Chapter 7 Absolute Value and Reciprocal Functions

Section 7.1 Absolute Value

Section 7.1 Page 363 Question 1

a) $|9| = 9$

b) $|0| = 0$

c) $|-7| = 7$

d) $|-4.728| = 4.728$

e) $|6.25| = 6.25$

f) $\left|-5\frac{1}{2}\right| = 5\frac{1}{2}$

Section 7.1 Page 363 Question 2

First, evaluate each number and express it in decimal form.

$|0.8| = 0.8$ 1.1 $|-2| = 2$, $\left|\frac{3}{5}\right| = 0.6$ –0.4 $\left|-1\frac{1}{4}\right| = 1.25$ –0.8

The numbers from least to greatest are –0.8, –0.4, $\left|\frac{3}{5}\right|$, $|0.8|$, 1.1, $\left|-1\frac{1}{4}\right|$, and $|-2|$.

Section 7.1 Page 363 Question 3

First, evaluate each number and express it in decimal form.

–2.4 $|1.3| = 1.3$ $\left|-\frac{7}{5}\right| = 1.4$ –1.9 $|-0.6| = 0.6$ $\left|1\frac{1}{10}\right| = 1.1$ 2.2

The numbers from greatest to least are 2.2, $\left|-\frac{7}{5}\right|$, $|1.3|$, $\left|1\frac{1}{10}\right|$, $|-0.6|$, –1.9, and –2.4.

Section 7.1 Page 363 Question 4

a) $|8 - 15| = |-7|$
$= 7$

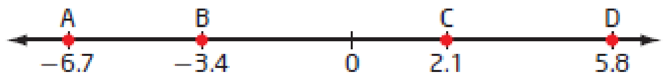
b) $|3| - |-8| = 3 - 8$
$= -5$

c) $|7 - (-3)| = |7 + 3|$
$= |10|$
$= 10$

d) $|2 - 5(3)| = |2 - 15|$
$= |-13|$
$= 13$

Section 7.1 Page 363 Question 5

a) distance AC:
$|2.1 - (-6.7)| = |2.1 + 6.7|$
$= |8.8|$
$= 8.8$



b) distance BD:
$|5.8 - (-3.4)| = |5.8 + 3.4|$
$= |9.2|$
$= 9.2$

c) distance CB:
$|-3.4 - 2.1| = |-5.5|$
$= 5.5$

d) distance DA:
$|-6.7 - 5.8| = |-12.5|$
$= 12.5$

Section 7.1 Page 363 Question 6

a) $2(|-6 - (-11)|) = 2(|-6 + 11|)$
$= 2|5|$
$= 2(5)$
$= 10$

b) $|-9.5| - |12.3| = 9.5 - 12.3$
$= -2.8$

c) $$3\left(\left|\frac{1}{2}\right|\right) + 5\left(\left|-\frac{3}{4}\right|\right) = 3\left(\frac{1}{2}\right) + 5\left(\frac{3}{4}\right)$$
$$= \frac{3}{2} + \frac{15}{4}$$
$$= \frac{6}{4} + \frac{15}{4}$$
$$= \frac{21}{4}$$

d) $|3(-2)^2 + 5(-2) + 7| = |12 - 10 + 7|$
$= |9|$
$= 9$

e) $|-4 + 13| + |6 - (-9)| - |8 - 17| + |-2| = |9| + |15| - |-9| + 2$
$= 9 + 15 - 9 + 2$
$= 17$

Section 7.1 Page 363 Question 7

Examples:

a) $|8 - 3| = |5|$
$= 5$

b) $|12 - (-8)| = |20|$
$= 20$

c) $|2 - 9| = |-7|$
$= 7$

d) $|-7 - 15| = |-22|$
$= 22$

e) $|a - b|$

f) $|m - n|$

Section 7.1 Page 364 Question 8

Let $T_1 = –11$, $T_2 = 7$, and $T_3 = –9$.

$|T_2 – T_1| + |T_3 – T_2|$
$= |7 – (–11)| + |–9 – 7|$
$= |18| + |–16|$
$= 18 + 16$
$= 34$

The total change in temperature for the day is 34 °C.

Section 7.1 Page 364 Question 9

Example:
Let $T_A = 0$, $T_B = 10$, $T_C = 17$, $T_D = 30$, $T_E = 42$, $T_F = 55$, $T_G = 24$, and $T_{GR} = 72$.

$|T_A – T_G| + |T_B – T_G| + |T_C – T_G| + |T_D – T_G| + |T_E – T_G| + |T_F – T_G| + |T_{GR} – T_G|$
$= |0 – 24| + |10 – 24| + |17 – 24| + |30 – 24| + |42 – 24| + |55 – 24| + |72 – 24|$
$= |–24| + |–14| + |–7| + |6| + |18| + |31| + |48|$
$= 24 + 14 + 7 + 6 + 18 + 31 + 48$
$= 148$

The total distance is 148 km.

Section 7.1 Page 364 Question 10

Let $D_{CL} = 51$, $D_{LR} = 496$, $D_W = 918$, $D_{BC} = 1202$, $D_{HJ} = 1016$, and $D_{DJ} = 1422$.

$|D_{LR} – D_{CL}| + |D_W – D_{LR}| + |D_{BC} – D_W| + |D_{HJ} – D_{BC}| + |D_{DJ} – D_{HJ}|$
$= |496 – 51| + |918 – 496| + |1202 – 918| + |1016 – 1202| + |1422 – 1016|$
$= |445| + |422| + |284| + |–186| + |406|$
$= 445 + 422 + 284 + 186 + 406$
$= 1743$

The total distance is 1743 miles.

Section 7.1 Page 365 Question 11

Let $B_1 = 359.22$, $B_2 = 310.45$, $B_3 = 295.78$, $B_4 = 513.65$, and $B_5 = 425.59$.

$|B_2 – B_1| + |B_3 – B_2| + |B_4 – B_3| + |B_5 – B_4|$
$= |310.45 – 359.22| + |295.78 – 310.45| + |513.65 – 295.78| + |425.59 – 513.65|$
$= |–48.77| + |–14.67| + |217.87| + |–88.06|$
$= 48.77 + 14.67 + 217.87 + 88.06$
$= 369.37$

The total change in Vanessa’s bank balance is \$369.37.

b) The net change is the change from the beginning point to the end point. The total change is all the changes in between added up.

$B_5 – B_1 = 425.59 – 359.22$
$= 66.37$

The net change in Vanessa’s bank balance is \$66.37.

Section 7.1 Page 365 Question 12

a) $\text{Amplitude} = \dfrac{|\text{crest height} - \text{trough height}|}{2}$

$= \dfrac{|17 - 2|}{2}$

$= \dfrac{|15|}{2}$

$= \dfrac{15}{2}$

b) $\text{Amplitude} = \dfrac{|\text{crest height} - \text{trough height}|}{2}$

$= \dfrac{|90 - (-90)|}{2}$

$= \dfrac{|180|}{2}$

$= \dfrac{180}{2}$

$= 90$

c) $\text{Amplitude} = \dfrac{|\text{crest height} - \text{trough height}|}{2}$

$= \dfrac{|1.25 - (-0.5)|}{2}$

$= \dfrac{|1.75|}{2}$

$= \dfrac{1.75}{2}$

$= 0.875$

Section 7.1 Page 365 Question 13

Let $C_1 = 0$, $C_2 = 500$, $C_3 = 900$, $C_4 = 1600$, and $C_5 = 2000$.

$|C_4 - C_1| + |C_3 - C_4| + |C_5 - C_3| + |C_2 - C_5|$
$= |1600 - 0| + |900 - 1600| + |2000 - 900| + |500 - 2000|$
$= |1600| + |-700| + |1100| + |-1500|$
$= 1600 + 700 + 1100 + 1500$
$= 4900$

The total change travelled by the race organizer is 4900 m or 4.9 km.

Section 7.1 Page 365 Question 14

a) Let $E_F = 440$ and $E_W = 2089$.
$E_W - E_F = 2089 - 440$
$= 1649$
The net change in elevation from Fairbanks to Whitehorse is 1649 ft.

b) Let $E_C = 935$, $E_{CC} = 597$, and $E_{DC} = 1050$.
$|E_C - E_F| + |E_{CC} - E_C| + |E_{DC} - E_{CC}| + |E_W - E_{DC}|$
$= |935 - 440| + |597 - 935| + |1050 - 597| + |2089 - 1050|$
$= |495| + |-338| + |453| + |1039|$
$= 495 + 338 + 453 + 1039$
$= 2325$
The total change in elevation from Fairbanks to Whitehorse is 2325 ft.

Section 7.1 Page 365 Question 15

Let $V_1 = 7.65$, $V_2 = 7.28$, $V_3 = 8.10$, $V_4 = x$, and $T = 1.55$.
$T = |V_2 - V_1| + |V_3 - V_2| + |V_4 - V_3|$
$1.55 = |7.28 - 7.65| + |8.10 - 7.28| + |x - 8.10|$
$1.55 = |-0.37| + |0.82| + |x - 8.10|$
$1.55 = 0.37 + 0.82 + |x - 8.10|$
$0.36 = |x - 8.10|$
The stock dropped $0.36.

Section 7.1 Page 366 Question 16

a) Let $D_1 = 2$, $D_2 = 7$, $D_3 = 3$, $D_4 = x$, and $T = 15$.
$T = |D_2 - D_1| + |D_3 - D_2| + |D_4 - D_3|$
$15 = |7 - 2| + |3 - 7| + |x - 3|$
$15 = |5| + |-4| + |x - 3|$
$15 = 5 + 4 + |x - 3|$
$6 = |x - 3|$
Toby travels 6 km in the last interval.

b) At the end of the scavenger hunt Toby is at 3 + 6, or the 9-km marker.

Section 7.1 Page 366 Question 17

a) Mikhala:
$|-2.5|^2 + |3|^2 + |-5|^2 + |7.1|^2$
$= 2.5^2 + 3^2 + 5^2 + 7.1^2$
$= 6.25 + 9 + 25 + 50.41$
$= 90.66$

Jocelyn:
$(-2.5)^2 + 3^2 + (-5)^2 + 7.1^2$
$= 6.25 + 9 + 25 + 50.41$
$= |6.25| + |9| + |25| + |50.41|$
$= 6.25 + 9 + 25 + 50.41$
$= 90.66$

b) It does not matter the order in which you square something and take the absolute value of it.

c) This is always true. The result of squaring a number is the same whether the original number is positive or negative.

Section 7.1 Page 366 Question 18

a) Michel has applied the definition of absolute value to $|x - 5|$ by writing it as a positive and a negative case.

b) i) $|x-7| = \begin{cases} x-7, \text{ if } x \geq 7 \\ 7-x, \text{ if } x < 7 \end{cases}$

ii) $|2x-1| = \begin{cases} 2x-1, \text{ if } x \geq \frac{1}{2} \\ 1-2x, \text{ if } x < \frac{1}{2} \end{cases}$

iii) $|3-x| = \begin{cases} 3-x, \text{ if } x \leq 3 \\ x-3, \text{ if } x > 3 \end{cases}$

iv) Since $|x^2 + 4|$ is always positive.

$|x^2 + 4| = x^2 + 4$

Section 7.1 Page 366 Question 19

a) Example:

$|-2| = -(-2)$
$= 2$

$|2| = -(2)$
$\neq -2$

To determine the absolute value of a number, keep the sign for positive values but change the sign for negative values.

Section 7.1 Page 366 Question 20

Let $D_1 = 45$, $D_2 = 67$, $D_3 = 32$, $D_4 = 58$.

$|D_2 - D_1| + |D_3 - D_2| + |D_4 - D_3|$
$= |67 - 45| + |32 - 67| + |58 - 32|$
$= |22| + |-35| + |26|$
$= 22 + 35 + 26$
$= 83$

The oil drop travels a total distance of 83 mm.

Section 7.1 Page 366 Question 21

Example: distance from a location and not the direction

Section 7.1 Page 366 Question 22

Example: I think a signed value would be more appropriate. It would then indicate direction as well: positive for up and negative for down. The velocity of the object at the top of its flight would have to be zero.

Section 7.1 Page 367 Question 23

a) mean $= \dfrac{172+181+178+175+180+168+177+175+178}{9}$

$= \dfrac{1584}{9}$

$= 176$

The mean height of the players is 176 cm.

b) |172 – 176| = 4 |181 – 176| = 5 |178 – 176| = 2 |175 – 176| = 1 |180 – 176| = 4
|168 – 176| = 8 |177 – 176| = 1 |175 – 176| = 1 |178 – 176| = 2
4 + 5 + 2 + 1 + 4 + 8 + 1 + 1 + 2 = 28

c) $\dfrac{28}{9} = 3.111....$

d) The result in part c) means that the typical player's height is within 3.11 cm of the mean height.

Section 7.1 Page 367 Question 24

a) i) Substitute $a = 2$, $p = -1$, and $q = -8$.

$x = p \pm \sqrt{\left|\dfrac{q}{a}\right|}$

$x = -1 \pm \sqrt{\left|\dfrac{-8}{2}\right|}$

$x = -1 \pm \sqrt{4}$

$x = -1 \pm 2$

The zeros are $x = 1$ and $x = -3$.

ii) Substitute $a = -1$, $p = -2$, and $q = 9$.

$x = p \pm \sqrt{\left|\dfrac{q}{a}\right|}$

$x = -2 \pm \sqrt{\left|\dfrac{9}{-1}\right|}$

$x = -2 \pm \sqrt{9}$

$x = -2 \pm 3$

The zeros are $x = 1$ and $x = -5$.

The zeros could be verified by substituting the values into the quadratic function or by graphing and determining the *x*-intercepts.

b) Using the equation form part a), substitute $a = 4$, $p = 3$, and $q = 16$.

$x = p \pm \sqrt{\left|\dfrac{q}{a}\right|}$

$x = 3 \pm \sqrt{\left|\dfrac{16}{4}\right|}$

$x = 3 \pm \sqrt{4}$

$x = 3 \pm 2$

$x = 5$ or $x = 1$

From the values of the parameters, the graph of this parabola opens upward and has its vertex above the x-axis. This means it has no x-intercepts, or zeros of the function. So, the equation from part a) can only be used for quadratic functions in vertex form that actually have zeros.

Section 7.1 Page 367 Question 25

Example: $\sqrt{x^2}$ is always positive, as is $|x|$.

Section 7.2 Absolution Value Functions

Section 7.2 Page 375 Question 1

a)

x	$y = f(x)$	$y = \|f(x)\|$
–2	–3	$\|-3\| = 3$
–1	–1	$\|-1\| = 1$
0	1	$\|1\| = 1$
1	3	$\|3\| = 3$
2	5	$\|5\| = 5$

b)

x	$y = f(x)$	$y = \|f(x)\|$
–2	0	$\|0\| = 0$
–1	–2	$\|-2\| = 2$
0	–2	$\|-2\| = 2$
1	0	$\|0\| = 0$
2	4	$\|4\| = 4$

Section 7.2 Page 375 Question 2

The corresponding point for (–5, –8) on the graph of $y = |f(x)|$ is (–5, 8).

Section 7.2 Page 75 Question 3

The corresponding x-intercept for (3, 0) on the graph of $y = |f(x)|$ is (3, 0). The corresponding y-intercept for (0, –4) on the graph of $y = |f(x)|$ is (0, 4).

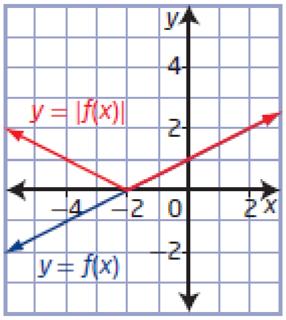
Section 7.2 Page 375 Question 4

The corresponding x-intercepts for (–2, 0) and (7, 0) on the graph of $y = |f(x)|$ is (–2, 0) and (7, 0). The corresponding y-intercept for $\left(0, -\frac{3}{2}\right)$ on the graph of $y = |f(x)|$ is $\left(0, \frac{3}{2}\right)$.

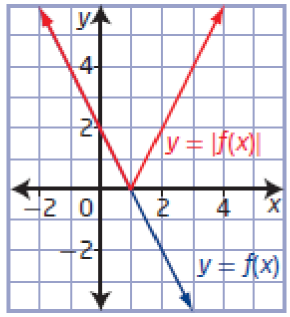
Section 7.2 Page 376 Question 5

Reflect in the x-axis the part of the graph that is below the x-axis.

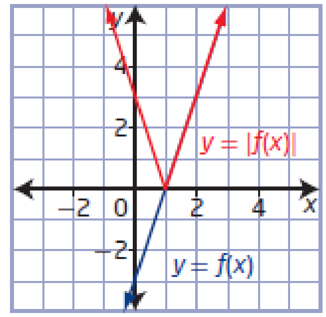
a)



b)



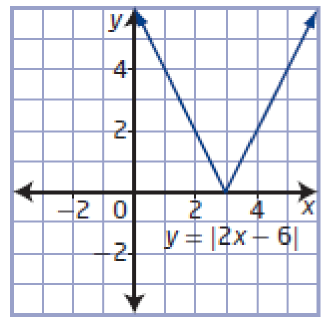
c)



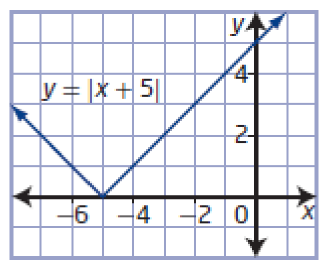
Section 7.2 Page 376 Question 6

Use a table of values to sketch each graph.

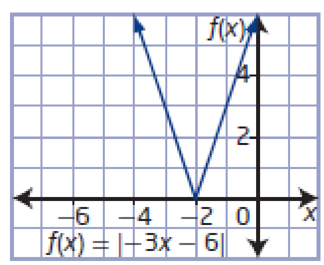
a) x-intercept: 3, y-intercept: 6
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



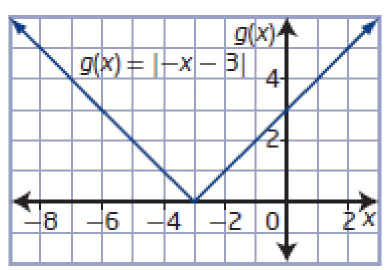
b) x-intercept: –5, y-intercept: 5
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



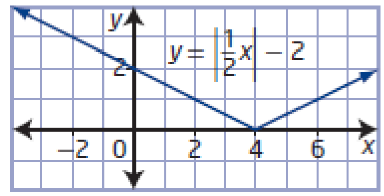
c) x-intercept: –2, y-intercept: 6
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



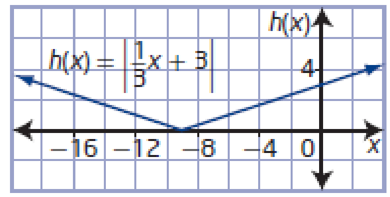
d) x-intercept: –3, y-intercept: 3
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



e) x-intercept: 4, y-intercept: 2
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \ge 0, y \in \mathrm{R}\}$



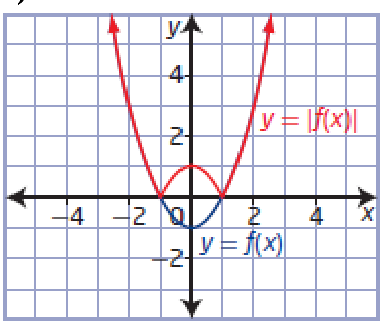
f) x-intercept: –9, y-intercept: 3
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \ge 0, y \in \mathrm{R}\}$



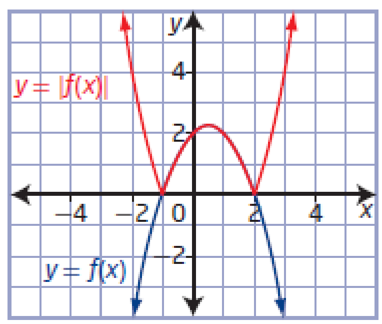
Section 7.2 Page 376 Question 7

Reflect in the x-axis the part of the graph that is below the x-axis.

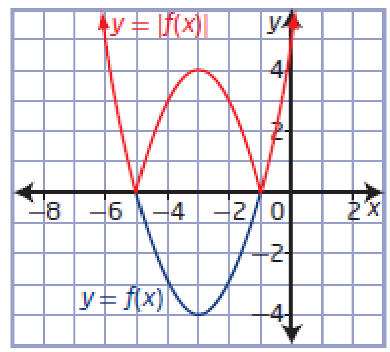
a)



b)



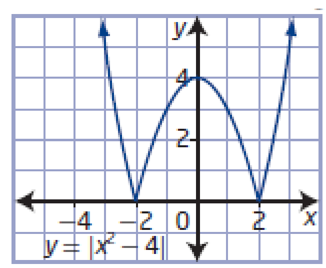
c)



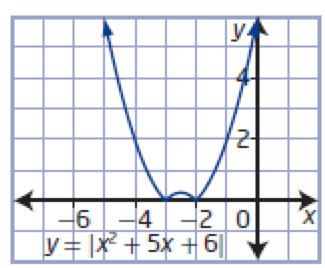
Section 7.2 Page 376 Question 8

Use a table of values to sketch each graph.

