

**b)** For $f(x) = \sqrt{x-3}$ and $g(x) = -x + 2$,
$y = (f + g)(x)$
$= f(x) + g(x)$
$= \sqrt{x-3} + (-x + 2)$
$= \sqrt{x-3} - x + 2$




The domain of $y = (f + g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \geq 3, x \in \mathrm{R}\}$.
From the graph, the range of $y = (f + g)(x)$ is approximately $\{y \mid y \geq -0.75, y \in \mathrm{R}\}$.

$y = (f - g)(x)$
$= f(x) - g(x)$
$= \sqrt{x-3} - (-x + 2)$
$= \sqrt{x-3} + x - 2$

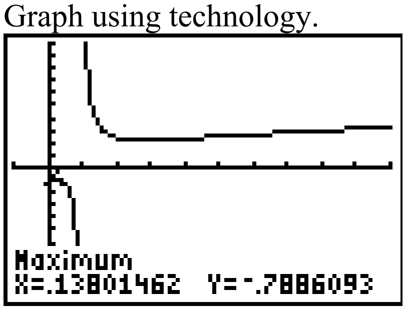
The domain of $y = (f - g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \geq 3, x \in \mathrm{R}\}$.
The range of $y = (f - g)(x)$ is $\{y \mid y \geq 1, y \in \mathrm{R}\}$.




**Chapter 10 Review Page 510 Question 4**

**a)** For $f(x) = \dfrac{1}{x-1}$ and $g(x) = \sqrt{x}$,
$(f + g)(x) = f(x) + g(x)$
$= \dfrac{1}{x-1} + \sqrt{x}$

The domain of $(f + g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \geq 0, x \neq 1, x \in \mathrm{R}\}$.
Graph using technology.




The range of $(f + g)(x)$ is approximately $\{y \mid y \leq -0.79 \text{ or } y \geq 2.2, y \in \mathrm{R}\}$.

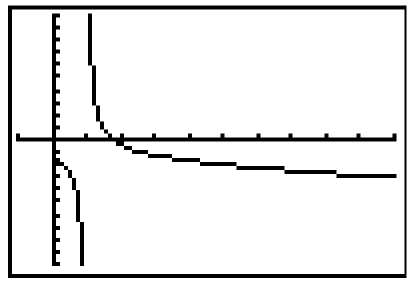
**b)** For $f(x) = \dfrac{1}{x-1}$ and $g(x) = \sqrt{x}$,

$(f - g)(x) = f(x) - g(x)$

$$= \frac{1}{x-1} - \sqrt{x}$$

The domain of $(f - g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \geq 0, x \neq 1, x \in \mathrm{R}\}$.

Graph using technology.



The range of $(f + g)(x)$ is $\{y \mid y \in \mathrm{R}\}$.

## Chapter 10 Review Page 510 Question 5

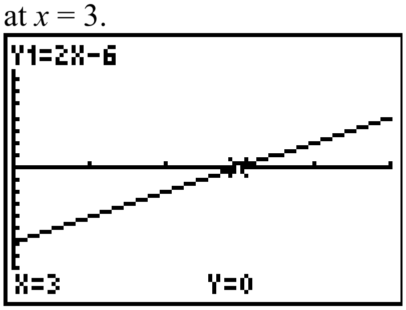
**a)** An expression that reflects the net change in population at any given time is

$P = b(x) - d(x)$

$= -4x + 78 - (-6x + 84)$

$= 2x - 6$

**b)** The net change in population will continue to increase, going from negative to positive at $x = 3$.




**c)** The population begins to increase after year 3, since the net change is then positive.

## Chapter 10 Review Page 510 Question 6

For $g(x) = x + 2$ and $h(x) = x^2 - 4$,

**a)** $f(x) = g(x)h(x)$

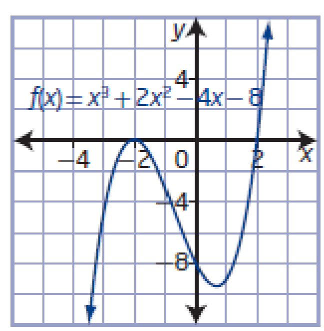
$= (x + 2)(x^2 - 4)$

$= x^3 + 2x^2 - 4x - 8$

The domain of $f(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$: $\{x \mid x \in \mathrm{R}\}$.

The range of $f(x)$ is $\{y \mid y \in \mathrm{R}\}$.

There are no asymptotes.

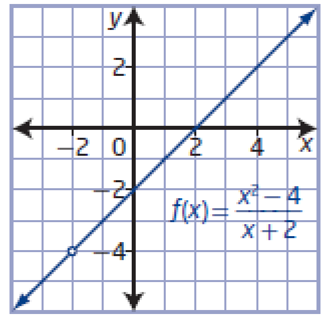


**b)** $f(x) = \frac{h(x)}{g(x)}$

$= \frac{x^2 - 4}{x + 2}$

$= \frac{(x+2)(x-2)}{x+2}$

$= x - 2, x \neq -2$



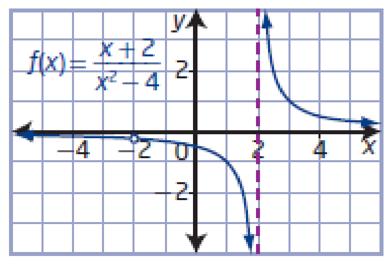

The domain of $f(x)$ consists of all values that are in both the domain of $h(x)$ and the domain of $g(x)$, excluding values of $x$ where $g(x) = 0$: $\{x \mid x \neq -2, x \in \mathrm{R}\}$.
The range of $f(x)$ is $\{y \mid y \neq -4, y \in \mathrm{R}\}$.
There are no asymptotes.

**c)** $f(x) = \frac{g(x)}{h(x)}$

$= \frac{x+2}{x^2 - 4}$

$= \frac{x+2}{(x+2)(x-2)}$

$= \frac{1}{x-2}, x \neq -2, 2$




The domain of $f(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$, excluding values of $x$ where $h(x) = 0$: $\{x \mid x \neq -2, 2, x \in \mathrm{R}\}$.
The range of $f(x)$ is $\{y \mid y \neq -0.25, 0, y \in \mathrm{R}\}$.
The equation of the asymptotes are $x = 2$ and $y = 0$.

**Chapter 10 Review Page 510 Question 7**

**a)** $f(x) = x^3 + 2x^2 - 4x - 8$
$f(-2) = (-2)^3 + 2(-2)^2 - 4(-2) - 8$
$f(-2) = -8 + 8 + 8 - 8$
$f(-2) = 0$

**b)** Since $f(x) = x - 2, x \neq -2, f(-2)$ does not exist.

**c)** Since $f(x) = \frac{1}{x-2}, x \neq -2, 2, f(-2)$ does not exist.

**Chapter 10 Review Page 511 Question 8**

For $g(x) = \dfrac{1}{x+4}$ and $h(x) = \dfrac{1}{x^2-16}$,

**a)** $f(x) = g(x)h(x)$

$$= \frac{1}{x+4}\left(\frac{1}{x^2-16}\right)$$

$$= \frac{1}{x^3+4x^2-16x-64},\ x \neq -4,\ 4$$

The domain of $f(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$: $\{x \mid x \neq -4, 4, x \in \mathrm{R}\}$.
The range of $f(x)$ is $\{y \mid y \neq 0, y \in \mathrm{R}\}$.

**b)** $f(x) = \dfrac{g(x)}{h(x)}$

$$= \frac{\frac{1}{x+4}}{\frac{1}{x^2-16}}$$

$$= \frac{x^2-16}{x+4}$$

$$= \frac{(x+4)(x-4)}{x+4}$$

$$= x-4,\ x \neq -4,\ 4$$

The domain of $f(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$, excluding values of $x$ where $h(x) = 0$: $\{x \mid x \neq -4, 4, x \in \mathrm{R}\}$.
The range of $f(x)$ is $\{y \mid y \neq -8, 0, y \in \mathrm{R}\}$.