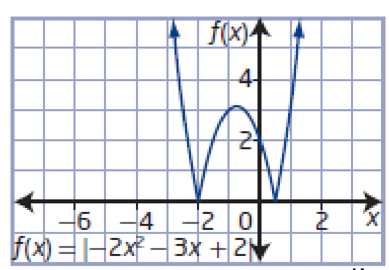
a) x-intercepts: –2 and 2, y-intercept: 4
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \ge 0, y \in \mathrm{R}\}$



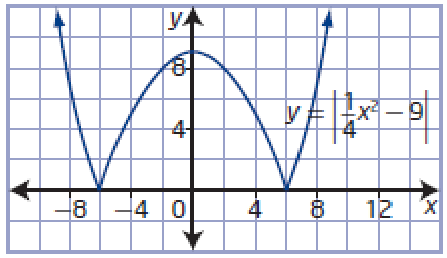
b) x-intercepts: –3 and –2, y-intercept: 6
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \ge 0, y \in \mathrm{R}\}$



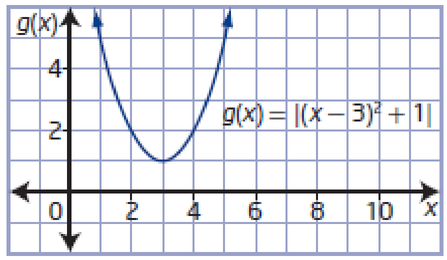
c) x-intercepts: –2 and $\frac{1}{2}$, y-intercept: 2
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \ge 0, y \in \mathrm{R}\}$



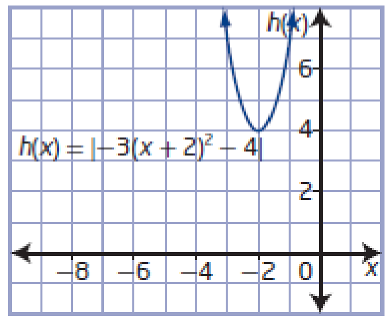
d) x-intercepts: –6 and 6, y-intercept: 9
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$



e) x-intercepts: none, y-intercept: 10
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 1, y \in \mathrm{R}\}$



f) x-intercepts: none, y-intercept: 16
domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 4, y \in \mathrm{R}\}$



Section 7.2 Page 377 Question 9

a) The x-intercept is $x = 1$. When $x \geq 1$, the graph of $y = |2x - 2|$ is the graph of $y = 2x - 2$. When $x < 1$, the graph of $y = |2x - 2|$ is the graph of $y = -(2x - 2)$ or $y = -2x + 2$. The absolute value function $y = |2x - 2|$ expressed as a piecewise function is

$$y = \begin{cases} 2x - 2, \text{ if } x \geq 1 \\ -2x + 2, \text{ if } x < 1 \end{cases}.$$

b) The x-intercept is $x = -2$. When $x \geq -2$, the graph of $y = |3x + 6|$ is the graph of $y = 3x + 6$. When $x < -2$, the graph of $y = |3x + 6|$ is the graph of $y = -(3x + 6)$ or $y = -3x - 6$. The absolute value function $y = |3x + 6|$ expressed as a piecewise function is

$$y = \begin{cases} 3x + 6, \text{ if } x \geq -2 \\ -3x - 6, \text{ if } x < -2 \end{cases}.$$

c) The x-intercept is $x = 2$. When $x \geq 2$, the graph of $y = \left|\frac{1}{2}x - 1\right|$ is the graph of

$y = \frac{1}{2}x - 1$. When $x < 2$, the graph of $y = \left|\frac{1}{2}x - 1\right|$ is the graph of $y = -\left(\frac{1}{2}x - 1\right)$ or

$y = -\frac{1}{2}x + 1$. The absolute value function $y = \left|\frac{1}{2}x - 1\right|$ expressed as a piecewise function is $y = \begin{cases} \frac{1}{2}x - 1, \text{ if } x \geq 2 \\ -\frac{1}{2}x + 1, \text{ if } x < 2 \end{cases}$.

Section 7.2 Page 377 Question 10

a) The x-intercepts are $x = -1$ and $x = 1$. When $x \leq -1$ or $x \geq 1$, the graph of $y = |2x^2 - 2|$ is the graph of $y = 2x^2 - 2$. When $-1 < x < 1$, the graph of $y = |2x^2 - 2|$ is the graph of $y = -(2x^2 - 2)$ or $y = -2x^2 + 2$. The absolute value function $y = |2x^2 - 2|$ expressed as a piecewise function is $y = \begin{cases} 2x^2 - 2, \text{ if } x \leq -1 \text{ or } x \geq 1 \\ -2x^2 + 2, \text{ if } -1 < x < 1 \end{cases}$.

b) The x-intercepts are $x = 1$ and $x = 2$. When $x \leq 1$ or $x \geq 2$, the graph of $y = |(x - 1.5)^2 - 0.25|$ is the graph of $y = (x - 1.5)^2 - 0.25$. When $1 < x < 2$, the graph of $y = |(x - 1.5)^2 - 0.25|$ is the graph of $y = -((x - 1.5)^2 - 0.25)$ or $y = -(x - 1.5)^2 + 0.25$. The absolute value function $y = |(x - 1.5)^2 - 0.25|$ expressed as a piecewise function is
$y = \begin{cases} (x - 1.5)^2 - 0.25, \text{ if } x \leq 1 \text{ or } x \geq 2 \\ -(x - 1.5)^2 + 0.25, \text{ if } 1 < x < 2 \end{cases}$.

c) The x-intercepts are $x = 1$ and $x = 3$. When $x \leq 1$ or $x \geq 3$, the graph of $y = |3(x - 2)^2 - 3|$ is the graph of $y = 3(x - 2)^2 - 3$. When $1 < x < 3$, the graph of $y = |3(x - 2)^2 - 3|$ is the graph of $y = -(3(x - 2)^2 - 3)$ or $y = -3(x - 2)^2 + 3$. The absolute value function $y = |3(x - 2)^2 - 3|$ expressed as a piecewise function is
$y = \begin{cases} 3(x - 2)^2 - 3, \text{ if } x \leq 1 \text{ or } x \geq 3 \\ -3(x - 2)^2 + 3, \text{ if } 1 < x < 3 \end{cases}$.

Section 7.2 Page 377 Question 11

a) First, find the x-intercept.
$y = |x - 4|$
$0 = x - 4$
$x = 4$
When $x \geq 4$, the graph of $y = |x - 4|$ is the graph of $y = x - 4$. When $x < 4$, the graph of $y = |x - 4|$ is the graph of $y = -(x - 4)$ or $y = -x + 4$. The absolute value function $y = |x - 4|$ expressed as a piecewise function is $y = \begin{cases} x - 4, \text{ if } x \geq 4 \\ -x + 4, \text{ if } x < 4 \end{cases}$.

b) First, find the x-intercept.
$y = |3x + 5|$
$0 = 3x + 5$
$x = -\frac{3}{5}$
When $x \geq -\frac{3}{5}$, the graph of $y = |3x + 5|$ is the graph of $y = 3x + 5$. When $x < -\frac{3}{5}$, the graph of $y = |3x + 5|$ is the graph of $y = -(3x + 5)$ or $y = -3x - 5$. The absolute value function $y = |3x + 5|$ expressed as a piecewise function is $y = \begin{cases} 3x+5, \text{ if } x \geq -\frac{3}{5} \\ -3x-5, \text{ if } x < -\frac{3}{5} \end{cases}$.

c) First, find the x-intercepts.
$y = |-x^2 + 1|$
$0 = -x^2 + 1$
$x^2 = 1$
$x = \pm 1$
When $-1 \leq x \leq 1$, the graph of $y = |-x^2 + 1|$ is the graph of $y = -x^2 + 1$. When $x < -1$ or $x > 1$, the graph of $y = |-x^2 + 1|$ is the graph of $y = -(-x^2 + 1)$ or $y = x^2 - 1$. The absolute value function $y = |-x^2 + 1|$ expressed as a piecewise function is
$y = \begin{cases} -x^2+1, \text{ if } -1 \leq x \leq 1 \\ x^2 - 1, \text{ if } x < -1 \text{ or } x > 1 \end{cases}$.

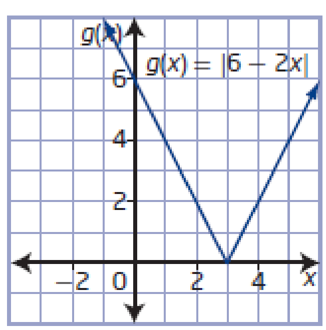
d) First, find the x-intercepts.
$y = |x^2 - x - 6|$
$0 = x^2 - x - 6$
$0 = (x - 3)(x + 2)$
$x - 3 = 0$ or $x + 2 = 0$
$x = 3$ $\quad x = -2$
When $x \leq -2$ or $x \geq 3$, the graph of $y = |x^2 - x - 6|$ is the graph of $y = x^2 - x - 6$. When $-2 < x < 3$, the graph of $y = |x^2 - x - 6|$ is the graph of $y = -(x^2 - x - 6)$ or $y = -x^2 + x + 6$. The absolute value function $y = |x^2 - x - 6|$ expressed as a piecewise function is
$y = \begin{cases} x^2 - x - 6, \text{ if } x \leq -2 \text{ or } x \geq 3 \\ -x^2 + x + 6, \text{ if } -2 < x < 3 \end{cases}$.

Section 7.2 Page 377 Question 12

a)

x	$g(x)$
−1	8
0	6
2	2
3	0
5	4

b)



c) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

d) The x-intercept is $x = 3$. When $x \leq 3$, the graph of $y = |6 - 2x|$ is the graph of $y = 6 - 2x$. When $x > 3$, the graph of $y = |6 - 2x|$ is the graph of $y = -(6 - 2x)$ or $y = -6 + 2x$. The absolute value function $y = |6 - 2x|$ expressed as a piecewise function is

$$y = \begin{cases} 6 - 2x, \text{ if } x \leq 3 \\ -6 + 2x, \text{ if } x > 3 \end{cases}.$$

Section 7.2 Page 377 Question 13

a) Determine the y-intercept by evaluating the function at $x = 0$.

$g(x) = |x^2 - 2x - 8|$
$g(0) = |0^2 - 2(0) - 8|$
$g(0) = |-8|$
$g(0) = 8$

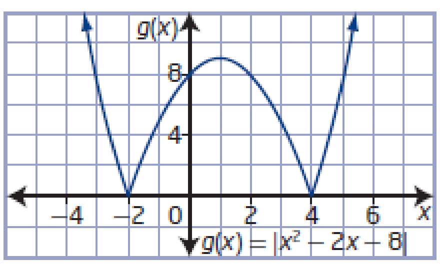
The y-intercept occurs at (0, 8).

To determine the x-intercepts, set $g(x) = 0$ and solve for x.

$g(x) = |x^2 - 2x - 8|$
$0 = x^2 - 2x - 8$
$0 = (x - 4)(x + 2)$
$x - 4 = 0$ or $x + 2 = 0$
$x = 4$ $\quad$ $x = -2$

The x-intercepts occur at (4, 0) and (–2, 0).

b)



c) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

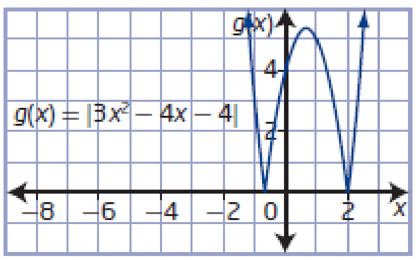
d) When $x \leq -2$ or $x \geq 4$, the graph of $y = |x^2 - 2x - 8|$ is the graph of $y = x^2 - 2x - 8$. When $-2 < x < 4$, the graph of $y = |x^2 - 2x - 8|$ is the graph of $y = -(x^2 - 2x - 8)$ or $y = -x^2 + 2x + 8$. The absolute value function $y = |x^2 - 2x - 8|$ expressed as a piecewise function is

$$y = \begin{cases} x^2 - 2x - 8, \text{ if } x \leq -2 \text{ or } x \geq 4 \\ -x^2 + 2x + 8, \text{ if } -2 < x < 4 \end{cases}.$$

Section 7.2 Page 377 Question 14

a) Determine the y-intercept by evaluating the function at $x = 0$.
$g(x) = |3x^2 – 4x – 4|$
$g(0) = |3(0)^2 – 4(0) – 4|$
$g(0) = |–4|$
$g(0) = 4$
The y-intercept occurs at (0, 4).
To determine the x-intercepts, set $g(x) = 0$ and solve for x.
$g(x) = |3x^2 – 4x – 4|$
$0 = 3x^2 – 4x – 4$
$0 = (3x + 2)(x – 2)$
$3x + 2 = 0$ or $x – 2 = 0$
$x = -\frac{2}{3}$ $\quad x = 2$
The x-intercepts occur at $\left(-\frac{2}{3}, 0\right)$ and (2, 0).

b)



c) The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

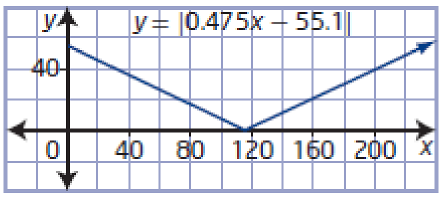
d) When $x \leq -\frac{2}{3}$ or $x \geq 2$, the graph of $y = |3x^2 – 4x – 4|$ is the graph of $y = 3x^2 – 4x – 4$.
When $-\frac{2}{3} < x < 4$, the graph of $y = |3x^2 – 4x – 4|$ is the graph of $y = –(3x^2 – 4x – 4)$ or $y = –3x^2 + 4x + 4$. The absolute value function $y = |3x^2 – 4x – 4|$ expressed as a piecewise function is $y = \begin{cases} 3x^2 - 4x - 4, \text{ if } x \leq -\frac{2}{3} \text{ or } x \geq 2 \\ -3x^2 + 4x + 4, \text{ if } -\frac{2}{3} < x < 2 \end{cases}$.

Section 7.2 Page 377 Question 15

The location of the vertex of $p(x) = 2x^2 - 9x + 10$ will determine who is correct.
$p(x) = 2(x^2 - 4.5x) + 10$
$p(x) = 2(x^2 - 4.5x + 5.0625 - 5.0625) + 10$
$p(x) = 2(x^2 - 4.5x + 5.0625) - 10.125 + 10$
$p(x) = 2(x - 2.25)^2 - 0.125$
Since the vertex is below the x-axis, the graphs and ranges will be different for $p(x) = 2x^2 - 9x + 10$ and $q(x) = |2x^2 - 9x + 10|$. So, Michael is correct.

Section 7.2 Page 378 Question 16

a)

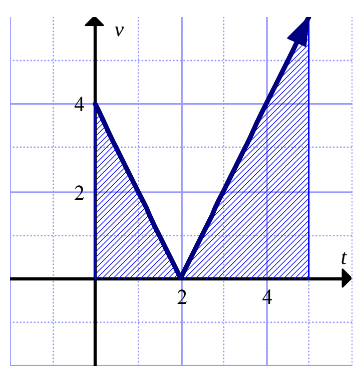


b) The puck ricochets of the side of the table at the x-intercept of $y = |0.475x - 55.1|$.
$y = |0.475x - 55.1|$
$0 = 0.475x - 55.1$
$x = \frac{55.1}{0.475}$
$x = 116$
The puck ricochets of the side of the table at (116, 0).

c) Determine the y-coordinate for an x-coordinate of 236.
$y = |0.475x - 55.1|$
$y = |0.475(236) - 55.1|$
$y = |57|$
$y = 57$
The puck reaches the end of the table at (236, 57). Since the centre of the goal is at 57 cm along the end of the table, the puck will go into the goal.

Section 7.2 Page 378 Question 17

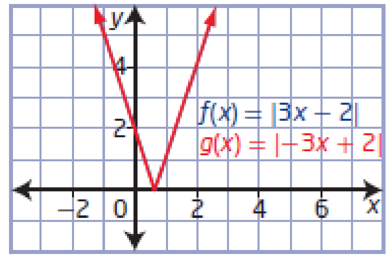
Graph $v(t) = |-2t + 4|$. Find the total area of the shaded regions.



$A_1 = \frac{1}{2}bh$	$A_2 = \frac{1}{2}bh$
$A_1 = \frac{1}{2}(2)(4)$	$A_2 = \frac{1}{2}(3)(6)$
$A_1 = \frac{1}{2}(8)$	$A_2 = \frac{1}{2}(18)$
$A_1 = 4$	$A_2 = 9$

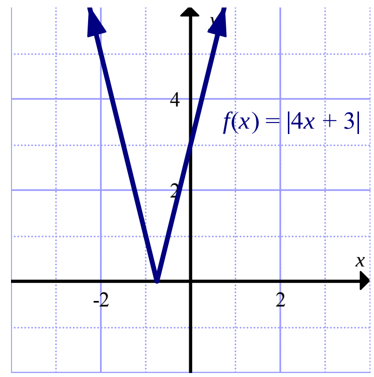
The distance travelled in the first 5 s is 4 + 9, or 13 m.

Section 7.2 Page 378 Question 18



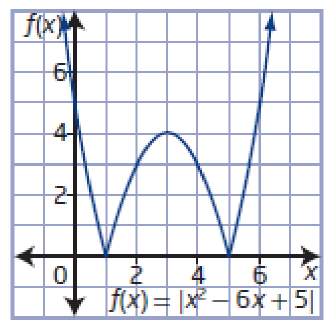
a) The two graphs are identical. They are identical because one is the negative of the other but since they are in absolute value brackets there is no change.

b) Following the example in part a), another absolute value function of the form $g(x) = |ax + b|$ with the same graph is $g(x) = |-(4x + 3)|$ or $g(x) = |-4x - 3)|$.



Section 7.2 Page 378 Question 19

Following the example in #18 part a), another absolute value function of the form $g(x) = |ax^2 + bx + c|$ with the same graph as $f(x) = |x^2 - 6x + 5|$ is
$g(x) = |-(x^2 - 6x + 5)|$or $g(x) = |-x^2 + 6x - 5|$.



Section 7.2 Page 378 Question 20

Use the given intercept points, $\left(\frac{3}{2}, 0\right)$ and (0, 6), to find the slope, *a*, of the line.

$$a = \frac{6-0}{0-\frac{3}{2}}$$

$$a = 6\left(-\frac{2}{3}\right)$$

$a = -4$
Substitute $a = -4$ and $b = 6$ into $f(x) = |ax + b|$.
$f(x) = |-4x + 6|$
From the example in #18 part a), another possibility is $a = 4$ and $b = -6$.
$f(x) = |4x - 6|$

Section 7.2 Page 378 Question 21

Use the given x-intercepts, (–6, 0) and (2, 0), to determine the values of b and c.
$f(x) = |(x + 6)(x - 2)|$
$f(x) = |x^2 + 4x - 12|$
So, $b = 4$ and $c = -12$.

Section 7.2 Page 378 Question 22

The graphs of $y = |x^2|$ and $y = x^2$ are identical because $y = x^2$ is always positive so taking the absolute value of the function does not change the range.

Section 7.2 Page 379 Question 23

For $x \geq 0$ and $y \geq 0$:
$|x| + |y| = x + y$

For $x < 0$ and $y < 0$:
$|x| + |y| = -x - y$

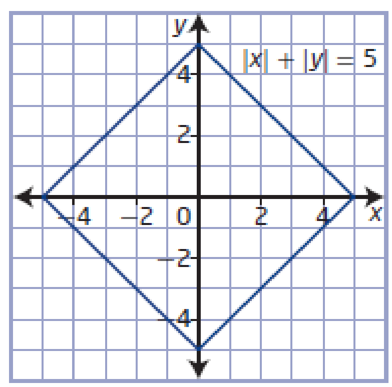
Write $|x + y|$ as a piecewise definition.

$$|x+y| = \begin{cases} x+y, \text{ if } x+y \geq 0 \\ -x-y, \text{ if } x+y < 0 \end{cases}$$

The statement $|x| + |y| = |x + y|$ is only true when x and y are the same sign.

Section 7.2 Page 379 Question 24

Use a table of values to help graph $|x| + |y| = 5$.
The values of x and y will range from –5 to 5.



Section 7.2 Page 379 Question 25

Write $|x|$ and $|y|$ as a piecewise definition.