**c)** $f(x) = \dfrac{h(x)}{g(x)}$

$$= \frac{\frac{1}{x^2-16}}{\frac{1}{x+4}}$$

$$= \frac{x+4}{x^2-16}$$

$$= \frac{x+4}{(x+4)(x-4)}$$

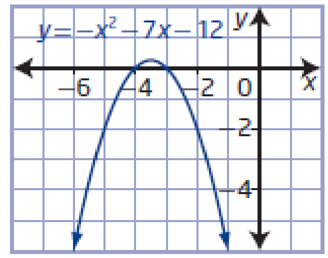
$$= \frac{1}{x-4},\ x \neq -4,\ 4$$

The domain of $f(x)$ consists of all values that are in both the domain of $h(x)$ and the domain of $g(x)$, excluding values of $x$ where $g(x) = 0$: $\{x \mid x \neq -4, 4, x \in \mathrm{R}\}$.
The range of $f(x)$ is $\{y \mid y \neq -0.125, 0, y \in \mathrm{R}\}$.

**Chapter 10 Review Page 511 Question 9**

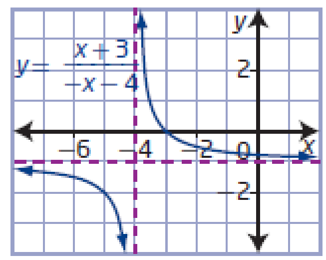
**a)** For $f(x) = x + 3$ and $g(x) = -x - 4$,

$$\begin{aligned} y &= (f \bullet g)(x) \\ &= f(x)g(x) \\ &= (x + 3)(-x - 4) \\ &= -x^2 - 7x - 12 \end{aligned}$$




The domain of $y = (f \bullet g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (f \bullet g)(x)$ is $\{y \mid y \le 0.25, y \in \mathrm{R}\}$.

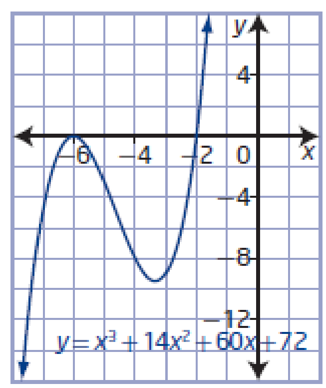
$$\begin{aligned} y &= \left(\frac{f}{g}\right)(x) \\ &= \frac{f(x)}{g(x)} \\ &= \frac{x+3}{-x-4}, x \neq -4 \end{aligned}$$




The domain of $y = \left(\frac{f}{g}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of $x$ where $g(x) = 0$: $\{x \mid x \neq -4, x \in \mathrm{R}\}$.
The range of $y = (f - g)(x)$ is $\{y \mid \neq -1, y \in \mathrm{R}\}$.

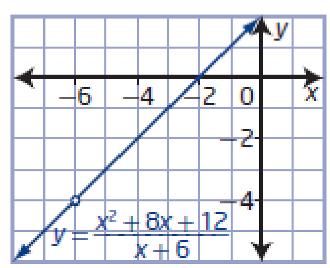
**b)** For $f(x) = x^2 + 8x + 12$ and $g(x) = x + 6$,

$$\begin{aligned} y &= (f \bullet g)(x) \\ &= f(x)g(x) \\ &= (x^2 + 8x + 12)(x + 6) \\ &= x^3 + 14x^2 + 60x + 72 \end{aligned}$$




The domain of $y = (f \bullet g)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$: $\{x \mid x \in \mathrm{R}\}$.
The range of $y = (f \bullet g)(x)$ is $\{y \mid y \in \mathrm{R}\}$.

$$\begin{aligned} y &= \left(\frac{f}{g}\right)(x) \\ &= \frac{f(x)}{g(x)} \\ &= \frac{x^2+8x+12}{x+6} \\ &= \frac{(x+2)(x+6)}{x+6} \\ &= x + 2, x \neq -6 \end{aligned}$$



The domain of $y = \left(\frac{f}{g}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of $x$ where $g(x) = 0$: $\{x \mid x \neq -6, x \in \mathrm{R}\}$.
The range of $y = (f - g)(x)$ is $\{y \mid \neq -4, y \in \mathrm{R}\}$.