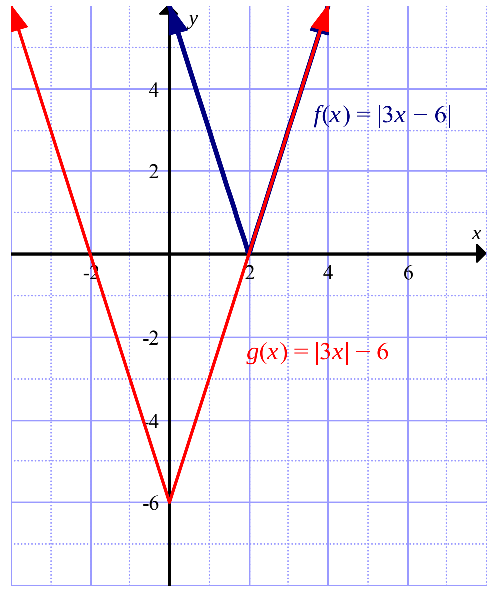
$$|x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases} \qquad |y| = \begin{cases} y, \text{ if } y \geq 0 \\ -y, \text{ if } y < 0 \end{cases}$$

Test the four possible cases.

Case	$\lvert x \rvert \lvert y \rvert$	$\lvert xy \rvert$
$x \geq 0, y \geq 0$	xy	xy
$x \geq 0, y < 0$	$x(-y)$	$-xy$
$x < 0, y \geq 0$	$(-x)y$	$-xy$
$x < 0, y < 0$	$(-x)(-y)$	xy

Section 7.2 Page 379 Question 26

Example: The graphs have the same shape but in different locations. The graph of $g(x)$ appears to be a translation of 2 units to the left and 6 units down of the graph of $f(x)$.



Section 7.2 Page 379 Question 27

Example: Graph both parts of the piecewise functions, taking care to allow them only in their specified domain.

Section 7.2 Page 379 Question 28

The graphs of $y = ax^2 + bx + c$ and $y = = |ax^2 + bx + c|$ will be equivalent when the quadratic function has $a > 0$ and a vertex on or above the x-axis. This means that the discriminant will be less than or equal to 0.

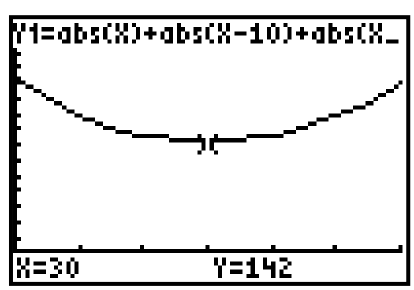
Section 7.2 Page 379 Question 29

Step 1 Example: Yes. Its location will be further east than its current planned location.

Step 2 Use absolute value because direction from the location is not relevant.
Let $T_A = 0$, $T_B = 10$, $T_C = 17$, $T_D = 30$, $T_E = 42$, $T_F = 55$, $T_G = x$, and $T_{GR} = 72$.

$|T_G - T_A| + |T_G - T_B| + |T_G - T_C| + |T_G - T_D| + |T_G - T_E| + |T_G - T_F| + |T_G - T_{GR}|$
$= |x - 0| + |x - 10| + |x - 17| + |x - 30| + |x - 42| + |x - 55| + |x - 72|$
$= |x| + |x - 10| + |x - 17| + |x - 30| + |x - 42| + |x - 55| + |x - 72|$

Step 3 Use window settings of x: [0, 60, 10] and y: [–30, 300, 20].



Step 4 **a)** The graph shows that there is a location that results in a minimum distance.
b) The minimum point is at (30, 142).
c) The point represents a place 30 km east of Allenby and results in a total distance from all towns of 142 km.

Section 7.2 Page 379 Question 30

a) Substitute $a = 1$, $p = 3$, and $q = 7$ into $y = |a(x - p)^2 + q|$: $y = |(x - 3)^2 + 7|$

b) Substitute $a = \frac{4}{5}$, $p = -3$, and $q = 0$ into $y = |a(x - p)^2 + q|$: $y = \left|\frac{4}{5}(x+3)^2\right|$

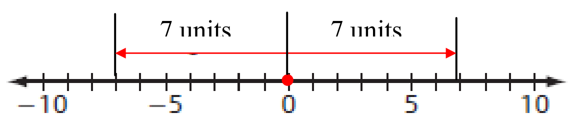
c) Substitute $a = -1$, $p = 0$, and $q = -6$ into $y = |a(x - p)^2 + q|$: $y = |-x^2 - 6|$

d) Substitute $a = 5$, $p = -3$, and $q = 3$ into $y = |a(x - p)^2 + q|$: $y = |5(x + 3)^2 + 3|$

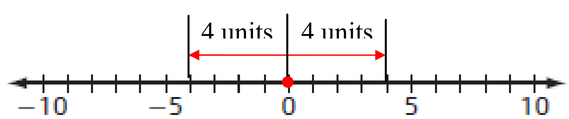
Section 7.3 Absolute Value Equations

Section 7.3 Page 389 Question 1

a) For $|x| = 7$, $x = -7$ or $x = 7$.



b) $|x| + 8 = 12$
$|x| = 4$
$x = -4$ or $x = 4$

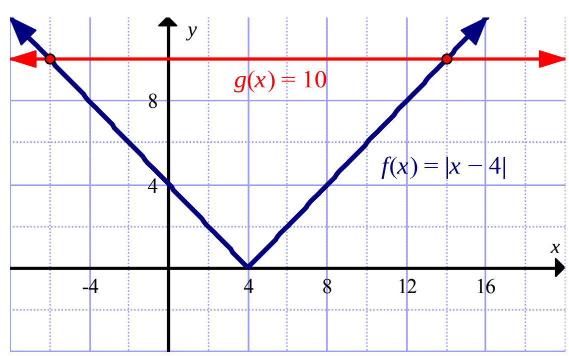


c) $|x| + 4 = 4$
$|x| = 0$
$x = 0$

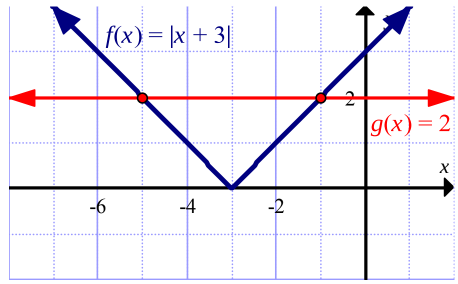
d) The equation $|x| = -6$ has not solution, since the absolute value is always positive.

Section 7.3 Page 389 Question 2

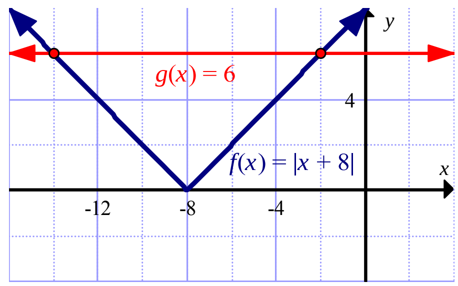
a) Graph $f(x) = |x - 4|$ and $g(x) = 10$ and find the points of intersection.
The graphs intersect at (-6, 10) and (14, 10).
This means that $x = -6$ and $x = 14$ are solutions to the equation $|x - 4| = 10$.



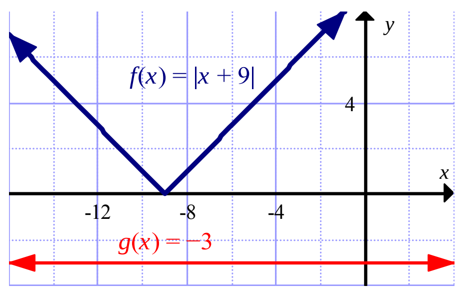
b) Graph $f(x) = |x + 3|$ and $g(x) = 2$ and find the points of intersection.
The graphs intersect at (-5, 2) and (–1, 2). This means that $x = –5$ and $x = –1$ are solutions to the equation $|x + 3| = 2$.



c) Graph $f(x) = |x + 8|$ and $g(x) = 6$ and find the points of intersection.
The graphs intersect at (-14, 6) and (–2, 6). This means that $x = –14$ and $x = –2$ are solutions to the equation $6 = |x + 8|$.



d) Graph $f(x) = |x + 9|$ and $g(x) = –3$ and find the points of intersection.
The graphs do not intersect. This means that there are no solutions to the equation $|x + 9| = –3$.



Section 7.3 Page 389 Question 3

a) The two solutions, –2 and 2, are centred about 0, so $a = 1$ and $b = 0$. The solutions are 2 units from 0, so $c = 2$. The absolute value equation in the form $|ax + b| = c$ is $|x + 0| = 2$, or $|x| = 2$.

b) The two solutions, –4 and 8, are centred about 2, so $b = –2$. The solutions are 6 units from 2, so $c = 6$. The absolute value equation is of form $|ax – 2| = 6$.
Substitute one of the solutions to determine the value of a. Use the solution on the right, $x = 8$, which represents when the expression $ax – 2 \geq 0$.

$$\begin{aligned} ax - 2 &= 6 \\ a(8) - 2 &= 6 \\ 8a &= 8 \\ a &= 1 \end{aligned}$$

The absolute value equation in the form $|ax + b| = c$ is $|x – 2| = 6$.

c) The two solutions, –1 and 9, are centred about 4, so $b = –4$. The solutions are 5 units from 4, so $c = 5$. The absolute value equation is of form $|ax – 4| = 5$.

Substitute one of the solutions to determine the value of a. Use the solution on the right, $x = 9$, which represents when the expression $ax - 4 \geq 0$.

$$ax - 4 = 5$$
$$a(9) - 4 = 5$$
$$9a = 9$$
$$a = 1$$

The absolute value equation in the form $|ax + b| = c$ is $|x - 4| = 5$.

Section 7.3 Page 389 Question 4

a) Examine the two cases.

Case 1

The expression $|x + 7|$ equals $x + 7$ when $x \geq -7$.

$$x + 7 = 12$$
$$x = 5$$

The value 5 satisfies the condition $x \geq -7$.

Case 2

The expression $|x + 7|$ equals $-(x + 7)$ when $x < -7$.

$$-(x + 7) = 12$$
$$x + 7 = -12$$
$$x = -19$$

The value -19 satisfies the condition $x < -7$.

The solution is $x = -19$ or $x = 5$.

b) First, isolate the absolute value expression.

$$|3x - 4| + 5 = 7$$
$$|3x - 4| = 2$$

Examine the two cases.

Case 1

The expression $|3x - 4|$ equals $3x - 4$ when $x \geq \frac{4}{3}$.

$$3x - 4 = 2$$
$$3x = 6$$
$$x = 2$$

The value 2 satisfies the condition $x \geq \frac{4}{3}$.

Case 2

The expression $|3x - 4|$ equals $-(3x - 4)$ when $x < \frac{4}{3}$.

$$-(3x - 4) = 2$$
$$3x - 4 = -2$$
$$3x = 2$$
$$x = \frac{2}{3}$$

The value $\frac{2}{3}$ satisfies the condition $x < \frac{4}{3}$.

The solution is $x = 2$ or $x = \frac{2}{3}$.

c) First, isolate the absolute value expression.

$2|x + 6| + 12 = –4$
$2|x + 6| = –16$
$|x + 6| = –8$

Since the absolute value of a number is always greater than or equal to zero, by inspection this equation has no solution.

d) First, isolate the absolute value expression.

$–6|2x – 14| = –42$
$|2x – 14| = 7$

Examine the two cases.

Case 1

The expression $|2x – 14|$ equals $2x – 14$ when $x \geq 7$.

$2x – 14 = 7$
$2x = 21$
$x = \frac{21}{2}$

The value $\frac{21}{2}$ satisfies the condition $x \geq 7$.

Case 2

The expression $|2x – 14|$ equals $–(2x – 14)$ when $x < 7$.

$–(2x –14) = 7$
$2x – 14 = –7$
$2x = 7$
$x = \frac{7}{2}$

The value $\frac{7}{2}$ satisfies the condition $x < 7$.

The solution is $x = \frac{21}{2}$ or $x = \frac{7}{2}$.

Section 7.3 Page 389 Question 5

a) Examine the two cases.

Case 1

The expression $|2a + 7|$ equals $2a + 7$ when $a \geq -\frac{7}{2}$.

$2a + 7 = a – 4$
$a = –11$

The value –11 does not satisfy the condition $a \geq -\frac{7}{2}$, so it is an extraneous solution.

Case 2

The expression |2*a* + 7| equals –(2*a* + 7) when $a < -\frac{7}{2}$.

$$\begin{aligned}-(2a + 7) &= a - 4\\ -2a - 7 &= a - 4\\ -3a &= 3\\ a &= -1\end{aligned}$$

The value –1 does not satisfy the condition $a < -\frac{7}{2}$, so it is an extraneous solution.

There are no solutions.

b) Examine the two cases.
Case 1

The expression |7 + 3*x*| equals 7 + 3*x* when $x \geq -\frac{7}{3}$.

$$\begin{aligned}7 + 3x &= 11 - x\\ 4x &= 4\\ x &= 1\end{aligned}$$

The value 1 satisfies the condition $x \geq -\frac{7}{3}$.

Case 2

The expression |7 + 3*x*| equals –(7 + 3*x*) when $x < -\frac{7}{3}$.

$$\begin{aligned}-(7 + 3x) &= 11 - x\\ -7 - 3x &= 11 - x\\ -2x &= 18\\ x &= -9\end{aligned}$$

The value –9 satisfies the condition $x < -\frac{7}{3}$.

The solution is *x* = 1 or *x* = –9.

c) Examine the two cases.
Case 1

The expression |1 – 2*m*| equals 1 – 2*m* when $m \leq \frac{1}{2}$.

$$\begin{aligned}1 - 2m &= m + 2\\ -3m &= 1\\ m &= -\frac{1}{3}\end{aligned}$$

The value $-\frac{1}{3}$ satisfies the condition $m \leq \frac{1}{2}$.

Case 2

The expression $|1 - 2m|$ equals $-(1 - 2m)$ when $m > \frac{1}{2}$.

$$
\begin{aligned}
-(1 - 2m) &= m + 2 \\
-1 + 2m &= m + 2 \\
m &= 3
\end{aligned}
$$

The value 3 satisfies the condition $m < \frac{1}{2}$.

The solution is $m = -\frac{1}{3}$ or $m = 3$.

d) Examine the two cases.

Case 1

The expression $|3x + 3|$ equals $3x + 3$ when $x \geq -1$.

$$
\begin{aligned}
3x + 3 &= 2x - 5 \\
x &= -8
\end{aligned}
$$

The value –8 does not satisfy the condition $x \geq -1$, so it is an extraneous solution.

Case 2

The expression $|3x + 3|$ equals $-(3x + 3)$ when $x < -1$.

$$
\begin{aligned}
-(3x + 3) &= 2x - 5 \\
-3x - 3 &= 2x - 5 \\
-5x &= -2 \\
x &= \frac{2}{5}
\end{aligned}
$$

The value $\frac{2}{5}$ does not satisfy the condition $x < -1$, so it is an extraneous solution.

There are no solutions.

e) First, isolate the absolute value expression.

$$
\begin{aligned}
3|2a + 7| &= 3a + 12 \\
|2a + 7| &= a + 4
\end{aligned}
$$

Examine the two cases.

Case 1

The expression $|2a + 7|$ equals $2a + 7$ when $a \geq -\frac{7}{2}$.

$$
\begin{aligned}
2a + 7 &= a + 4 \\
a &= -3
\end{aligned}
$$

The value –3 satisfies the condition $a \geq -\frac{7}{2}$.

Case 2

The expression $|2a + 7|$ equals $-(2a + 7)$ when $a < -\frac{7}{2}$.

$$
\begin{aligned}
-(2a + 7) &= a + 4 \\
-2a - 7 &= a + 4
\end{aligned}
$$

$-3a = 11$

$a = -\frac{11}{3}$

The value $-\frac{11}{3}$ satisfies the condition $a < -\frac{7}{2}$.

The solution is $a = -3$ or $a = -\frac{11}{3}$.

Section 7.3 Page 389 Question 6

a) Examine the two cases.

Case 1

The expression $|x|$ equals x when $x \geq 0$.

$x = x^2 + x - 3$

$0 = x^2 - 3$

$3 = x^2$

$x = \pm\sqrt{3}$

Only the value $\sqrt{3}$ satisfies the condition $x \geq 0$.

Case 2

The expression $|x|$ equals $-x$ when $x < 0$.

$-x = x^2 + x - 3$

$0 = x^2 + 2x - 3$

$0 = (x + 3)(x - 1)$

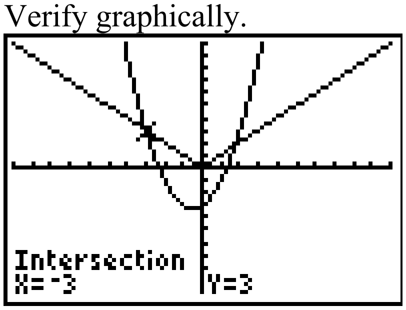
$x + 3 = 0$ or $x - 1 = 0$

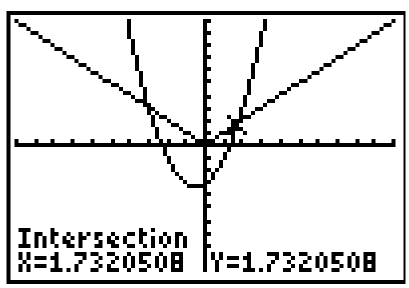
$x = -3$ $\quad$ $x = 1$

Only the value -3 satisfies the condition $x < 0$.

The solution is $x = -3$ or $x = \sqrt{3}$, or $x = 1.7320…$.

Verify graphically.





b) The graph of $y = x^2 - 2x + 2$ is always positive.

The expression $|x^2 - 2x + 2|$ equals $x^2 - 2x + 2$ when $x^2 - 2x + 2 \geq 0$.

$x^2 - 2x + 2 = 3x - 4$

$x^2 - 5x + 6 = 0$

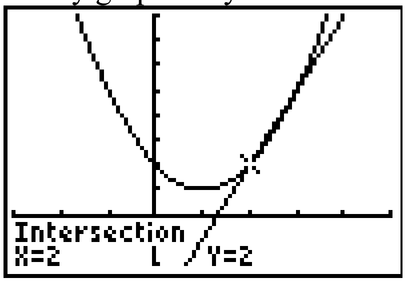
$(x - 2)(x - 3) = 0$

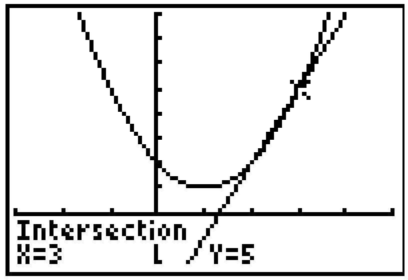
$x - 2 = 0$ or $x - 3 = 0$

$x = 2$ $\quad$ $x = 3$

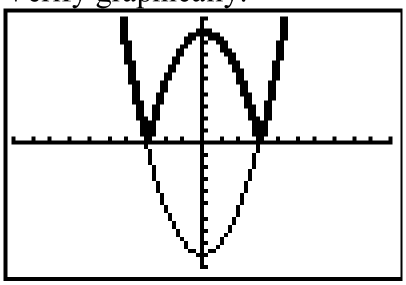
The solution is $x = 2$ or $x = 3$.

Verify graphically.





c) The expression $|x^2 - 9|$ equals $x^2 - 9$ when $x \le -3$ or $x \ge 3$.
Verify graphically.



d) Examine the two cases.

Case 1

The expression $|x^2 - 1|$ equals $x^2 - 1$ when $x \le -1$ or $x \ge 1$.

$x^2 - 1 = x$

$x^2 - x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{5}}{2}$$

$x = \frac{1+\sqrt{5}}{2}$ or $x = \frac{1-\sqrt{5}}{2}$

$x = 1.6180\ldots$ $\quad$ $x = -0.6180\ldots$

Only the value $\frac{1+\sqrt{5}}{2}$ satisfies the conditions.

Case 2

The expression $|x^2 - 1|$ equals $-(x^2 - 1)$ when $-1 < x < 1$.

$-(x^2 - 1) = x$

$0 = x^2 + x - 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{5}}{2}$$

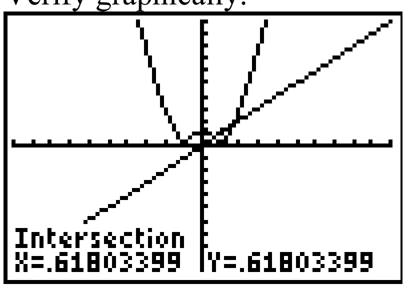
$x = \frac{-1 + \sqrt{5}}{2}$ or $x = \frac{-1 - \sqrt{5}}{2}$

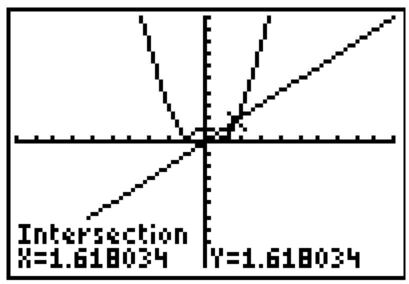
$x = 0.6180\ldots$ $\qquad$ $x = -1.6180\ldots$

Only the value of $\frac{-1 + \sqrt{5}}{2}$ satisfies the conditions.

The solution is $x = \frac{-1 + \sqrt{5}}{2}$ or $x = \frac{1 + \sqrt{5}}{2}$.

Verify graphically.





e) Examine the two cases.

Case 1

The expression $|x^2 - 2x - 16|$ equals $x^2 - 2x - 16$ when $x \le 1 - \sqrt{17}$ or $x \ge 1 + \sqrt{17}$.

$x^2 - 2x - 16 = 8$

$x^2 - 2x - 24 = 0$

$(x - 6)(x + 4) = 0$

$x - 6 = 0$ or $x + 4 = 0$

$x = 6$ $\qquad$ $x = -4$

Both values of –4 and 6 satisfy the conditions.

Case 2

The expression $|x^2 - 2x - 16|$ equals $-(x^2 - 2x - 16)$ when $1 - \sqrt{17} < x < 1 + \sqrt{17}$.

$-(x^2 - 2x - 16) = 8$

$x^2 - 2x - 16 = -8$

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

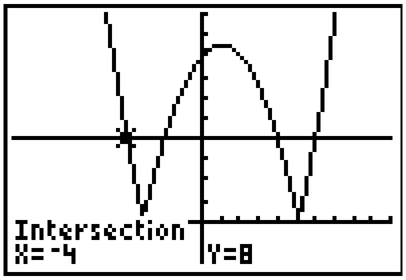
$x - 4 = 0$ or $x + 2 = 0$

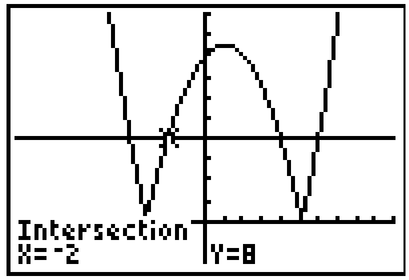
$x = 4$ $\qquad$ $x = -2$

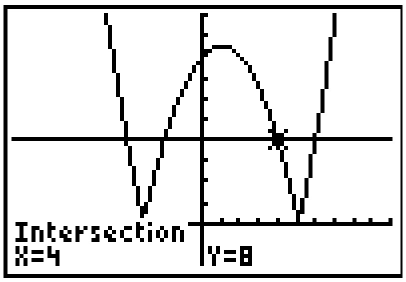
Both values of –2 and 4 satisfy the conditions.

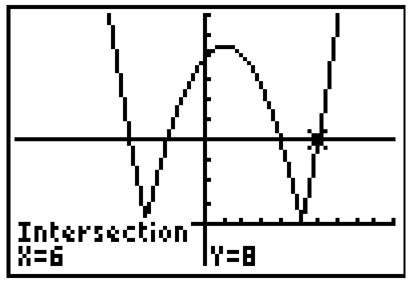
The solution is $x = -4$, $x = -2$, $x = 4$, or $x = 6$.

Verify graphically.









Section 7.3 Page 389 Question 7

a) Substitute $a = 18$ and $b = 0.5$ into $|d - a| = b$.
The absolute value equation that describes the acceptance limits for the bolts is $|d - 18| = 0.5$.

b) Examine the two cases.
Case 1
The expression $|d - 18|$ equals $d - 18$ when $d \geq 18$.
$d - 18 = 0.5$
$d = 18.5$
The value 18.5 satisfies the condition.
Case 2
The expression $|d - 18|$ equals $-(d - 18)$ when $d < 18$.
$-(d - 18) = 0.5$
$d - 18 = -0.5$
$d = 17.5$
The value 18.5 satisfies the condition.
The maximum diameter of the bolts is 18.5 mm and the minimum diameter of the bolts is 17.5 mm.

Section 7.3 Page 389 Question 8

a) Substitute $a = 299\ 792\ 456.2$ and $b = 1.1$ into $|c - a| = b$.
The absolute value equation that describes the measured speed of light is
$|c - 299\ 792\ 456.2| = 1.1$.

b) Examine the two cases.
Case 1
The expression $|c - 299\ 792\ 456.2|$ equals $c - 299\ 792\ 456.2$ when $c \geq 299\ 792\ 456.2$.

$c - 299\ 792\ 456.2 = 1.1$
$c = 299\ 792\ 457.3$
The value 299 792 457.3 satisfies the condition.
Case 2
The expression $|c - 299\ 792\ 456.2|$ equals $-(c - 299\ 792\ 456.2)$ when $c < 299\ 792\ 456.2$.
$-(c - 299\ 792\ 456.2) = 1.1$
$c - 299\ 792\ 456.2 = -1.1$
$c = 299\ 792\ 455.1$
The value 299 792 455.1 satisfies the condition.
The maximum value for the speed of light is 299 792 457.3 m/s and the minimum value for the speed of light is 299 792 455.1 m/s.

Section 7.3 Page 389 Question 9

a) Substitute $a = 50\ 000$ and $b = 2000$ into $|V - a| = b$.
The absolute value equation that describes the limits for the volume of fuel is $|V - 50\ 000| = 2000$.

b) Examine the two cases.
Case 1
The expression $|V - 50\ 000|$ equals $V - 50\ 000$ when $V \geq 50\ 000$.
$V - 50\ 000 = 2000$
$V = 52\ 000$
The value 52 000 satisfies the condition.
Case 2
The expression $|V - 50\ 000|$ equals $-(V - 50\ 000)$ when $V < 50\ 000$.
$-(V - 50\ 000) = 2000$
$V - 50\ 000 = -2000$
$V = 48\ 000$
The value 48 000 L satisfies the condition.
The maximum volume of fuel is 50 000 m/s and the minimum value of fuel is 48 000 L.

Section 7.3 Page 390 Question 10

a) The statement represents two values of x: 7 + 4.8, or 11.8, and 7 – 4.8, or 2.2.

b) The statement $x = 7 \pm 4.8$ written as an absolute value equation is $|x - 7| = 4.8$.

Section 7.3 Page 390 Question 11

a) The actual mass of the substance is 66.5 g (64 + 2.5 or 69 – 2.5).

b) The least possible measure for the volume is 258 – 7, or 251 mL. The greatest measure for the volume is 258 + 7, or 256 mL.

Section 7.3 Page 390 Question 12

a) Examine the two cases.
Case 1
The expression $|d - 381\ 550|$ equals $d - 381\ 550$ when $d \geq 381\ 550$.

$$d - 381\ 550 = 25\ 150$$
$$d = 406\ 700$$

The value 406 700 satisfies the condition.
Case 2
The expression $|d - 381\ 550|$ equals $-(d - 381\ 550)$ when $d < 381\ 550$.

$$-(d - 381\ 550) = 25\ 150$$
$$d - 381\ 550 = -25\ 150$$
$$d = 356\ 400$$

The value 356 400 satisfies the condition.
The perigee is 356 400 km and the apogee is 406 700 km.

Section 7.3 Page 390 Question 13

a) Isolate the absolute value expression.

$$n + |-n| = 2n$$
$$|-n| = n$$

The expression $|-n|$ equals $-n$ when $n \leq 0$.
The expression $|-n|$ equals $-(-n)$ or n when $n > 0$.
So, $n \geq 0$ would make the equation true.

b) Isolate the absolute value expression.

$$n + |-n| = 0$$
$$|-n| = -n$$

The expression $|-n|$ equals $-n$ when $n \leq 0$.
The expression $|-n|$ equals $-(-n)$ or n when $n > 0$.
So, $n \leq 0$ would make the equation true.

Section 7.3 Page 390 Question 14

a) Isolate the absolute value expression.

$$|ax| - b = c$$
$$|ax| = b + c,\ b + c \geq 0$$

Examine the two cases.
Case 1
The expression $|ax|$ equals ax when $x \geq 0$.

$$ax = b + c$$
$$x = \frac{b + c}{a},\ a \neq 0$$

Case 2
The expression $|ax|$ equals $-(ax)$ when $x < 0$.

$$-ax = b + c$$

$x = \frac{-(b+c)}{a}$, $a \neq 0$

b) For $|x - b| = c$, $c \geq 0$. Examine the two cases.
Case 1
The expression $|x - b|$ equals $x - b$ when $x \geq b$.
$x - b = c$
$x = b + c$
Case 2
The expression $|x - b|$ equals $-(x - b)$ when $x < b$.
$-(x - b) = c$
$x - b = -c$
$x = b - c$

Section 7.3 Page 390 Question 15

Andrea's solution is correct. Erin made a mistake in the third line by setting $x + 4$ equal to 4 instead of $x - 4$.

Section 7.3 Page 390 Question 16

Let T represent the water temperature, in degrees Celsius. Then, an equation that describes the limits of the water temperature is $|T - 11.5| = 2.5$.
Examine the two cases.
Case 1
The expression $|T - 11.5|$ equals $T - 11.5$ when $T \geq 11.5$.
$T - 11.5 = 2.5$
$T = 14$
The value 14 satisfies the condition.
Case 2
The expression $|T - 11.5|$ equals $-(T - 11.5)$ when $T < 11.5$.
$-(T - 11.5) = 2.5$
$T - 11.5 = -2.5$
$T = 9$
The value 9 satisfies the condition.
The limits of the ideal temperature range are 9 °C to 14 °C.

Section 7.3 Page 391 Question 17

a) Let a represent the amount of ASA per tablet, in milligrams. The tolerance is 20% of 81, or 16.2 mg. Then, an equation that describes the limits of ASA per tablet is $|a - 81| = 16.2$.
Examine the two cases.
Case 1
The expression $|a - 81|$ equals $a - 81$ when $a \geq 81$.
$a - 81 = 16.2$

$a = 97.2$

The value 97.2 satisfies the condition.

Case 2

The expression $|a - 81|$ equals $-(a - 81)$ when $a < 81$.

$-(a - 81) = 16.2$

$a - 81 = -16.2$

$a = 64.8$

The value 64.8 satisfies the condition.

The maximum and minimum amounts of ASA are 64.8 mg and 97.2 mg, respectively.

b) Example: The drug company might lean toward the 97.2-mg limit because it would better regulate and reduce heart attack risk.

Section 7.3 Page 391 Question 18

Let t represent the time, in hours. Then, an equation that describes the earliest and latest acceptable times for launch is $|t - 10| = 2$.

Section 7.3 Page 391 Question 19

a) Since the value of $|x + 1|$ is always greater than or equal to zero, the statement $|x + 1| > 0$ is sometimes true. It will equal zero when $x = -1$.

b) Since the solution to $|x + a| = 0$ is $x = -a$, the statement $|x + a| = 0$ is sometimes true. When x has any other value it will not be true.

c) Since the value of $|x + a| + a$, is always greater than zero, the statement $|x + a| + a > 0$ is always true.

Section 7.3 Page 391 Question 20

Examples:

a) An absolute value equation with solutions –2 and 8 is $|x - 3| = 5$.

b) An absolute value equation with no solution is $|x - 3| = -5$.

c) An absolute value equation with one integral solution is $|x - 3| + 2 = 2$.

d) An absolute value equation with two integral solutions is $|x - 3| = 2$.

Section 7.3 Page 391 Question 21

The absolute value equation $|ax + b| = 0$ will always have solution $x = -\frac{b}{a}$, $a \neq 0$.

Section 7.3 Page 391 Question 22

a) The graph shows the absolute value function $y = |x - 3|$ and the linear function $y = 4$. So, the equation being solved is $|x - 3| = 4$.

b) The graph shows the absolute value function $y = |x^2 - 4|$ and the linear function $y = 5$. So, the equation being solved is $|x^2 - 4| = 5$.

Section 7.3 Page 391 Question 23

Example: The equation $|3x + 1| = -2$ has no solution because the expression on the left side is always non-negative. Isolating the absolute value expression in $|3x + 1| - 4 = -2$ gives $|3x + 1| = 2$, which has solutions of $x = -1$ and $x = \frac{1}{3}$.

Section 7.3 Page 391 Question 24

Example: When solving each case, the solutions generated are for the domain $\{x \mid x \in \mathrm{R}\}$. However, since each case is only valid for a specific domain, solutions outside of that domain are extraneous.

Section 7.4 Reciprocal Functions

Section 7.4 Page 403 Question 1

a) The reciprocal function of $y = -x + 2$ is $y = \dfrac{1}{-x+2}$.

b) The reciprocal function of $y = 3x - 5$ is $y = \dfrac{1}{3x-5}$.

c) The reciprocal function of $y = x^2 - 9$ is $y = \dfrac{1}{x^2-9}$.

d) The reciprocal function of $y = x^2 - 7x + 10$ is $y = \dfrac{1}{x^2-7x+10}$.

Section 7.4 Page 403 Question 2

a) i) Find the zero of $f(x) = x + 5$ by setting $f(x) = 0$ and solving.

$x + 5 = 0$

$x = -5$

ii) The reciprocal function of $f(x) = x + 5$ is $y = \dfrac{1}{x+5}$.

iii) Find the non-permissible value of $\frac{1}{x+5}$ by setting the denominator equal to 0 and solving.

$x + 5 = 0$
$x = -5$

iv) The zero of the original function is the non-permissible value of the reciprocal function.

v) Since the reciprocal function is undefined at the non-permissible value, the equation of the vertical asymptote is $x = -5$.

b) i) Find the zero of $g(x) = 2x + 1$ by setting $g(x) = 0$ and solving.

$2x + 1 = 0$
$x = -\frac{1}{2}$

ii) The reciprocal function of $g(x) = 2x + 1$ is $y = \frac{1}{2x+1}$.

iii) Find the non-permissible value of $\frac{1}{2x+1}$ by setting the denominator equal to 0 and solving.

$2x + 1 = 0$
$x = -\frac{1}{2}$

iv) The zero of the original function is the non-permissible value of the reciprocal function.

v) Since the reciprocal function is undefined at the non-permissible value, the equation of the vertical asymptote is $x = -\frac{1}{2}$.

c) i) Find the zero of $h(x) = x^2 - 16$ by setting $g(x) = 0$ and solving.

$x^2 - 16 = 0$
$(x - 4)(x + 4) = 0$
$x - 4 = 0$ or $x + 4 = 0$
$x = 4$ $x = -4$

ii) The reciprocal function of $h(x) = x^2 - 16$ is $y = \frac{1}{x^2 - 16}$.

iii) Find the non-permissible value of $\frac{1}{x^2 - 16}$ by setting the denominator equal to 0 and solving.

$x^2 - 16 = 0$
$(x - 4)(x + 4) = 0$
$x - 4 = 0$ or $x + 4 = 0$
$x = 4$ $x = -4$

iv) The zeros of the original function are the non-permissible values of the reciprocal function.

v) Since the reciprocal function is undefined at the non-permissible values, the equations of the vertical asymptotes are $x = 4$ and $x = –4$.

d) **i)** Find the zero of $t(x) = x^2 + x – 12$ by setting $g(x) = 0$ and solving.

$x^2 + x – 12 = 0$
$(x + 4)(x – 3) = 0$
$x + 4 = 0$ or $x – 3 = 0$
$x = –4$ $\quad$ $x = 3$

ii) The reciprocal function of $t(x) = x^2 + x – 12$ is $y = \frac{1}{x^2 + x - 12}$.

iii) Find the non-permissible value of $\frac{1}{x^2 + x - 12}$ by setting the denominator equal to 0 and solving.

$x^2 + x – 12 = 0$
$(x + 4)(x – 3) = 0$
$x + 4 = 0$ or $x – 3 = 0$
$x = –4$ $\quad$ $x = 3$

iv) The zeros of the original function are the non-permissible values of the reciprocal function.

v) Since the reciprocal function is undefined at the non-permissible values, the equations of the vertical asymptotes are $x = –4$ and $x = 3$.

Section 7.4 Page 404 Question 3

The equations of the vertical asymptotes are determined by the non-permissible values of the rational expression.

a) For $f(x) = \frac{1}{5x - 10}$, solve $5x – 10 = 0$.

$5x – 10 = 0$
$x = 2$

The equation of the vertical asymptote is $x = 2$.

b) For $f(x) = \frac{1}{3x + 7}$, solve $3x + 7 = 0$.

$3x + 7 = 0$
$x = -\frac{7}{3}$

The equation of the vertical asymptote is $x = -\frac{7}{3}$.

c) For $f(x) = \frac{1}{(x-2)(x+4)}$, solve $(x – 2)(x + 4) = 0$.

$(x – 2)(x + 4) = 0$
$x – 2 = 0$ or $x + 4 = 0$
$x = 2$ $x = –4$
The equations of the vertical asymptotes are $x = 2$ and $x = –4$.

d) For $f(x) = \frac{1}{x^2-9x+20}$, solve $x^2 – 9x + 20 = 0$.

$x^2 – 9x + 20 = 0$
$(x – 4)(x – 5) = 0$
$x – 4 = 0$ or $x – 5 = 0$
$x = 4$ $x = 5$
The equations of the vertical asymptotes are $x = 4$ and $x = 5$.

Section 7.4 Page 404 Question 4

The undefined statement occurs for $x = 3$, which is the non-permissible value for the function
$f(x) = \frac{1}{x-3}$.

Section 7.4 Page 404 Question 5

Since the value 0 is not in the range of these reciprocal functions, there are no *x*-intercepts.
To find the *y*-intercept, substitute $x = 0$.

a) For $f(x) = \frac{1}{x+5}$, the *y*-intercept is $\frac{1}{5}$.

b) For $f(x) = \frac{1}{3x-4}$, the *y*-intercept is $-\frac{1}{4}$.

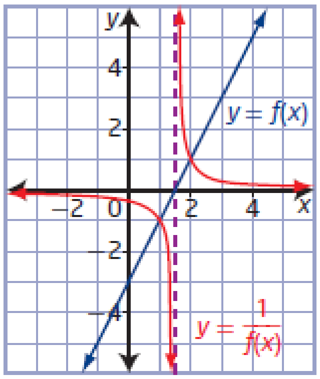
c) For $f(x) = \frac{1}{x^2-9}$, the *y*-intercept is $-\frac{1}{9}$.

d) For $f(x) = \frac{1}{x^2+7x+12}$, the *y*-intercept is $\frac{1}{12}$.

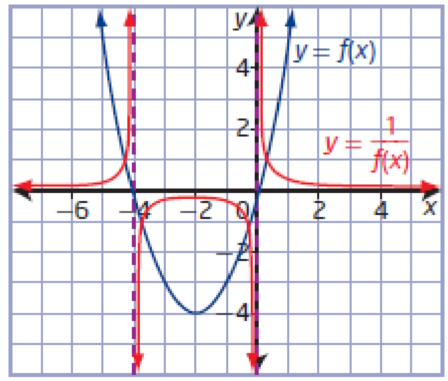
Section 7.4 Page 404 Question 6

Example: Locate zeros of the original function (where $y = 0$) and draw the vertical asymptotes. Locate the invariant points (where $f(x)$ has a value of 1 or –1). Use these points to help sketch the graph of the reciprocal function.

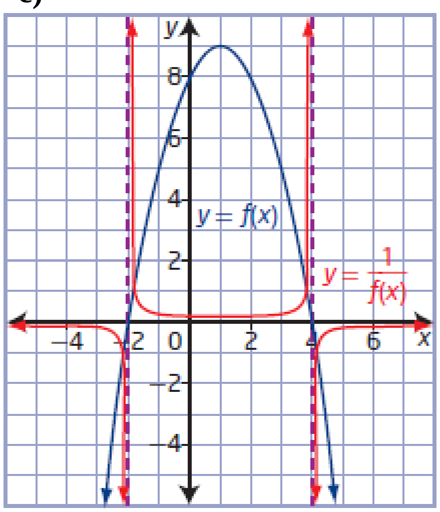
a)



b)



c)



Section 7.4 Page 404 Question 7

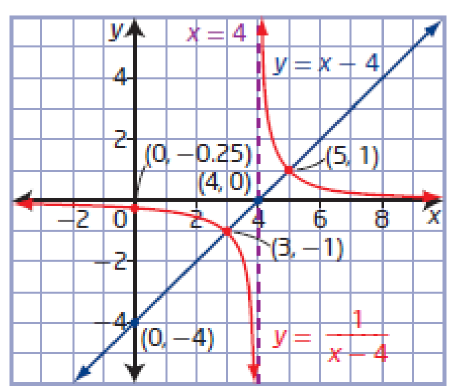
a) To sketch the graph of the function $f(x) = x - 4$ use the y-intercept of –4 and slope of 1.
To sketch the graph of the reciprocal function, consider the following characteristics:

- The zero of $f(x)$ is 4, so the reciprocal function has vertical asymptote $x = 4$.
- Determine the invariant points by solving $f(x) = \pm 1$.

$x - 4 = 1$ $\quad$ $x - 4 = -1$
$x = 5$ $\quad$ $x = 3$

Invariant points are (5, 1) and (3, –1).

- The y-intercept of $y = \dfrac{1}{x-4}$ is $-\dfrac{1}{4}$ or –0.25.