**Chapter 10 Review Page 511 Question 10**

Given: $f(x) = x^2$ and $g(x) = x + 1$.

**a)** Determine $g(-2)$.

$$g(x) = x + 1$$
$$g(-2) = -2 + 1$$
$$g(-2) = -1$$

Substitute $g(-2) = -1$ into $f(x)$.

$$\begin{aligned} f(g(-2)) &= f(-1) \\ &= (-1)^2 \\ &= 1 \end{aligned}$$

**b)** Determine $f(-2)$.

$$f(x) = x^2$$
$$f(-2) = (-2)^2$$
$$f(-2) = 4$$

Substitute $f(-2) = 4$ into $g(x)$.

$$\begin{aligned} g(f(-2)) &= g(4) \\ &= 4 + 1 \\ &= 5 \end{aligned}$$

**Chapter 10 Review Page 511 Question 11**

Given: $f(x) = 2x^2$ and $g(x) = \frac{4}{x}$,

**a)**
$$\begin{aligned} f(g(x)) &= f\left(\frac{4}{x}\right) \\ &= 2\left(\frac{4}{x}\right)^2 \\ &= 2\left(\frac{16}{x^2}\right) \\ &= \frac{32}{x^2}, x \neq 0 \end{aligned}$$

**b)**
$$\begin{aligned} g(f(x)) &= g(2x^2) \\ &= \frac{4}{2x^2} \\ &= \frac{2}{x^2}, x \neq 0 \end{aligned}$$

**c)**
$$g(f(x)) = \frac{2}{x^2}$$
$$g(f(-2)) = \frac{2}{(-2)^2}$$
$$g(f(-2)) = \frac{1}{2}$$

## Chapter 10 Review Page 511 Question 12

**a)** For $f(x) = -\frac{2}{x}$ and $g(x) = \sqrt{x}$,

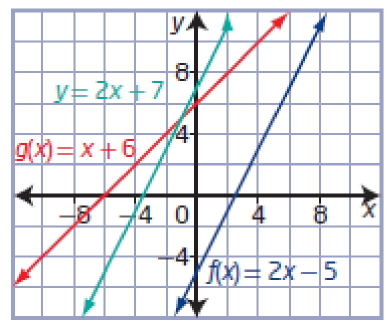
$$\begin{aligned} y &= f(g(x)) \\ &= f\left(\sqrt{x}\right) \\ &= -\frac{2}{\sqrt{x}}, x > 0 \end{aligned}$$

**b)** The domain of $y = f(g(x))$ is the set of all values of $x$ in the domain of $g$ for which $g(x)$ is in the domain of $f$: $\{x \mid x > 0, x \in \mathrm{R}\}$. The range of $y = f(g(x))$ is $\{y \mid y < 0, y \in \mathrm{R}\}$.

## Chapter 10 Review Page 511 Question 13

For $f(x) = 2x - 5$ and $g(x) = x + 6$,

$$\begin{aligned} y &= (f \circ g)(x) \\ &= f(x + 6) \\ &= 2(x + 6) - 5 \\ &= 2x + 7 \end{aligned}$$




## Chapter 10 Review Page 511 Question 14

First, determine a model for travel down the shaft: $d = 5t$. The temperature as a function of time can be modelled by

$$\begin{aligned} T(d(t)) &= T(5t) \\ &= 0.01(5t) + 20 \\ &= 0.05t + 20 \end{aligned}$$

## Chapter 10 Review Page 511 Question 15

**a)** Let $x$ represent the current price of the tablet. Then, the price, $d$, of the tablet after the discount is $d(x) = 0.75x$.
The price, $c$, of the tablet after the coupon as a function of the current price is $c(x) = x - 10$.