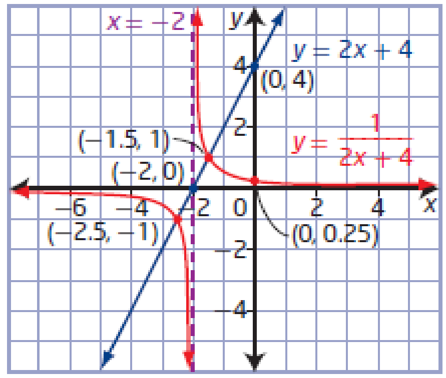


b) To sketch the graph of the function $f(x) = 2x + 4$ use the y-intercept of 4 and slope of 2.
To sketch the graph of the reciprocal function, consider the following characteristics:
• The zero of $f(x)$ is –2, so the reciprocal function has vertical asymptote $x = –2$.
• Determine the invariant points by solving $f(x) = \pm 1$.

$2x + 4 = 1$ $\qquad$ $2x + 4 = –1$

$x = -\frac{3}{2}$ $\qquad$ $x = -\frac{5}{2}$

Invariant points are (–1.5, 1) and (–2.5, –1).
• The y-intercept of $y = \frac{1}{2x+4}$ is $\frac{1}{4}$ or 0.25.

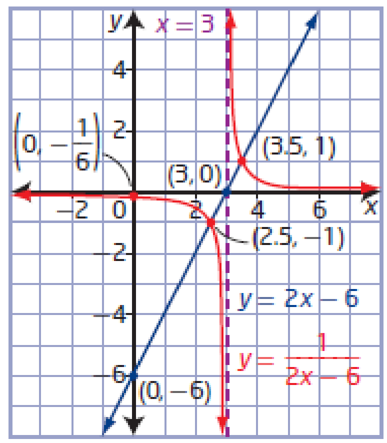


c) To sketch the graph of the function $f(x) = 2x – 6$ use the y-intercept of –6 and slope of 2.
To sketch the graph of the reciprocal function, consider the following characteristics:
• The zero of $f(x)$ is 3, so the reciprocal function has vertical asymptote $x = 3$.
• Determine the invariant points by solving $f(x) = \pm 1$.

$2x – 6 = 1$ $\qquad$ $2x – 6 = –1$

$x = \frac{7}{2}$ $\qquad$ $x = -\frac{5}{2}$

Invariant points are (3.5, 1) and (–2.5, –1).
• The y-intercept of $y = \frac{1}{2x-6}$ is $-\frac{1}{6}$.

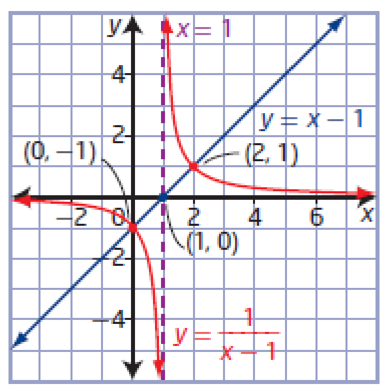


d) To sketch the graph of the function $f(x) = x – 1$ use the y-intercept of –1 and slope of 1.
To sketch the graph of the reciprocal function, consider the following characteristics:
• The zero of $f(x)$ is 1, so the reciprocal function has vertical asymptote $x = 1$.
• Determine the invariant points by solving $f(x) = \pm 1$.

$x – 1 = 1$ $\qquad$ $x – 1 = –1$

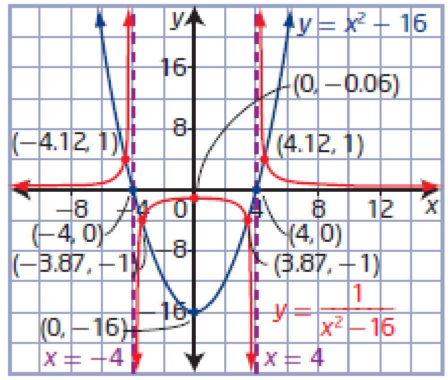
$x = 2$ $\qquad$ $x = 0$

Invariant points are (2, 1) and (0, –1).
• The y-intercept of $y = \frac{1}{x-1}$ is –1.



Section 7.4 Page 404 Question 8

a) Use the location of the vertex and intercepts to plot the graph of $f(x)$.



• For $f(x) = x^2 - 16$, the coordinates of the vertex are (0, –16).

• Determine the zeros of the function, or x-intercepts of the graph, by solving $f(x) = 0$.

$$x^2 - 16 = 0$$
$$(x - 4)(x + 4) = 0$$
$$x - 4 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = 4 \qquad\qquad x = -4$$

• The y-intercept is –16.

To sketch the graph of the reciprocal function, consider the following characteristics:

• The reciprocal function has vertical asymptotes $x = 4$ and $x = -4$.

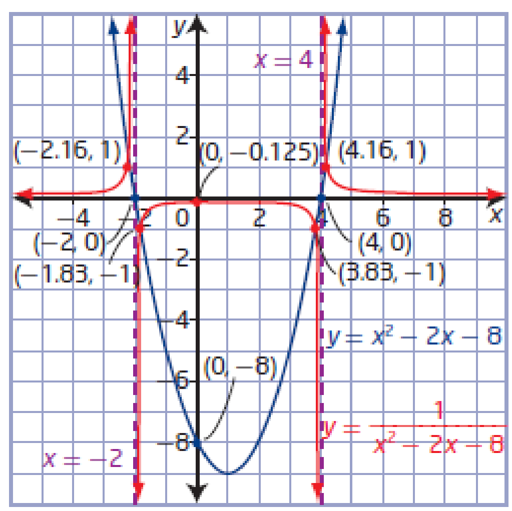
• Determine the invariant points by solving $f(x) = \pm 1$.

$$x^2 - 16 = 1 \qquad x^2 - 16 = -1$$
$$x^2 = 17 \qquad x^2 = 15$$
$$x = \pm\sqrt{17} \qquad x = \pm\sqrt{15}$$
$$x \approx \pm 4.12 \qquad x \approx \pm 3.87$$

Invariant points are (4.12, 1), (–4.12, 1), (3.87, –1), and (–3.87, –1).

The y-intercept of $y = \dfrac{1}{x^2 - 16}$ is $-\dfrac{1}{16}$ or –0.0625.

b) Use the location of the vertex and intercepts to plot the graph of $f(x)$.



• Determine the coordinates of the vertex for $f(x) = x^2 - 2x - 8$ using $x = -\dfrac{b}{2a}$ and its corresponding value of y. The vertex is located at (1, 9).

• Determine the zeros of the function, or x-intercepts of the graph, by solving $f(x) = 0$.

$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$
$$x - 4 = 0 \quad \text{or} \quad x + 2 = 0$$
$$x = 4 \qquad\qquad x = -2$$

• The y-intercept is –8.

To sketch the graph of the reciprocal function, consider the following characteristics:

• The reciprocal function has vertical asymptotes $x = 4$ and $x = -2$.

• Determine the invariant points by solving $f(x) = \pm 1$.

$$x^2 - 2x - 8 = 1 \qquad\qquad x^2 - 2x - 8 = -1$$
$$x^2 - 2x - 9 = 0 \qquad\qquad x^2 - 2x - 7 = 0$$

In each case, use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-9)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{40}}{2}$

$x \approx 4.16$ or $x \approx -2.16$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-7)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{32}}{2}$

$x \approx 3.83$ or $x \approx -1.83$

Invariant points are (4.16, 1), (–2.16, 1), (3.83, –1), and (–1.83, –1).

The y-intercept of $y = \dfrac{1}{x^2 - 2x - 8}$ is $-\dfrac{1}{8}$ or –0.125.

c) Use the location of the vertex and intercepts to plot the graph of $f(x)$.

• Determine the coordinates of the vertex for $f(x) = x^2 - x - 2$ using $x = -\dfrac{b}{2a}$ and its corresponding value of y. The vertex is located at (0.5, –2.25).

• Determine the zeros of the function, or x-intercepts of the graph, by solving $f(x) = 0$.

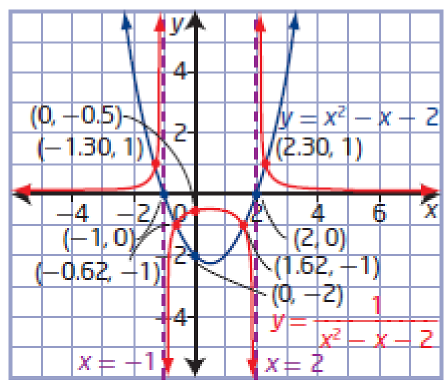
$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x - 2 = 0$ or $x + 1 = 0$

$x = 2$ $\quad$ $x = -1$

• The y-intercept is –2.



To sketch the graph of the reciprocal function, consider the following characteristics:

• The reciprocal function has vertical asymptotes $x = 2$ and $x = -1$.

• Determine the invariant points by solving $f(x) = \pm 1$.

$x^2 - x - 2 = 1$

$x^2 - x - 3 = 0$

$x^2 - x - 2 = -1$

$x^2 - x - 1 = 0$

In each case, use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$

$x = \dfrac{1 \pm \sqrt{13}}{2}$

$x \approx 2.30$ or $x \approx -1.30$

$x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$

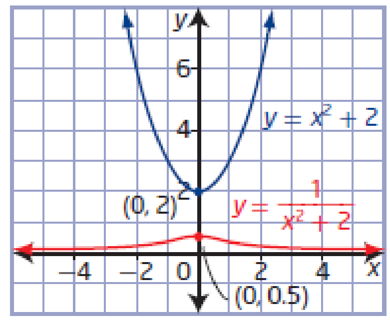
$x = \dfrac{1 \pm \sqrt{5}}{2}$

$x \approx 1.62$ or $x \approx -0.62$

Invariant points are (2.30, 1), (–1.30, 1), (1.62, –1), and (–0.62, –1).

The y-intercept of $y = \dfrac{1}{x^2 - x - 2}$ is $-\dfrac{1}{2}$ or –0.5.

d) Use the location of the vertex and intercepts to plot the graph of $f(x)$.
• For $f(x) = x^2 + 2$, the coordinates of the vertex are (0, 2).
• Since $a > 0$ and the vertex is above the x-axis, there are no x-intercepts.
• The y-intercept is 2.
To sketch the graph of the reciprocal function, consider the following characteristics:
• The reciprocal function has no vertical asymptotes.
• Since the vertex is located at (0, 2) and the parabola opens upward, $f(x) \neq \pm 1$ and there are no invariant points.



The y-intercept of $y = \frac{1}{x^2 + 2}$ is $\frac{1}{2}$ or 0.5.

Section 7.4 Page 405 Question 9

a) This is the graph of a linear function with x-intercept of 2 and y-intercept of –2. The graph of its reciprocal will have a vertical asymptote of $x = 2$ and a y-intercept of –0.5. This description matches graph **D**.

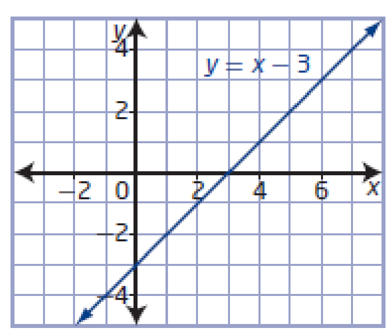
b) This is the graph of a quadratic function with x-intercept of 2 and y-intercept of 4. The graph of its reciprocal will have a vertical asymptote of $x = 2$ and a y-intercept of 0.25. This description matches graph **C**.

c) This is the graph of a quadratic function with x-intercepts of –1 and 2 and y-intercept of –2. The graph of its reciprocal will have vertical asymptotes of $x = –1$ and $x = 2$ and a y-intercept of –0.5. This description matches graph **A**.

d) This is the graph of a linear function with x-intercept of –1 and y-intercept of 1. The graph of its reciprocal will have a vertical asymptote of $x = –1$ and a y-intercept of 1. This description matches graph **B**.

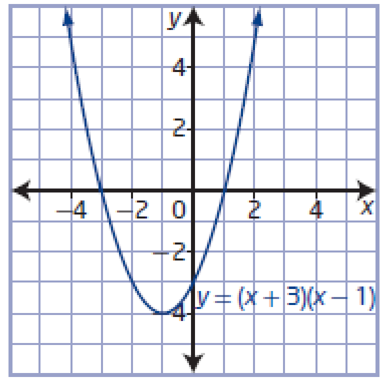
Section 7.4 Page 406 Question 10

a) i) and **ii)** Since the graph of the reciprocal function has a vertical asymptote of $x = 3$, the graph of the original function will have x-intercept at (3, 0). The given point, (4, 1), is an invariant point so it is also a point on the graph of the original function. Plot the two points and draw the line.



iii) Use the coordinates of the two known points to determine that the slope, m, is 1. Substitute the coordinates of one of the points into $y = x + b$ and solve for b: $b = –3$.
The original function is $y = x – 3$.

b) i) and **ii)** Since the graph of the reciprocal function has vertical asymptotes of $x = -3$ and $x = 1$, the graph of the original function will have x-intercepts at (–3, 0) and (1, 0). The given point, (–1, –0.25), corresponds to the point (–1, –4) on the graph of the original function, which is the vertex. Plot the three points and draw the parabola.
iii) Use the coordinates of the vertex to write the function in the form $y = a(x + 1)^2 - 4$. Substitute the coordinates of one of the points and solve for a: $a = 1$.
The original function is $y = (x + 1)^2 - 4$, or $y = x^2 + 2x - 3$.



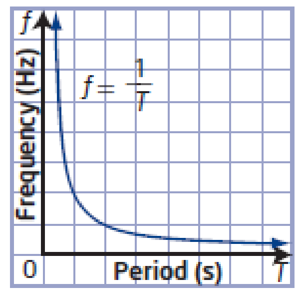
Section 7.4 Page 406 Question 11

a) Use a table of values to help sketch the graph.

b) The reciprocal function is $y = T$.

c) Substitute $T = 2.5$ into $f = \frac{1}{T}$.

$$f = \frac{1}{2.5}$$
$$f = 0.4$$
The frequency of a pendulum with a period of 2.5 s is 0.4 Hz.

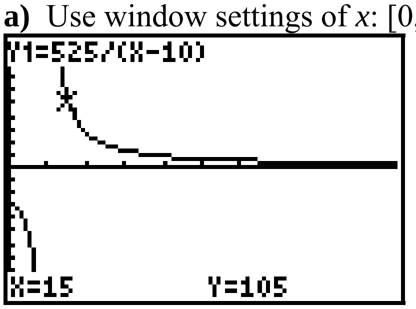


d) Substitute $f = 1.6$ into $f = \frac{1}{T}$.

$$1.6 = \frac{1}{T}$$
$$T = \frac{1}{1.6}$$
$$T = 0.625$$
The period of a pendulum with a frequency of 1.6 Hz is 0.625 s.

Section 7.4 Page 406 Question 12

a) Use window settings of x: [0, 100, 10] and y: [–200, 200, 20].



b) Since the asymptote is $d = 10$ and d must be positive, choose domain $\{d \mid d > 10, d \in \mathrm{R}\}$.

c) Substitute $d = 40$ into $t = \frac{525}{d-10}$.

$t = \frac{525}{40-10}$

$t = 17.5$

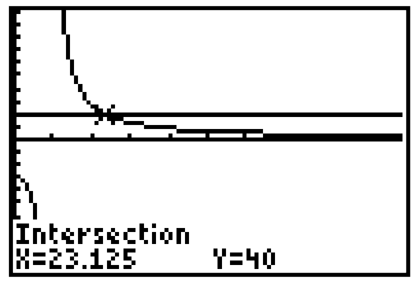
The maximum time without stopping for a scuba diver who is 40 m deep is 17.5 min.

d) The point of intersection is (23.125, 40). This means that the diver has a maximum of 40 min at a depth of 23.125 m.

Check.

Left Side	Right Side
t	$\frac{525}{d-10}$
$= 40$	$= \frac{525}{23.125-10}$
	$= 40$

Left Side = Right Side

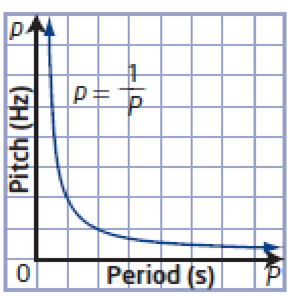


e) Yes. The horizontal asymptote is $t = 0$. At great depths it is almost impossible to not stop for decompression.

Section 7.4 Page 407 Question 13

a) A function for pitch, p, in terms of period, P is $p = \frac{1}{P}$.

b)



c) Substitute $P = 0.048$ into $p = \frac{1}{P}$.

$p = \frac{1}{0.048}$

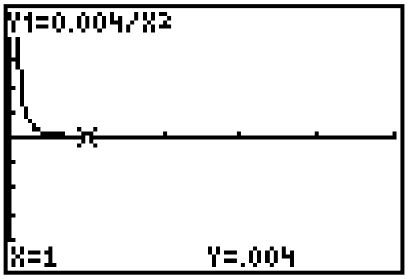
$p \approx 20.8$

The pitch, to the nearest 0.1 Hz, for a musical note with period 0.048 s is 20.8 Hz.

Section 7.4 Page 407 Question 14

a) A function for I in terms of d to represent this relationship is $I = \frac{0.004}{d^2}$.

b)



c) Substitute $d = 5$ into $I = \frac{0.004}{d^2}$.

$$I = \frac{0.004}{5^2}$$
$$I = 0.000\ 16$$

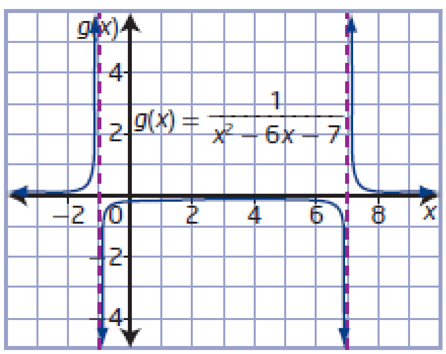
The intensity of a car horn for a person standing 5 m from the car is 0.000 16 W/m^2.

Section 7.4 Page 407 Question 15

a) Example: Determine the coordinates of the vertex for $f(x) = x^2 - 6x - 7$ using $x = -\frac{b}{2a}$ and its corresponding value of y or use completing the square to change the function to vertex form.

b) Example: Knowing the location of the vertex helps with the location of the maximum for the U-shaped section of the graph of $g(x) = \frac{1}{x^2 - 6x - 7}$.

c)

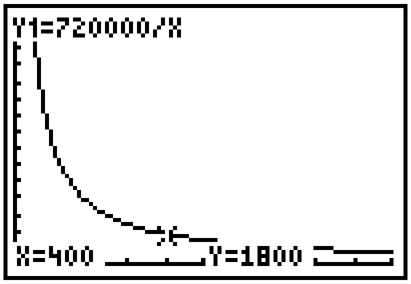


Section 7.4 Page 407 Question 16

a) Substitute $t = 720$ and $n = 1000$ into $t = \frac{k}{n}$.

$$t = \frac{k}{n}$$
$$720 = \frac{k}{1000}$$
$$k = 720\ 000$$

b) Graph $t = \frac{720\ 000}{n}$ using technology.



c) Using the graph, if only 400 workers were on the job, it would have taken 1800 days to complete.

d) Substitute $t = 500$ into $t = \dfrac{720\,000}{n}$ and solve for n.

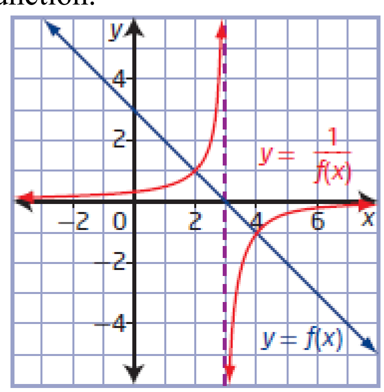
$$t = \frac{720\,000}{n}$$
$$500 = \frac{720\,000}{n}$$
$$n = 1440$$

You would need 1440 workers to complete the job in 500 days.

Section 7.4 Page 408 Question 17

Use the table provided to graph both $y = f(x)$ and $y = \dfrac{1}{f(x)}$. From the intervals, the x-intercept of the graph of $y = f(x)$ is 3 and the vertical asymptote is located at $x = 3$. Use the sign and direction information for $f(x)$ to deduce that it is a line with a negative slope. Similarly, complete the branches of the reciprocal function.

Interval of x	$x < 3$	$x > 3$
Sign of $f(x)$	+	−
Direction of $f(x)$	decreasing	decreasing
Sign of $\dfrac{1}{f(x)}$	+	−
Direction of $\dfrac{1}{f(x)}$	increasing	increasing



Section 7.4 Page 408 Question 18

a) The graph of $y = \dfrac{1}{f(x)}$ always has a vertical asymptote.

This statement is false. The graph of $y = \frac{1}{f(x)}$ will have a vertical asymptote only if the graph of $f(x)$ has an x-intercept. For example, the graph of $f(x) = x^2 + 2$ has no x-intercepts so the graph of $y = \frac{1}{f(x)}$ has no vertical asymptote.

b) A function of the form of $y = \frac{1}{f(x)}$ always has at least one value for which it is not defined.

This statement is false. The function of the form $y = \frac{1}{f(x)}$ will be undefined only if $f(x)$ has a zero. For example, the function $f(x) = x^2 + 2$ has no zeros so the function $y = \frac{1}{f(x)}$ is defined for all values in its domain.

c) The domain of $y = \frac{1}{f(x)}$ is always the same as the domain of $y = f(x)$.

This statement is false. The domain of $y = \frac{1}{f(x)}$ will differ from the domain of $y = f(x)$ when $f(x)$ has a zero because the zero is the non-permissible value for $\frac{1}{f(x)}$.

Section 7.4 Page 408 Question 19

a) Both students have made correct assumptions. The non-permissible values are the roots of the corresponding equation.

b) The assumptions made hold true for both a linear and a quadratic function.

Section 7.4 Page 408 Question 20

a) Substitute $u = 300$ and $f = 50$ into $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$.

$$\begin{aligned}\frac{1}{u} + \frac{1}{v} &= \frac{1}{f} \\ \frac{1}{300} + \frac{1}{v} &= \frac{1}{50} \\ \frac{1}{v} &= \frac{5}{300} \\ v &= 60\end{aligned}$$

The distance between the lens and the image is 60 mm.

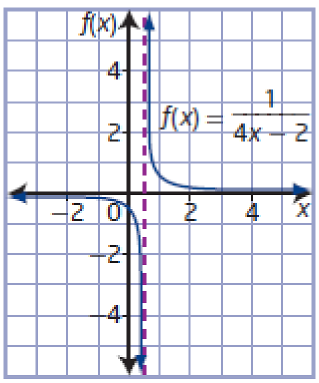
b) Substitute $u = 10\ 000$ and $v = 210$ into $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$.

$$\begin{aligned}\frac{1}{u}+\frac{1}{v}&=\frac{1}{f}\\ \frac{1}{10\,000}+\frac{1}{210}&=\frac{1}{f}\\ \frac{10\,210}{2\,100\,000}&=\frac{1}{f}\\ f&=205.680...\end{aligned}$$

The focal length of the lens is approximately 205.68 mm.

Section 7.4 Page 408 Question 21

Step 1



Step 2a)

x	$f(x)$
0	−0.5
0.4	−2.5
0.45	−5
0.47	−8.33
0.49	−25
0.495	−50
0.499	−250

x	$f(x)$
0	0.5
0.6	2.5
0.55	5
0.53	8.33
0.51	25
0.505	50
0.501	250

b) As the value of x approaches the asymptote from either the right or the left, the function approaches negative infinity or infinity, respectively. The function will always approach infinity or negative infinity at an asymptote.

Step 3a)

x	$f(x)$
−10	$-\frac{1}{42}$
−100	$-\frac{1}{402}$
−1000	$-\frac{1}{4002}$
−10 000	$-\frac{1}{40\,002}$
−100 000	$-\frac{1}{400\,002}$

x	$f(x)$
10	$\frac{1}{38}$
100	$\frac{1}{398}$
1000	$\frac{1}{3998}$
10 000	$\frac{1}{39\,998}$
100 000	$\frac{1}{399\,998}$

b) As the values of x approach $\pm\infty$, the graph of the reciprocal function approaches the line $y = 0$.

Section 7.4 Page 409 Question 22

$y = f(x)$	$y = \frac{1}{f(x)}$
The absolute value of the function gets very large.	The absolute value of the function gets very small.
Function values are positive.	Reciprocal values are positive.
Function values are negative.	Reciprocal values are negative.
The zeros of the function are the x-intercepts of the graph.	The zeros of the function are the vertical asymptotes of the graph.
The value of the function is 1.	The value of the reciprocal function is 1.
The absolute value of the function approaches zero.	The absolute value of the reciprocal approaches infinity or negative infinity.
The value of the function is −1.	The value of the reciprocal function is −1.

Chapter 7 Review

Chapter 7 Review Page 410 Question 1

a) $|-5| = 5$

b) $\left|2\frac{3}{4}\right| = 2\frac{3}{4}$

c) $|-6.7| = 6.7$

Chapter 7 Review Page 410 Question 2

First, evaluate each number and express it in decimal form.

-4 $\sqrt{9} = 3$ $|-3.5| = 3.5$ -2.7 $\left|-\frac{9}{2}\right| = 4.5$ $|-1.6| = 1.6$ $\left|1\frac{1}{2}\right| = 1.5$

The numbers from least to greatest are -4, -2.7, $\left|1\frac{1}{2}\right|$, $|-1.6|$, $\sqrt{9}$, $|-3.5|$, and $\left|-\frac{9}{2}\right|$.

Chapter 7 Review Page 410 Question 3

a) $|-7 - 2| = |-9|$
$= 9$

b) $|-3 + 11 - 6| = |2|$
$= 2$

c) $5|-3.75| = 5(3.75)$
$= 18.75$

d) $|5^2 - 7| + |-10 + 2^3| = |25 - 7| + |-10 + 8|$
$= |18| + |-2|$
$= 18 + 2$
$= 20$

Chapter 7 Review Page 410 Question 4

Let $D_1 = 0.0$, $D_2 = 4.2$, $D_3 = 10.5$, $D_4 = 19.6$, $D_5 = 21.9$, and $D_6 = 15.0$.
$|D_2 - D_1| + |D_3 - D_2| + |D_4 - D_3| + |D_5 - D_4| + |D_6 - D_5| + |D_1 - D_5|$
$= |4.2 - 0.0| + |10.5 - 4.2| + |19.6 - 10.5| + |21.9 - 19.6| + |15.0 - 21.9| + |0.0 - 15.0|$
$= |4.2| + |6.3| + |9.1| + |2.3| + |-6.9| + |-15.0|$
$= 4.2 + 6.3 + 9.1 + 2.3 + 6.9 + 15$
$= 43.8$
The total distance hiked was 43.8 km.

Chapter 7 Review Page 410 Question 5

a) The net change in the closing value of this stock is the change from the start of the week to the end of the week: \$6.40 – \$4.28 = \$2.12.

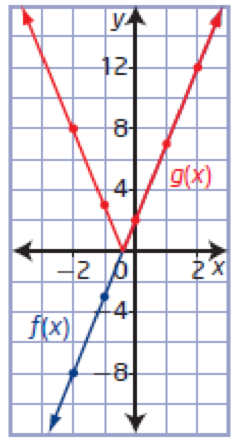
b) Let $V_1 = 4.28$, $V_2 = 5.17$, $V_3 = 4.79$, $V_4 = 7.15$, and $V_5 = 6.40$.
$|V_2 - V_1| + |V_3 - V_2| + |V_4 - V_3| + |V_5 - V_4|$
$= |5.17 - 4.28| + |4.79 - 5.17| + |7.15 - 4.79| + |6.40 - 7.15|$
$= |0.89| + |-0.38| + |2.36| + |-0.75|$
$= 0.89 + 0.38 + 2.36 + 0.75$
$= 4.38$
The total change in the closing value of this stock is \$4.38.

Chapter 7 Review Page 410 Question 6

a)

x	$f(x)$	$g(x)$
−2	−8	8
−1	−3	3
0	2	2
1	7	7
2	12	12

b)



c) For $f(x)$: The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $g(x)$: The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

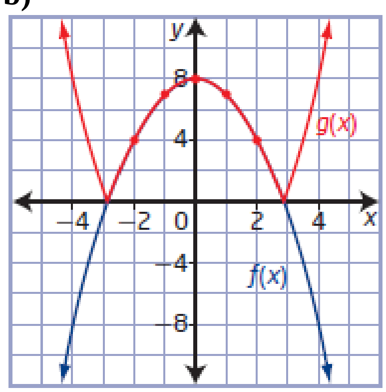
d) Example: They are the same graph for values of x where $f(x) \geq 0$. Otherwise, the graph of $g(x)$ is a reflection of the graph of $f(x)$ in the x-axis.

Chapter 7 Review Page 410 Question 7

a)

x	$f(x)$	$g(x)$
−2	4	4
−1	7	7
0	8	8
1	7	7
2	4	4

b)



c) For $f(x)$: The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \leq 8, y \in \mathrm{R}\}$.
For $g(x)$: The domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

d) Example: They are the same graph for values of x where $f(x) \geq 0$. Otherwise, the graph of $g(x)$ is a reflection of the graph of $f(x)$ in the x-axis.

Chapter 7 Review Page 411 Question 8

a) The x-intercept is $x = 2$. When $x \geq 2$, the graph of $y = |2x - 4|$ is the graph of $y = 2x - 4$. When $x < 2$, the graph of $y = |2x - 4|$ is the graph of $y = -(2x - 4)$ or $y = -2x + 4$. The absolute value function $y = |2x - 4|$ expressed as a piecewise function is

$$y = \begin{cases} 2x - 4, \text{ if } x \geq 2 \\ -2x + 4, \text{ if } x < 2 \end{cases}.$$

b) The x-intercepts are $x = -1$ and $x = 1$. When $x \leq -1$ or $x \geq 1$, the graph of $y = |x^2 - 1|$ is the graph of $y = x^2 - 1$. When $-1 < x < 1$, the graph of $y = |x^2 - 1|$ is the graph of $y = -(x^2 - 1)$ or $y = -x^2 + 1$. The absolute value function $y = |x^2 - 1|$ expressed as a piecewise function is $y = \begin{cases} x^2 - 1, \text{ if } x \leq -1 \text{ or } x \geq 1 \\ -x^2 + 1, \text{ if } -1 < x < 1 \end{cases}$.

Chapter 7 Review Page 411 Question 9

a) The functions $f(x) = 3x^2 + 7x + 2$ and $g(x) = |3x^2 + 7x + 2|$ have different graphs because the initial graph goes below the x-axis. The absolute value brackets reflect anything below the x-axis above the x-axis.

b) The functions $f(x) = 3x^2 + 4x + 2$ and $g(x) = |3x^2 + 4x + 2|$ have the same graphs because the initial function is always positive (above the x-axis).

Chapter 7 Review Page 411 Question 10

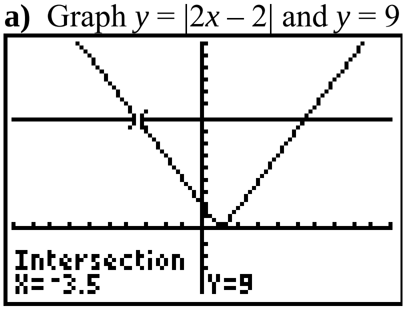
Determine the equation of the line in the form $f(x) = ax + b$ using the two given points: $\left(-\frac{2}{3}, 0\right)$ and (0, 10).

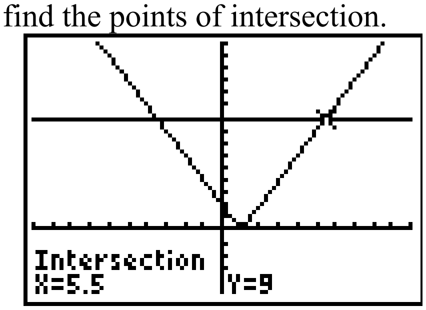
$f(x) = 15x + 10$

Therefore, $a = 15$ and $b = 10$.

Chapter 7 Review Page 411 Question 11

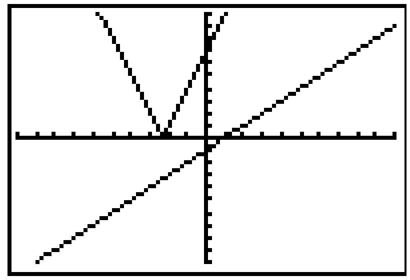
a) Graph $y = |2x - 2|$ and $y = 9$ and find the points of intersection.



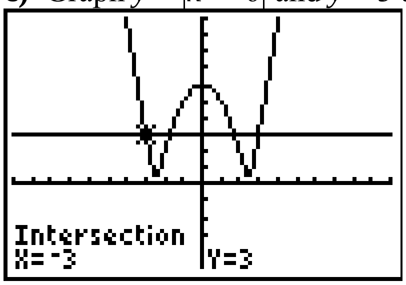


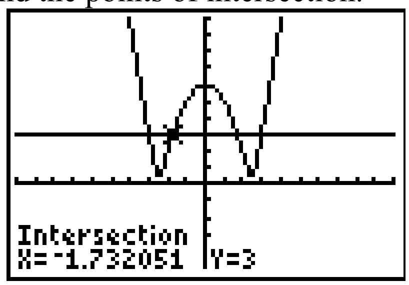
The solutions are $x = -3.5$ and $x = 5.5$.

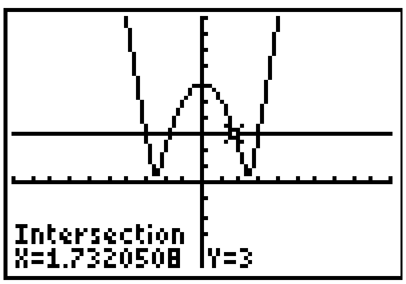
b) Graph $y = |7 + 3x|$ and $y = x - 1$ and find the points of intersection.
Since the graphs do not intersect, there is no solution.

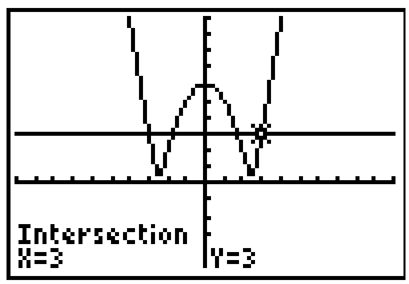


c) Graph $y = |x^2 - 6|$ and $y = 3$ and find the points of intersection.



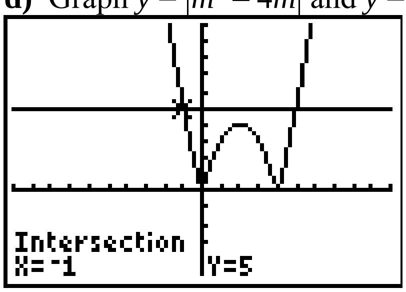


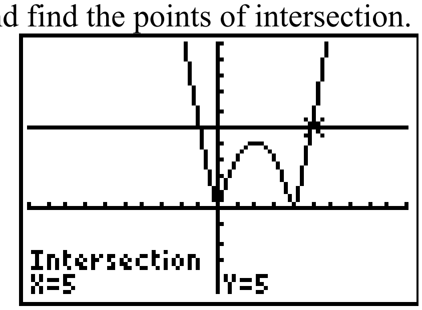




The solutions are $x = -3$, $x \approx -1.7$, $x \approx 1.7$, and $x = 3$.

d) Graph $y = |m^2 - 4m|$ and $y = 5$ and find the points of intersection.





The solutions are $m = -1$ and $m = 5$.

Chapter 7 Review Page 411 Question 12

a) Examine the two cases.
Case 1
The expression $|q + 9|$ equals $q + 9$ when $q \geq -9$.
$q + 9 = 2$
$q = -7$
The value -7 satisfies the condition $q \geq -9$.

Case 2
The expression $|q + 9|$ equals $-(q + 9)$ when $q < -9$.
$-(q + 9) = 2$
$q + 9 = -2$
$q = -11$
The value -11 satisfies the condition $q < -9$.
The solution is $q = -7$ or $q = -11$.

b) Examine the two cases.
Case 1
The expression $|7x - 3|$ equals $7x - 3$ when $x \geq \frac{3}{7}$.
$7x - 3 = x + 1$
$6x = 4$
$x = \frac{2}{3}$
The value $\frac{2}{3}$ satisfies the condition $x \geq \frac{3}{7}$.
Case 2
The expression $|7x - 3|$ equals $-(7x - 3)$ when $x < \frac{3}{7}$.
$-(7x - 3) = x + 1$
$-7x + 3 = x + 1$
$-8x = -2$
$x = \frac{1}{4}$
The value $\frac{1}{4}$ satisfies the condition $x < \frac{3}{7}$.
The solution is $x = \frac{2}{3}$ or $x = \frac{1}{4}$.

c) Examine the two cases.
Case 1
The expression $|x^2 - 6x|$ equals $x^2 - 6x$ when $x \leq 0$ or $x \geq 6$.
$x^2 - 6x = x$
$x^2 - 7x = 0$
$x(x - 7) = 0$
$x = 0$ or $x - 7 = 0$
$x = 7$
Both values 0 and 7 satisfy the conditions.
Case 2
The expression $|x^2 - 6x|$ equals $-(x^2 - 6x)$ when $0 < x < 6$.
$-(x^2 - 6x) = x$
$0 = x^2 - 5x$
$0 = x(x - 5)$

$x = 0$ or $x - 5 = 0$

$x = 5$

Only the value 5 satisfies the conditions.
The solution is $x = 0$, $x = 5$, or $x = 7$.

d) Examine the two cases.

Case 1

The expression $|4x^2 - x - 4|$ equals $4x^2 - x - 4$ when $x \le \frac{1-\sqrt{65}}{8}$ or $x \ge \frac{1+\sqrt{65}}{8}$ or approximately $x \le -0.88$ or $x \ge 1.13$.

$$3x - 1 = 4x^2 - x - 4$$
$$0 = 4x^2 - 4x - 3$$
$$0 = (2x + 1)(2x - 3)$$

$2x + 1 = 0$ or $2x - 3 = 0$

$x = -\frac{1}{2}$ $\quad x = \frac{3}{2}$

Only the value $\frac{3}{2}$ satisfies the conditions.

Case 2

The expression $|4x^2 - x - 4|$ equals $-(4x^2 - x - 4)$ when $\frac{1-\sqrt{65}}{8} < x < \frac{1+\sqrt{65}}{8}$ or approximately $-0.88 < x < 1.13$.

$$3x - 1 = -(4x^2 - x - 4)$$
$$3x - 1 = -4x^2 + x + 4$$
$$4x^2 + 2x - 5 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(4)(-5)}}{2(4)}$$

$$x = \frac{-2 \pm \sqrt{84}}{8}$$

$$x = \frac{-1 \pm \sqrt{21}}{4}$$

$x = \frac{-1+\sqrt{21}}{4}$ or $x = \frac{-1-\sqrt{21}}{4}$

$x = 0.8956\ldots$ $\quad x = -1.3956\ldots$

Only the value of $\frac{-1+\sqrt{21}}{4}$ satisfies the conditions.

The solution is $x = \frac{3}{2}$ or $x = \frac{-1+\sqrt{21}}{4}$.

Chapter 7 Review Page 411 Question 13

a) Examine the two cases.

Case 1

The expression $|d - 4.075|$ equals $d - 4.075$ when $d \geq 4.075$.

$d - 4.075 = 1.665$

$d = 5.74$

The value 5.74 satisfies the condition.

Case 2

The expression $|d - 4.075|$ equals $-(d - 4.075)$ when $d < 4.075$.

$-(d - 4.075) = 1.665$

$d - 4.075 = -1.665$

$d = 2.41$

The value 2.41 satisfies the condition.

The solution is $d = 5.74$ or $d = 2.41$.

The depth of the water at high tide is 5.74 m, and the depth of the water at low tide is 2.41 m.

b) Let $D_1 = 2.94$, $D_2 = 5.71$, $D_3 = 2.28$, and $D_4 = 4.58$.

$|D_2 - D_1| + |D_3 - D_2| + |D_4 - D_3|$

$= |5.71 - 2.94| + |2.28 - 5.71| + |4.58 - 2.28|$

$= |2.77| + |-3.43| + |2.3|$

$= 2.77 + 3.43 + 2.3$

$= 8.5$

The total change in water depth that day was 8.5 m.

Chapter 7 Review Page 412 Question 14

Examine the two cases.

Case 1

The expression $|m - 35.932|$ equals $m - 35.932$ when $m \geq 35.932$.

$m - 35.932 = 11.152$

$m = 47.084$

The value 47.084 satisfies the condition.

Case 2

The expression $|m - 35.932|$ equals $-(m - 35.932)$ when $m < 35.932$.

$-(m - 35.932) = 11.152$

$m - 35.932 = -11.152$

$m = 24.78$

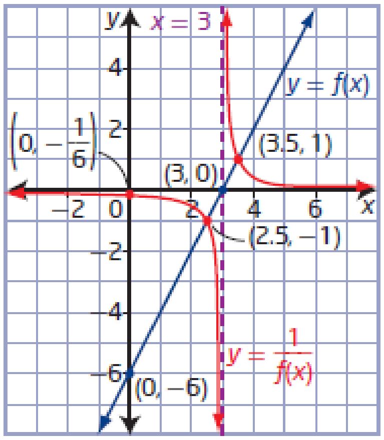
The value 24.78 satisfies the condition.

The solution is $m = 47.084$ or $m = 24.78$.

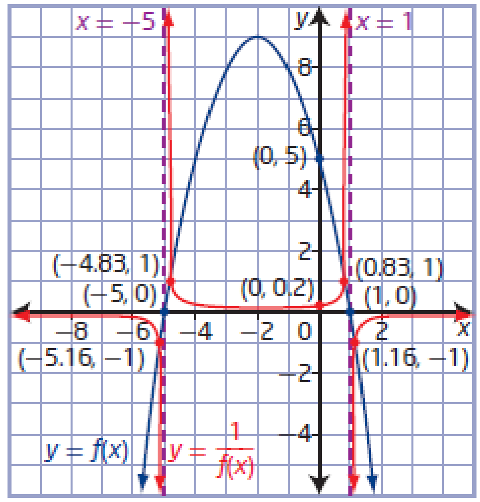
The two masses are 24.78 kg and 47.084 kg.