**b)** $c(d(x)) = c(0.75x)$
$= 0.75x - 10$
This represents using the coupon after the discount.

**c)** $d(c(x)) = d(x - 10)$
$= 0.75(x - 10)$
$= 0.75x - 7.5$
This represents applying the discount after the coupon.

**d)** Comparing the functions from part b) and c), $c(d(x)) = 0.75x - 10$ using the coupon after the discount, will result in the lower sale price of \$290.

## Chapter 10 Practice Test

### Chapter 10 Practice Test Page 512 Question 1

For $f(x) = (x + 3)^2$ and $g(x) = x + 4$,

$$\begin{aligned} h(x) &= f(x) + g(x) \\ &= (x + 3)^2 + x + 4 \\ &= x^2 + 6x + 9 + x + 4 \\ &= x^2 + 7x + 13 \end{aligned}$$

Choice **B**.

### Chapter 10 Practice Test Page 512 Question 2

For $f(x) = x + 8$ and $g(x) = 2x^2 - 128$,

$$\begin{aligned} y &= \frac{g(x)}{f(x)} \\ &= \frac{2x^2 - 128}{x + 8} \\ &= \frac{2(x^2 - 64)}{x + 8} \\ &= \frac{2(x - 8)(x + 8)}{x + 8} \\ &= 2(x - 8) \\ &= 2x - 16, x \neq -8 \end{aligned}$$

The domain of $y = \frac{g(x)}{f(x)}$ consists of all values that are in both the domain of $g(x)$ and the domain of $f(x)$, excluding values of $x$ where $f(x) = 0$: $\{x \mid x \neq -8, x \in \mathrm{R}\}$.

Choice **D**.

### Chapter 10 Practice Test Page 512 Question 3

From the graph, $f(x) = x^2 + 2$ and $g(x) = x$.

$$\begin{aligned} g(x) - f(x) &= x - (x^2 + 2) \\ &= -x^2 + x - 2 \end{aligned}$$

The equation of the parabola in vertex form is $y = \left(x - \frac{1}{2}\right)^2 - \frac{7}{4}$. Since this parabola opens downward and has vertex $\left(\frac{1}{2}, -\frac{7}{4}\right)$, the entire parabola is below the $x$-axis.

Therefore, $g(x) - f(x) < 0$ is true for $x \in \mathrm{R}$: choice **A**.

The other options can be shown to be false.
For choice B, consider $x = –1$. From the graph $f(–1) = 3$ and $g(–1) = –1$, so $\frac{f(-1)}{g(-1)} = \frac{3}{-1}$ or $-3$, which is less than 1. Therefore choice B is not true.
Choice C is not true by observing the graph. For all value of $x$, $f(x) > g(x)$.
For choice D, consider $x = –1$. $g(–1) + f(–1) = 3 + (–1)$ or 2. So $g(x) + f(x) > 0$ at $x = –1$.
Choice D is not true.

**Chapter 10 Practice Test Page 512 Question 4**

For $f(x) = 5 – x$ and $g(x) = 2\sqrt{3x}$,

$$\begin{aligned} f(g(x)) &= f(2\sqrt{3x}) \\ &= 5 - 2\sqrt{3x} \\ f(g(3)) &= 5 - 2\sqrt{3(3)} \\ &= 5 - 2(3) \\ &= -1 \end{aligned}$$

Choice **C**.

**Chapter 10 Practice Test Page 512 Question 5**

For $f(x) = x + 5$ and $g(x) = x^2$,

$$\begin{aligned} y &= f(g(x)) \\ &= f(x^2) \\ &= x^2 + 5 \end{aligned}$$

Choice **A**.