Chapter 7 Review Page 412 Question 15

a)

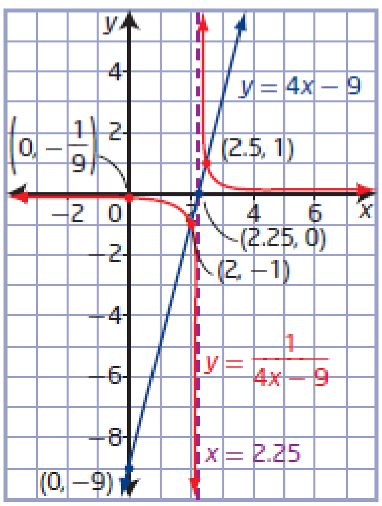


b)

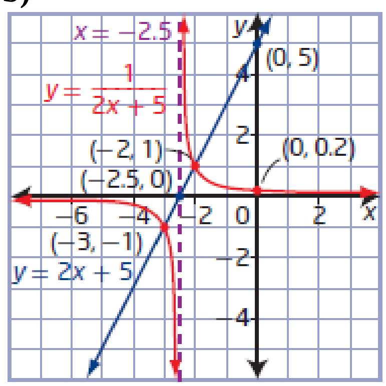


Chapter 7 Review Page 412 Question 16

a)



b)



Chapter 7 Review Page 412 Question 17

a) i) The reciprocal function for $f(x) = x^2 - 25$ is $y = \dfrac{1}{x^2 - 25}$.

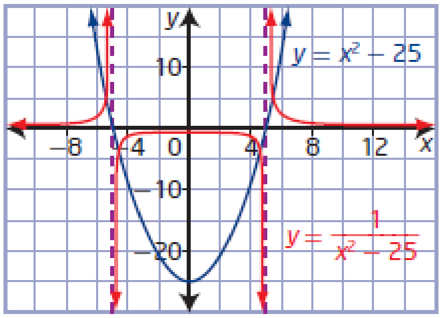
ii) Find the non-permissible values of $\dfrac{1}{x^2 - 25}$ by setting the denominator equal to 0 and solving.

$$x^2 - 25 = 0$$
$$(x - 5)(x + 5) = 0$$
$$x - 5 = 0 \quad \text{or} \quad x + 5 = 0$$
$$x = 5 \qquad\qquad x = -5$$

The non-permissible values and the equations of the asymptotes are $x = 5$ and $x = -5$.

iii) The reciprocal function has no x-intercepts. The y-intercept is $-\frac{1}{25}$.

iv)



b) i) The reciprocal function for $f(x) = x^2 - 6x + 5$ is $y = \frac{1}{x^2 - 6x + 5}$.

ii) Find the non-permissible values of $\frac{1}{x^2 - 6x + 5}$ by setting the denominator equal to 0 and solving.

$x^2 - 6x + 5 = 0$

$(x - 5)(x - 1) = 0$

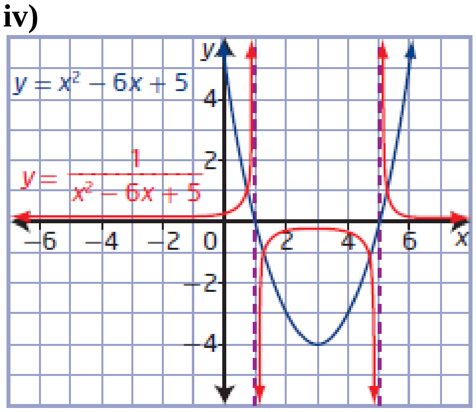
$x - 5 = 0$ or $x - 1 = 0$

$x = 5$ $\quad$ $x = 1$

The non-permissible values and the equations of the asymptotes are $x = 5$ and $x = 1$.

iii) The reciprocal function has no x-intercepts. The y-intercept is $\frac{1}{5}$.

iv)



Chapter 7 Review Page 412 Question 18

a) Substitute $d = 2.5$ into $F = \frac{600}{d}$.

$$F = \frac{600}{d}$$
$$F = \frac{600}{2.5}$$
$$F = 240$$

The force required is 240 N.

b) Substitute $F = 450$ into $F = \frac{600}{d}$.

$$F = \frac{600}{d}$$
$$450 = \frac{600}{d}$$
$$d = 1.333...$$

The distance from the fulcrum is approximately 1.33 m.

c) If the distance is doubled:

$$F_d = \frac{600}{2d}$$
$$F_d = \frac{1}{2}\left(\frac{600}{d}\right)$$
$$F_d = \frac{1}{2}F$$

If the distance is tripled:

$$F_t = \frac{600}{3d}$$
$$F_t = \frac{1}{3}\left(\frac{600}{d}\right)$$
$$F_t = \frac{1}{3}F$$

If the distance is doubled the force is halved. If the distance is tripled only a third of the force is needed.

Chapter 7 Practice Test

Chapter 7 Practice Test Page 413 Question 1

$|-9 - 3| - |5 - 2^3| + |-7 + 1 - 4|$
$= |-12| - |-3| + |-10|$
$= 12 - 3 + 10$
$= 19$

The answer is **B**.

Chapter 7 Practice Test Page 413 Question 2

The range of the absolute value of a linear function is $\{y \mid y \geq 0, y \in \mathrm{R}\}$. The range of $f(x) = |x - 3|$ is $\{y \mid y \geq 0, y \in \mathrm{R}\}$. The answer is **C**.

Chapter 7 Practice Test Page 413 Question 3

Examine the two cases.
Case 1
The expression $|1 - 2x|$ equals $1 - 2x$ when $x \leq \frac{1}{2}$.

$1 - 2x = 9$
$x = -4$

The value –4 satisfies the condition.
Case 2
The expression $|1 - 2x|$ equals $-(1 - 2x)$ when $x > \frac{1}{2}$.

$-(1 - 2x) = 9$
$1 - 2x = -9$
$x = 5$

The value 5 satisfies the condition.
The solution is $x = -4$ or $x = 5$.
The answer is **D**.

Chapter 7 Practice Test Page 413 Question 4

Since the asymptotes occur at $x = -2$ and $x = 1$, the original quadratic function has zeros –2 and 1.
$f(x) = (x + 2)(x - 1)$
$f(x) = x^2 + x - 2$
The answer is **A**.

Chapter 7 Practice Test Page 413 Question 5

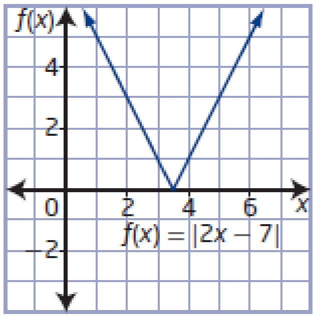
Vertical asymptotes occur at non-permissible values. Set the denominator equal to 0 and solve for x.

$x^2 - 16 = 0$
$(x - 4)(x + 4) = 0$
$x - 4 = 0$ or $x + 4 = 0$
$x = 4$ $x = -4$

The equations of the vertical asymptotes are $x = 4$ and $x = -4$.
The answer is **B**.

Chapter 7 Practice Test Page 413 Question 6

a) Use a table of values to sketch the graph.



b) The x-intercept is 3.5 and the y-intercept is 7.

c) The domain is $\{x \mid x \in R\}$ and the range is $\{y \mid y \geq 0, y \in R\}$.

d) The x-intercept is $x = 3.5$. When $x \geq 3.5$, the graph of $y = |2x - 7|$ is the graph of $y = 2x - 7$. When $x < 3.5$, the graph of $y = |2x - 7|$ is the graph of $y = -(2x - 7)$ or $y = -2x + 7$. The absolute value function $y = |2x - 7|$ expressed as a piecewise function is

$$y = \begin{cases} 2x - 7, \text{ if } x \geq 3.5 \\ -2x + 7, \text{ if } x < 3.5 \end{cases}.$$

Chapter 7 Practice Test Page 413 Question 7

Examine the two cases.

Case 1

The expression $|3x^2 - x|$ equals $3x^2 - x$ when $x \leq 0$ or $x \geq \frac{1}{3}$.

$$3x^2 - x = 4x - 2$$
$$3x^2 - 5x + 2 = 0$$
$$(3x - 2)(x - 1) = 0$$
$$3x - 2 = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = \frac{2}{3} \qquad\qquad x = 1$$

Both values $\frac{2}{3}$ and 1 satisfy the conditions.

Case 2

The expression $|3x^2 - x|$ equals $-(3x^2 - x)$ when $0 < x < \frac{1}{3}$.

$$-(3x^2 - x) = 4x - 2$$
$$0 = 3x^2 + 3x - 2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{-3 \pm \sqrt{33}}{6}$$

$$x = \frac{-3 + \sqrt{33}}{6} \quad \text{or} \quad x = \frac{-3 - \sqrt{33}}{6}$$

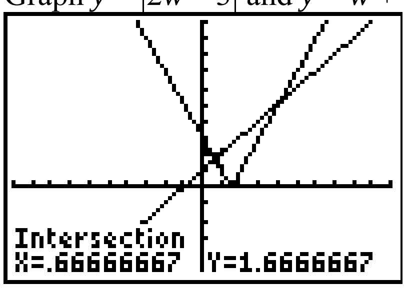
$x = 0.4574\ldots$ $\qquad$ $x = -1.4574\ldots$

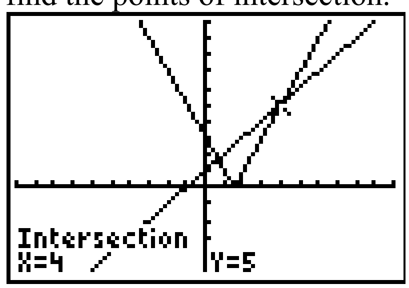
Neither value satisfies the conditions.

The solution is $x = \frac{2}{3}$ or $x = 1$.

Chapter 7 Practice Test Page 413 Question 8

Graph $y = |2w - 3|$ and $y = w + 1$ and find the points of intersection.





The solutions are $x = \frac{2}{3}$ and $x = 4$.

Chapter 7 Practice Test Page 413 Question 9

The first error is in line 1 of Case 1: $x + 4$ should be $x - 4$. Then, the corrected solution for Case 1 when the expression $|x - 4|$ equals $x - 4$ when $x \geq 4$:

Case 1

$x - 4 = x^2 + 4x$

$0 = x^2 + 3x + 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{-7}}{2}$$

No solutions.

The next error is in line 1 of Case 2: $-x - 4$ should be $-x + 4$. Then, the corrected solution for Case 2 when the expression $|x - 4|$ equals $-(x - 4)$ when $x < 4$:

Case 2

$$-x + 4 = x^2 + 4x$$
$$0 = x^2 + 5x - 4$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(-4)}}{2(1)}$$
$$x = \frac{-5 \pm \sqrt{41}}{2}$$
$$x = \frac{-5 + \sqrt{41}}{2} \quad \text{or} \quad x = \frac{-5 - \sqrt{41}}{2}$$
$$x = 0.7015\ldots \qquad x = -5.7015\ldots$$

Both values satisfy the condition.

The solutions are $x = \frac{-5 + \sqrt{41}}{2}$ and $x = \frac{-5 - \sqrt{41}}{2}$ or approximately $x = 0.7$ and $x = -5.7$.

Chapter 7 Practice Test Page 414 Question 10

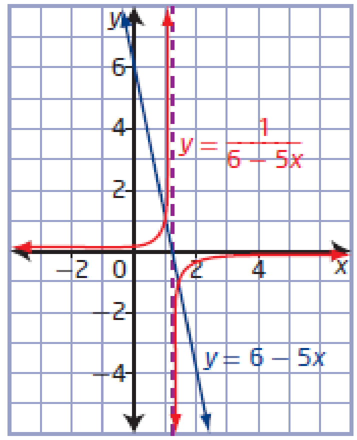
a) The reciprocal function for $f(x) = 6 - 5x$ is $y = \frac{1}{6 - 5x}$.

b) Find the non-permissible values of $\frac{1}{6 - 5x}$ by setting the denominator equal to 0 and solving.

$$6 - 5x = 0$$
$$x = \frac{6}{5}$$

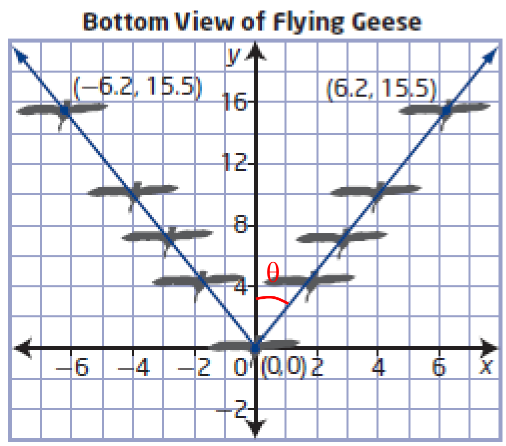
The non-permissible value and the equation of the vertical asymptote is $x = \frac{6}{5}$.

c) Example: Use the asymptote and the invariant points to sketch the graph of the reciprocal function.



Chapter 7 Practice Test Page 414 Question 11

a) Using the given points, (0, 0) and (6.2, 15.5), the equation of the absolute value function is $y = |2.5x|$.



b) First determine the angle between one half of the vee formation and the *y*-axis using a trigonometric ratio.

$$\tan \theta = \frac{6.2}{15.5}$$
$$\theta = \tan^{-1} 0.4$$
$$\theta = 21.801\ldots$$

Then, the angle between the legs of the vee formation is 2(21.80) or 43.6°.

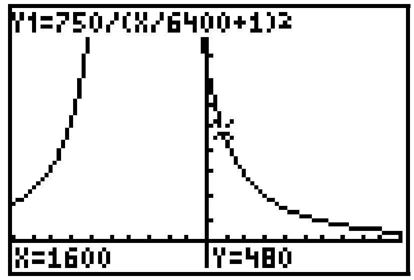
c) For $y = |2.8x|$, the angle between one half of the vee formation and the *y*-axis is

$$\tan \theta = \frac{1}{2.8}$$
$$\theta = \tan^{-1}\left(\frac{1}{2.8}\right)$$
$$\theta = 19.653\ldots$$

Then, the angle between the legs of the vee formation is 2(19.65) or 39.3°.

Chapter 7 Practice Test Page 414 Question 12

a) Substitute $W_e = 750$ into $W_h = \dfrac{W_e}{\left(\dfrac{h}{6400}+1\right)^2}$.



Graph $W_h = \dfrac{750}{\left(\dfrac{h}{6400}+1\right)^2}$.

b) i) Substitute $h = 8$ into $W_h = \dfrac{750}{\left(\dfrac{h}{6400}+1\right)^2}$.

$$W_h = \frac{750}{\left(\frac{h}{6400}+1\right)^2}$$

$$W_h = \frac{750}{\left(\frac{8}{6400}+1\right)^2}$$

$$W_h = \frac{750(6400)^2}{6408^2}$$

$$W_h = 748.128...$$

The astronaut's weight at a height of 8 km is approximately 748.13 N.

ii) Substitute $h = 2000$ into $W_h = \dfrac{750}{\left(\dfrac{h}{6400}+1\right)^2}$.

$$W_h = \frac{750}{\left(\frac{h}{6400}+1\right)^2}$$

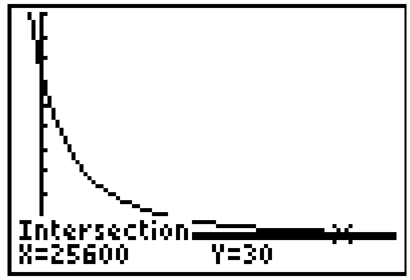
$$W_h = \frac{750}{\left(\frac{2000}{6400}+1\right)^2}$$

$$W_h = \frac{750(6400)^2}{8400^2}$$

$$W_h = 435.374...$$

The astronaut's weight at a height of 2000 km is approximately 435.37 N.

c) Graph $W_h = \dfrac{750}{\left(\dfrac{h}{6400}+1\right)^2}$ and $W_h = 30$ and find the point of intersection.



So, $h > 25\ 600$ or a height of more than 25 600 km will result in the astronaut having a weight of less than 30 N.

Cumulative Review, Chapters 5–7

Cumulative Review Page 416 Question 1

$$\begin{aligned}3xy^3\sqrt{2x} &= \sqrt{(3xy^3)^2(2x)}\\ &= \sqrt{3^2x^2y^6(2x)}\\ &= \sqrt{18x^3y^6}\end{aligned}$$

Cumulative Review Page 416 Question 2

$$\begin{aligned}\sqrt{48a^3b^2c^5} &= \sqrt{16(3)(a^2)(a)(b^2)(c^4)(c)}\\ &= 4abc^2\sqrt{3ac}\end{aligned}$$

Cumulative Review Page 416 Question 3

$3\sqrt{6} = \sqrt{9(6)} = \sqrt{54}$ $\sqrt{36}$ $2\sqrt{3} = \sqrt{4(3)} = \sqrt{12}$ $\sqrt{18}$ $2\sqrt{9} = \sqrt{4(9)} = \sqrt{36}$ $\sqrt[3]{8} = 2 = \sqrt{4}$

The numbers from least to greatest are $\sqrt[3]{8}, 2\sqrt{3}, \sqrt{18}, \sqrt{36}, 2\sqrt{9},$ and $3\sqrt{6}$ or $\sqrt[3]{8}, 2\sqrt{3}, \sqrt{18}, 2\sqrt{9}, \sqrt{36},$ and $3\sqrt{6}$.

Cumulative Review Page 416 Question 4

a) $4\sqrt{2a} + 5\sqrt{2a} = 9\sqrt{2a},\ a \ge 0$

b) $$\begin{aligned}10\sqrt{20x^2} - 3x\sqrt{45} &= 20x\sqrt{5} - 9x\sqrt{5}\\ &= 11x\sqrt{5}\end{aligned}$$

Cumulative Review Page 416 Question 5

a) $2\sqrt[3]{4}\left(-4\sqrt[3]{6}\right) = -8\sqrt[3]{24}$

$= -16\sqrt[3]{3}$

b) $\sqrt{6}\left(\sqrt{12} - \sqrt{3}\right) = \sqrt{72} - \sqrt{18}$

$= 6\sqrt{2} - 3\sqrt{2}$

$= 3\sqrt{2}$

c) $\left(6\sqrt{a} + \sqrt{3}\right)\left(2\sqrt{a} - \sqrt{4}\right) = 12a - 6\sqrt{4a} + 2\sqrt{3a} - \sqrt{12}$

$= 12a - 12\sqrt{a} + 2\sqrt{3a} - 2\sqrt{3}, a \geq 0$

Cumulative Review Page 416 Question 6

a) $\frac{\sqrt{12}}{\sqrt{4}} = \frac{\sqrt{12}}{\sqrt{4}}\left(\frac{\sqrt{4}}{\sqrt{4}}\right)$

$= \frac{\sqrt{48}}{4}$

$= \frac{4\sqrt{3}}{4}$

$= \sqrt{3}$

b) $\frac{2}{2+\sqrt{3}} = \frac{2}{2+\sqrt{3}}\left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right)$

$= \frac{4-2\sqrt{3}}{4-3}$

$= 4 - 2\sqrt{3}$

c)

$\frac{\sqrt{7}+\sqrt{28}}{\sqrt{7}-\sqrt{14}} = \frac{\sqrt{7}+\sqrt{28}}{\sqrt{7}-\sqrt{14}}\left(\frac{\sqrt{7}+\sqrt{14}}{\sqrt{7}+\sqrt{14}}\right)$

$= \frac{7+\sqrt{7(14)}+\sqrt{7(28)}+\sqrt{28(14)}}{7-14}$

$= \frac{7+7\sqrt{2}+14+14\sqrt{2}}{-7}$

$= \frac{21+21\sqrt{2}}{-7}$

$= -3 - 3\sqrt{2}$

Cumulative Review Page 416 Question 7

$\sqrt{x+6} = x,\ x \geq -6$

$\left(\sqrt{x+6}\right)^2 = x^2$

$x + 6 = x^2$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x - 3 = 0$ or $x + 2 = 0$

$x = 3$ $\quad$ $x = -2$

Check for $x = 3$.

Left Side	Right Side
$\sqrt{x+6}$	x
$= \sqrt{3+6}$	$= 3$
$= 3$	

Left Side = Right Side

The solution is $x = 3$.

Check for $x = -2$.

Left Side	Right Side
$\sqrt{x+6}$	x
$= \sqrt{-2+6}$	$= -2$
$= 2$	

Left Side $\neq$ Right Side

Cumulative Review Page 416 Question 8

a) Substitute $v = 20$ and $r = 15$ into $v = \sqrt{h-2r}$ and solve for h.