**Chapter 10 Practice Test Page 512 Question 6**

For $f(x) = \sin x$ and $g(x) = 2x^2$,

**a)** $$\begin{aligned} h(x) &= (f + g)(x) \\ &= f(x) + g(x) \\ &= \sin x + 2x^2 \end{aligned}$$

**b)** $$\begin{aligned} h(x) &= (f - g)(x) \\ &= f(x) - g(x) \\ &= \sin x - 2x^2 \end{aligned}$$

**c)** $$\begin{aligned} h(x) &= (f \bullet g)(x) \\ &= f(x)g(x) \\ &= (\sin x)2x^2 \end{aligned}$$

**d)** $$\begin{aligned} h(x) &= \left(\frac{f}{g}\right)(x) \\ &= \frac{f(x)}{g(x)} \\ &= \frac{\sin x}{2x^2}, x \neq 0 \end{aligned}$$

## Chapter 10 Practice Test Page 512 Question 7

Use mental math.

| | $g(x)$ | $f(x)$ | $(f+g)(x)$ | $(f \circ g)(x)$ |
|---|---|---|---|---|
| **a)** | $x - 8$ | $\sqrt{x}$ | $\sqrt{x} + x - 8$ | $\sqrt{x-8}$ |
| **b)** | $x + 3$ | $4x$ | $5x + 3$ | $4x + 12$ |
| **c)** | $x^2$ | $\sqrt{x-4}$ | $\sqrt{x-4} + x^2$ | $\sqrt{x^2-4}$ |
| **d)** | $\dfrac{1}{x}$ | $\dfrac{1}{x}$ | $\dfrac{2}{x}$ | $x$ |

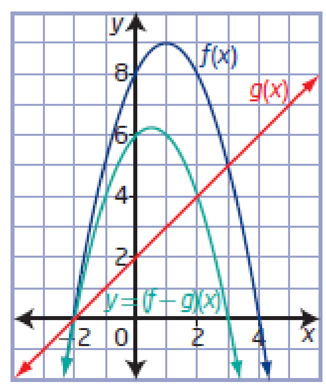
## Chapter 10 Practice Test Page 512 Question 8

For $g(x) = \dfrac{1}{1+x}$ and $h(x) = \dfrac{1}{3+2x}$,

$$\begin{aligned} y &= g(x)h(x) \\ &= \frac{1}{1+x}\left(\frac{1}{3+2x}\right) \\ &= \frac{1}{2x^2+5x+3}, x \neq -1.5, -1 \end{aligned}$$

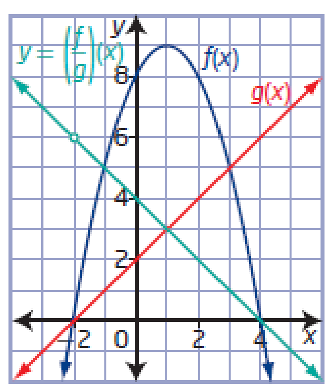
The domain of $y = g(x)h(x)$ consists of all values that are in both the domain of $g(x)$ and the domain of $h(x)$: $\{x \mid x \neq -1.5, -1, x \in \mathrm{R}\}$.

## Chapter 10 Practice Test Page 513 Question 9




From the graph, $f(x) = -(x - 1)^2 + 9$ and $g(x) = x + 2$.

**a)**
$$\begin{aligned} y &= (f - g)(x) \\ &= f(x) - g(x) \\ &= -(x-1)^2 + 9 - (x + 2) \\ &= -x^2 + 2x - 1 + 9 - x - 2 \\ &= -x^2 + x + 6 \end{aligned}$$




**b)**
$$\begin{aligned} y &= \left(\frac{f}{g}\right)(x) \\ &= \frac{f(x)}{g(x)} \\ &= \frac{-(x-1)^2+9}{x+2} \\ &= \frac{-x^2+2x+8}{x+2} \\ &= \frac{-(x-4)(x+2)}{x+2} \\ &= -x + 4, x \neq -2 \end{aligned}$$

## Chapter 10 Practice Test Page 513 Question 10

**a)** For $f(x) = 3 - x$ and $g(x) = |x + 3|$,

$$\begin{aligned} g(f(x)) &= g(3 - x) \\ &= |3 - x + 3| \\ &= |6 - x| \end{aligned}$$

The domain of $f(x)$ is $\{x \mid x \in \mathrm{R}\}$. The domain of $g(x)$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(f(x))$ is the set of all values of $x$ in the domain of $f$ for which $f(x)$ is in the domain of $g$: $\{x \mid x \in \mathrm{R}\}$. The range of $g(f(x))$ is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

**b)** For $f(x) = 4^x$ and $g(x) = x + 1$,

$$\begin{aligned} g(f(x)) &= g(4^x) \\ &= 4^x + 1 \end{aligned}$$

The domain of $f(x)$ is $\{x \mid x \in \mathrm{R}\}$. The domain of $g(x)$ is $\{x \mid x \in \mathrm{R}\}$.
The domain of $g(f(x))$ is the set of all values of $x$ in the domain of $f$ for which $f(x)$ is in the domain of $g$: $\{x \mid x \in \mathrm{R}\}$. The range of $g(f(x))$ is $\{y \mid y \geq 1, y \in \mathrm{R}\}$.

**c)** For $f(x) = x^4$ and $g(x) = \sqrt{x}$,

$$\begin{aligned} g(f(x)) &= g(x^4) \\ &= \sqrt{x^4} \\ &= x^2 \end{aligned}$$

The domain of $f(x)$ is $\{x \mid x \in \mathrm{R}\}$. The domain of $g(x)$ is $\{x \mid x \geq 0, x \in \mathrm{R}\}$.
The domain of $g(f(x))$ is the set of all values of $x$ in the domain of $f$ for which $f(x)$ is in the domain of $g$: $\{x \mid x \in \mathrm{R}\}$. The range of $g(f(x))$ is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

## Chapter 10 Practice Test Page 513 Question 11

**a)** Let $x$ represent Becky's earnings per pay period.
Then, her income, $r$, after the retirement deduction as a function of her earnings per pay period can be modelled by $r(x) = x - 200$.
Her income, $t$, after federal taxes as a function of her earnings per pay period can be modelled by $t(x) = 0.72x$.

**b)**

$$\begin{aligned} t(r(x)) &= t(x - 200) \\ &= 0.72(x - 200) \\ &= 0.72x - 144 \end{aligned}$$

This represents determining federal income taxes after Becky's retirement deduction.

**c)** Substitute $x = 2700$.

$$\begin{aligned} t(r(x)) &= 0.72x - 144 \\ t(r(2700)) &= 0.72(2700) - 144 \\ t(r(2700)) &= 1800 \end{aligned}$$

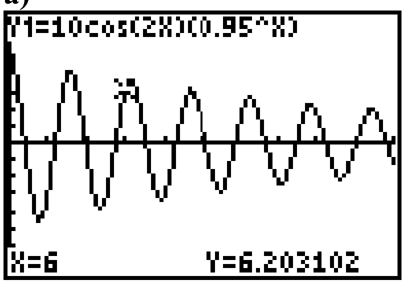
Becky's net income is \$1800.

**d)** $r(t(x)) = r(0.72x)$
$= 0.72x - 200$
Substitute $x = 2700$.
$r(t(x)) = 0.72x - 200$
$r(t(2700)) = 0.72(2700) - 200$
$r(t(2700)) = 1744$
Becky's net income is $1744.

**e)** The calculation order changes the net income. If federal income tax is calculated using the income after the retirement deduction, Becky receives more money.

**Chapter 10 Practice Test Page 513 Question 12**

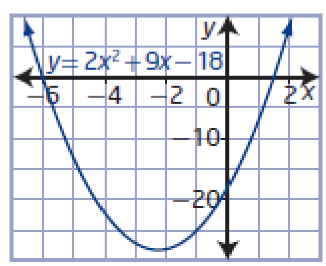
**a)**



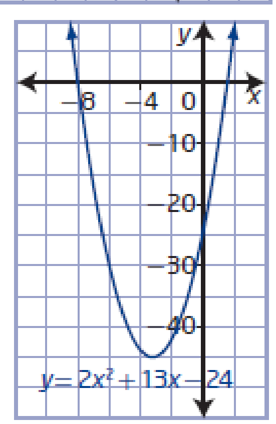

**b)** $x(t) = (10 \cos 2t)(0.95^t)$
$= f(t)g(t)$
Then, $f(t) = 10 \cos 2t$ and $g(t) = 0.95^t$.
The function $f(t)$ is responsible for the periodic motion, while the function $g(t)$ is responsible for the exponential decay of the amplitude.

**Chapter 10 Practice Test Page 513 Question 13**

For $f(x) = 2x^2 + 11x - 21$ and $g(x) = 2x - 3$,
**a)** $y = f(x) - g(x)$
$= 2x^2 + 11x - 21 - (2x - 3)$
$= 2x^2 + 9x - 18$




**b)** $y = f(x) + g(x)$
$= 2x^2 + 11x - 21 + 2x - 3$
$= 2x^2 + 13x - 24$

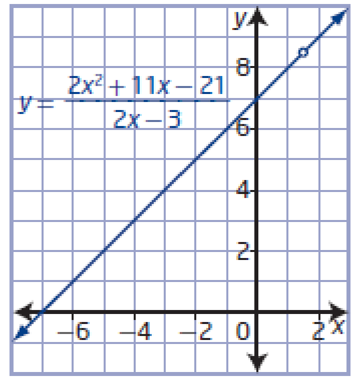


**c)** $y = \frac{f(x)}{g(x)}$

$= \frac{2x^2 + 11x - 21}{2x - 3}$

$= \frac{(2x-3)(x+7)}{2x-3}$

$= x + 7, x \neq 1.5$



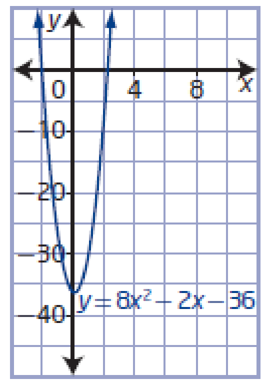

**d)** $y = f(g(x))$

$= f(2x - 3)$

$= 2(2x - 3)^2 + 11(2x - 3) - 21$

$= 2(4x^2 - 12x + 9) + 22x - 33 - 21$

$= 8x^2 - 2x - 36$




**Chapter 10 Practice Test Page 513 Question 14**

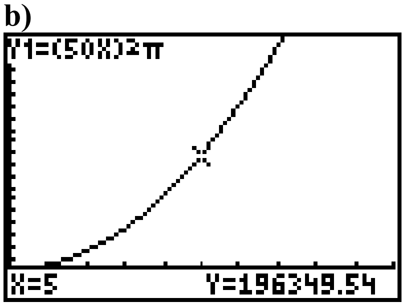
**a)** Let $r$ represent the radius of the circular ripple. Then, the area of the circular ripple is given by $A(r) = \pi r^2$. The radius changes with time according to $r(t) = 50t$.
An equation that represents the area of the circle as a function of time is the composite function $A(r(t))$.

$A(r(t)) = A(50t)$

$= \pi(50t)^2$

$= 2500\pi t^2$

**b)**




**c)** Substitute $t = 5$.

$A(r(t)) = 2500\pi t^2$

$A(r(5)) = 2500\pi(5)^2$

$A(r(5)) = 196\ 349.540\ldots$

The area of the circle after 5 s is approximately 196 350 cm$^2$.

**d)** Example: No. In 30 s, the radius would be 1500 cm. Most likely the circular ripples would no longer be visible on the surface of the water due to turbulence.