$v = \sqrt{h-2r}$

$20 = \sqrt{h-2(15)}$

$20^2 = \left(\sqrt{h-30}\right)^2$

$400 = h - 30$

$h = 430$

The height of the fill is 430 ft.

b) Example: The velocity would decrease with an increasing radius because the expression $\sqrt{h-2r}$ would decrease.

Cumulative Review Page 416 Question 9

a) $\dfrac{12a^3b}{48a^2b^4} = \dfrac{a}{4b^3},\quad a \neq 0, b \neq 0$

b) $\dfrac{4-x}{x^2-8x+16} = \dfrac{-(x-4)}{(x-4)(x-4)}$

$= -\dfrac{1}{x-4},\quad x \neq 4$

c) $\dfrac{(x-3)(x+5)}{x^2-1} \div \dfrac{x+2}{x-3} = \dfrac{(x-3)(x+5)}{(x-1)(x+1)}\left(\dfrac{x-3}{x+2}\right)$

$$= \frac{(x-3)^2(x+5)}{(x-1)(x+1)(x+2)}, \quad x \neq -2, -1, 1, 3$$

d) $\dfrac{5x-10}{6x} \times \dfrac{3x}{15x-30} = \dfrac{\cancel{5}\,\cancel{(x-2)}}{\underset{2}{\cancel{6x}}}\left(\dfrac{\cancel{3x}}{\underset{3}{\cancel{15}}\,\cancel{(x-2)}}\right)$

$$= \frac{1}{6}, \quad x \neq 0, 2$$

e) $\left(\dfrac{x+2}{x-3}\right)\left(\dfrac{x^2-9}{x^2-4}\right) \div \left(\dfrac{x+3}{x-2}\right) = \left(\dfrac{\cancel{x+2}}{\cancel{x-3}}\right)\left(\dfrac{\cancel{(x-3)}\,\cancel{(x+3)}}{\cancel{(x-2)}\,\cancel{(x+2)}}\right)\left(\dfrac{\cancel{x-2}}{\cancel{x+3}}\right)$

$$= 1, \quad x \neq -3, -2, 2, 3$$

Cumulative Review Page 416 Question 10

a) $\dfrac{10}{a+2} + \dfrac{a-1}{a-7} = \dfrac{10(a-7)}{(a+2)(a-7)} + \dfrac{(a-1)(a+2)}{(a-7)(a+2)}$

$$= \frac{10a - 70 + a^2 + a - 2}{(a+2)(a-7)}$$

$$= \frac{a^2 + 11a - 72}{(a+2)(a-7)}$$

$$= \frac{a^2 + 11a - 72}{(a+2)(a-7)}, \quad a \neq -2, 7$$

b) $\dfrac{3x+2}{x+4} - \dfrac{x-5}{x^2-4} = \dfrac{(3x+2)(x-2)(x+2)}{(x+4)(x-2)(x+2)} - \dfrac{(x-5)(x+4)}{(x-2)(x+2)(x+4)}$

$$= \frac{3x^3 - 12x + 2x^2 - 8 - x^2 + x + 20}{(x+4)(x-2)(x+2)}$$

$$= \frac{3x^3 + x^2 - 11x + 12}{(x+4)(x-2)(x+2)}, \quad x \neq -4, -2, 2$$

c) $\frac{2x}{x^2-25}-\frac{3}{x^2-4x-5}=\frac{2x(x+1)}{(x+5)(x-5)(x+1)}-\frac{3(x+5)}{(x-5)(x+1)(x+5)}$

$$=\frac{2x^2+2x-3x-15}{(x+5)(x-5)(x+1)}$$

$$=\frac{2x^2-x-15}{(x+5)(x-5)(x+1)},\quad x\neq -5,-1,5 \quad \text{or}$$

$$=\frac{(2x+5)(x-3)}{(x+5)(x-5)(x+1)},\quad x\neq -5,-1,5$$

Cumulative Review Page 416 Question 11

Example: I disagree with Sandra. The expression $\frac{(x+2)(x+5)}{x+5}$ is not equivalent to $x + 2$. She forgot to state the restriction on the variable. The expression $\frac{(x+2)(x+5)}{x+5}$ is equivalent to $x + 2$, $x \neq -5$.

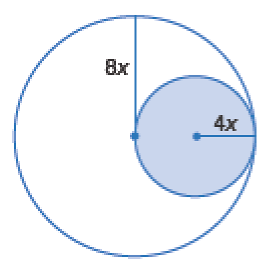
Cumulative Review Page 416 Question 12

$$\frac{x+4}{4}=\frac{x}{3}$$
$$12\left(\frac{x+4}{4}\right)=\left(\frac{x}{3}\right)12$$
$$3(x+4)=4x$$
$$12=x$$

The value of x is 12.

Cumulative Review Page 417 Question 13

$$P=\frac{\text{area of shaded region}}{\text{area of entire figure}}$$
$$P=\frac{\pi(4x)^2}{\pi(8x)^2}$$
$$P=\frac{16\pi x^2}{64\pi x^2}, x\neq 0$$
$$P=\frac{1}{4}$$



The probability that the point is in the shaded region is $\frac{1}{4}$.

Cumulative Review Page 417 Question 14

First, evaluate each number and express it in decimal form.

$|-5| = 5$

$|4 - 6| = |-2| = 2$

$|2(-4) - 5| = |-8 - 5| = |-13| = 13$

$|8.4| = 8.4$

The numbers from least to greatest are $|4 - 6|$, $|-5|$, $|8.4|$, and $|2(-4) - 5|$.

Cumulative Review Page 417 Question 15

a) The x-intercept is $x = 2$. When $x \geq 2$, the graph of $y = |3x - 6|$ is the graph of $y = 3x - 6$. When $x < 2$, the graph of $y = |3x - 6|$ is the graph of $y = -(3x - 6)$ or $y = -3x + 6$. The absolute value function $y = |3x - 6|$ expressed as a piecewise function is

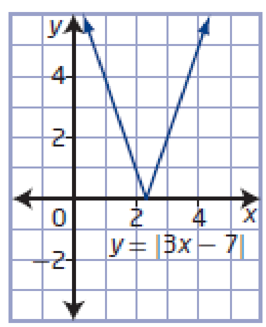
$$y = \begin{cases} 3x - 6, \text{ if } x \geq 2 \\ -3x + 6, \text{ if } x < 2 \end{cases}.$$

b) The x-intercepts are $x = -1$ and $x = 5$. When $x \leq -1$ or $x \geq 5$, the graph of $y = \left|\frac{1}{3}(x-2)^2 - 3\right|$ is the graph of $y = \frac{1}{3}(x-2)^2 - 3$. When $-1 < x < 5$, the graph of $y = \left|\frac{1}{3}(x-2)^2 - 3\right|$ is the graph of $y = -\left(\frac{1}{3}(x-2)^2 - 3\right)$ or $y = -\frac{1}{3}(x-2)^2 + 3$. The absolute value function $y = \left|\frac{1}{3}(x-2)^2 - 3\right|$ expressed as a piecewise function is

$$y = \begin{cases} \frac{1}{3}(x-2)^2 - 3, \text{ if } x \leq -1 \text{ or } x \geq 5 \\ -\frac{1}{3}(x-2)^2 + 3, \text{ if } -1 < x < 5 \end{cases}.$$

Cumulative Review Page 417 Question 16

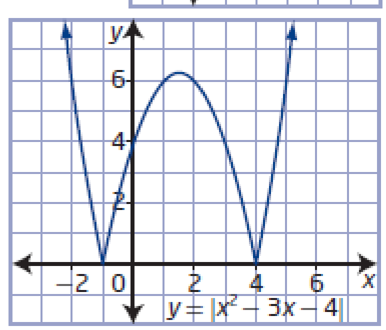
a) i) Use a table of values to sketch the graph.



ii) x-intercept: $\frac{7}{3}$, y-intercept: 7

iii) domain: $\{x \mid x \in \mathbb{R}\}$, range: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

b) i) Use a table of values to sketch the graph.



ii) x-intercepts: –1 and 4, y-intercept: 4

iii) domain: $\{x \mid x \in \mathbb{R}\}$, range: $\{y \mid y \geq 0, y \in \mathbb{R}\}$

Cumulative Review Page 417 Question 17

a) Examine the two cases.
Case 1

The expression $|2x - 1|$ equals $2x - 1$ when $x \geq \frac{1}{2}$.

$2x - 1 = 9$
$2x = 10$
$x = 5$

The value 5 satisfies the condition $x \geq \frac{1}{2}$.

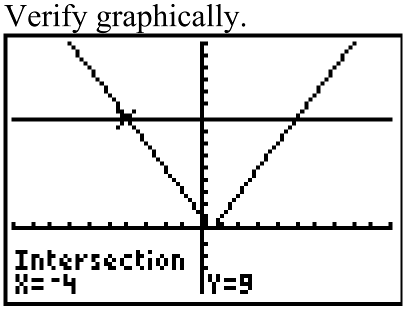
Case 2

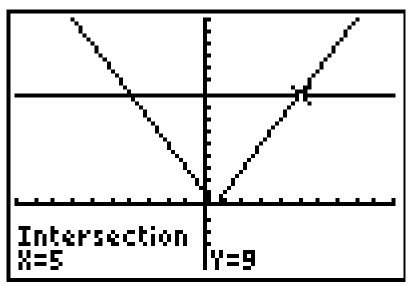
The expression $|2x - 1|$ equals $-(2x - 1)$ when $x < \frac{1}{2}$.

$-(2x - 1) = 9$
$2x - 1 = -9$
$2x = -8$
$x = -4$

The value –4 satisfies the condition $x < \frac{1}{2}$.

The solution is $x = -4$ or $x = 5$.
Verify graphically.





b) Examine the two cases.
Case 1

The expression $|2x^2 - 5|$ equals $2x^2 - 5$ when $x \leq -\sqrt{\frac{5}{2}}$ or $x \geq \sqrt{\frac{5}{2}}$ or approximately $x \leq -1.58$ or $x \geq 1.58$.

$2x^2 - 5 = 13$
$2x^2 = 18$
$x^2 = 9$
$x = \pm 3$

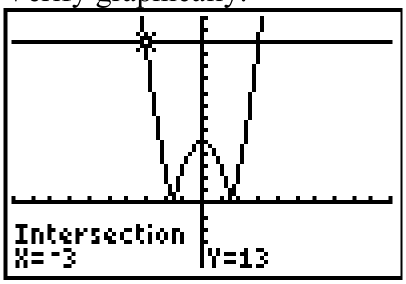
Both values satisfy the conditions.
Case 2

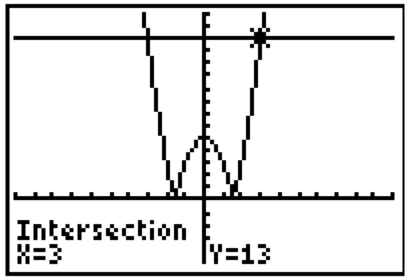
The expression $|2x^2 - 5|$ equals $-(2x^2 - 5)$ when $-\sqrt{\frac{5}{2}} < x < \sqrt{\frac{5}{2}}$.

$-(2x^2 - 5) = 13$
$2x^2 - 5 = -13$
$2x^2 = -8$
$x^2 = -4$

There are no solutions.

The solution is $x = -3$ or $x = 3$.
Verify graphically.





Cumulative Review Page 417 Question 18

a) Example: Absolute value must be used in the formula for area because area cannot be negative.

b) Substitute $a = -5$, $b = 2$, $c = -3$, and $d = 4$ into $A = \frac{1}{2}|ad - bc|$.

$A = \frac{1}{2}|ad - bc|$

$A = \frac{1}{2}|-5(4) - 2(-3)|$

$A = \frac{1}{2}|-20 + 6|$

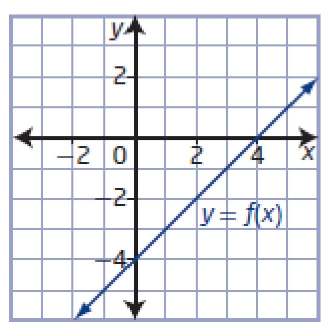
$A = \frac{1}{2}|-14|$

$A = \frac{1}{2}(14)$

$A = 7$

The area of the triangle is 7 square units.

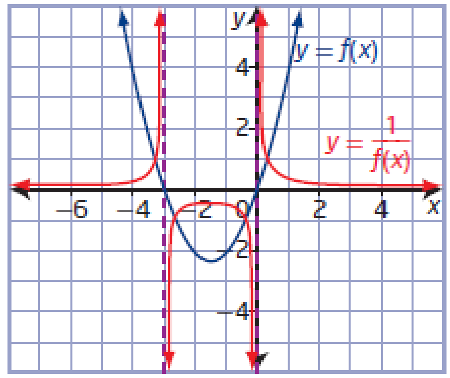
Cumulative Review Page 417 Question 19



Since the graph of the reciprocal function has a vertical asymptote of $x = 4$, the graph of the original function will have x-intercept at (4, 0). The given point, (5, 1), is an invariant point so it is also a point on the graph of the original function. Plot the two points and draw the line.
$f(x) = x - 4$

Cumulative Review Page 417 Question 20

Example: Locate zeros of the original function (where $y = 0$) and draw the vertical asymptotes. Locate the invariant points (where $f(x)$ has a value of 1 or –1). Use these points to help sketch the graph of the reciprocal function.



Cumulative Review Page 417 Question 21

Determine characteristics of the graph of $f(x)$.

- For $f(x) = (x + 2)^2$, the coordinates of the vertex are (–2, 0).
- Determine the zeros of the function, or x-intercepts of the graph, by solving $f(x) = 0$.

$(x + 2)^2 = 0$
$x + 2 = 0$
$x = –2$

- The y-intercept is 0.

To sketch the graph of the reciprocal function, consider the following characteristics:

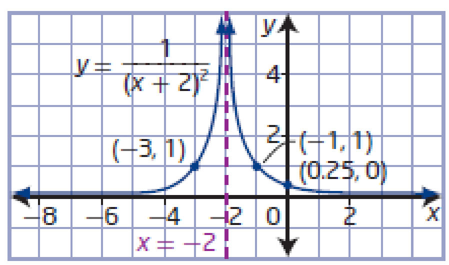
- The reciprocal function has vertical asymptote $x = –2$.
- Determine the invariant points by solving $f(x) = \pm 1$.

$(x + 2)^2 = 1$
$x + 2 = \pm 1$
$x + 2 = –1$ or $x + 2 = 1$
$x = –3$ $x = –1$

$(x + 2)^2 = –1$
No solutions.

Invariant points are (–3, 1) and (–1, 1).

The y-intercept of $y = \dfrac{1}{(x+2)^2}$ is $\dfrac{1}{4}$ or 0.25.



Cumulative Review Page 417 Question 22

a) Example: The shape, range, and y-intercept will be different for $y = |f(x)|$ from $f(x) = 3x – 1$.

For $f(x) = 3x – 1$, the graph is a line, the range is $\{y \mid y \in R\}$, and the y-intercept is –1.

For $y = |3x – 1|$, the graph is vee-shaped, the range is $\{y \mid y \geq 0, y \in R\}$, and the y-intercept is 1.

b) Example: The graph of the reciprocal function has a horizontal asymptote, a vertical asymptote, and no x-intercept.

For $f(x) = 3x - 1$, the graph is a line, the x-intercept is $\frac{1}{3}$, and the y-intercept is –1.

For $y = \frac{1}{3x-1}$, the graph has two branches, a horizontal asymptote at $y = 0$, and a vertical asymptote at $x = \frac{1}{3}$, and the y-intercept is –1.

Unit 3 Test

Unit 3 Test Page 418 Question 1

$$\begin{aligned} 2\sqrt[3]{-27} &= \sqrt[3]{2^3(-27)} \\ &= \sqrt[3]{8(-27)} \\ &= \sqrt[3]{-216} \end{aligned}$$

The best answer is **C**.

Unit 3 Test Page 418 Question 2

$$\begin{aligned} \frac{4\sqrt{72x^5}}{x\sqrt{8}} &= \frac{4}{x}\left(\sqrt{\frac{72x^4(x)}{8}}\right) \\ &= \frac{4x^2}{x}\left(\sqrt{9x}\right) \\ &= 12x\left(\sqrt{x}\right) \end{aligned}$$

The best answer is **D**.

Unit 3 Test Page 418 Question 3

$$\begin{aligned} x+2 &= \sqrt{x^2+3} \\ (x+2)^2 &= x^2+3 \\ x^2+4x+4 &= x^2+3 \\ 4x &= -1 \\ x &= -\frac{1}{4} \end{aligned}$$

The best answer is **B**.

Unit 3 Test **Page 418** **Question 4**

$$\begin{aligned}\frac{9x^4 - 27x^6}{3x^3} &= \frac{9x^4(1-3x^2)}{3x^3}\\ &= 3x(1-3x^2)\\ &= 3x - 9x^3\end{aligned}$$

The best answer is **C**.

Unit 3 Test **Page 418** **Question 5**

An expression for the length of the line segment between (4, –3) and (–6, –3) is |–6 – 4|.
The best answer is **D**.

Unit 3 Test **Page 418** **Question 6**

First, evaluate each number and express it in decimal form.

$$\begin{aligned}|4-11| &= |-7|\\ &= 7\end{aligned}\qquad \begin{aligned}\frac{1}{5}|-5| &= \frac{1}{5}(5)\\ &= 1\end{aligned}\qquad \begin{aligned}\left|1-\frac{1}{4}\right| &= \left|\frac{3}{4}\right|\\ &= 0.75\end{aligned}\qquad \begin{aligned}|2|-|4| &= 2-4\\ &= -2\end{aligned}$$

The numbers from least to greatest are $|2| - |4|$, $\left|1-\frac{1}{4}\right|$, $\frac{1}{5}|-5|$, and $|4 - 11|$.

The best answer is **B**.

Unit 3 Test **Page 418** **Question 7**

$$\sqrt{n}\sqrt{m} = \sqrt{nm}\qquad \begin{aligned}\frac{\sqrt{18}}{\sqrt{36}} &= \sqrt{\frac{18}{36}}\\ &= \sqrt{\frac{1}{2}}\end{aligned}\qquad \begin{aligned}\frac{\sqrt{7}}{\sqrt{8n}} &= \frac{\sqrt{7}}{\sqrt{8n}}\left(\frac{\sqrt{8n}}{\sqrt{8n}}\right)\\ &= \frac{\sqrt{56n}}{8n}\\ &= \frac{2\sqrt{14n}}{8n}\\ &= \frac{\sqrt{14n}}{4n}\end{aligned}\qquad \sqrt{m^2+n^2} \neq m+n$$

The best answer is **D**.

Unit 3 Test **Page 418** **Question 8**

If the graph of $y = \frac{1}{f(x)}$ has vertical asymptotes at $x = -2$ and $x = 5$ and a horizontal asymptote at $y = 0$, then the graph of $y = f(x)$ has x-intercepts of –2 and 5. The equation of $f(x)$ is $f(x) = (x + 2)(x - 5)$ or $f(x) = x^2 - 3x - 10$.
The best answer is **B**.

Unit 3 Test　　Page 418　　Question 9

The restriction on the variable for $\sqrt{3x-9}$ is $3x - 9 \geq 0$ or $x \geq 3$.
The radical $\sqrt{3x-9}$ results in real numbers when $x \geq \mathbf{3}$.

Unit 3 Test　　Page 418　　Question 10

$$\begin{aligned}\frac{\sqrt{5}}{3\sqrt{2}} &= \frac{\sqrt{5}}{3\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)\\ &= \frac{\sqrt{10}}{6}\end{aligned}$$

When the denominator of the expression $\frac{\sqrt{5}}{3\sqrt{2}}$ is rationalized, the expression becomes $\mathbf{\frac{\sqrt{10}}{6}}$.

Unit 3 Test　　Page 419　　Question 11

$$\begin{aligned}\frac{3x-7}{x+11} - \frac{x-k}{x+11} &= \frac{2x+21}{x+11}\\ \frac{2x-7+k}{x+11} &= \frac{2x+21}{x+11}\\ 2x-7+k &= 2x+21\\ k &= 28\end{aligned}$$

The expression $\frac{3x-7}{x+11} - \frac{x-k}{x+11}$, $x \neq -11$, simplifies to $\frac{2x+21}{x+11}$ when the value of k is **28**.

Unit 3 Test　　Page 419　　Question 12

Examine the two cases.
Case 1
The expression $|1 - 4x|$ equals $1 - 4x$ when $x \leq \frac{1}{4}$.

$$\begin{aligned}1 - 4x &= 9\\ -4x &= 8\\ x &= -2\end{aligned}$$

The value -2 satisfies the condition $x \leq \frac{1}{4}$.

Case 2

The expression |1 – 4*x*| equals –(1 – 4*x*) when $x > \frac{1}{4}$.

$$
\begin{aligned}
-(1 - 4x) &= 9 \\
1 - 4x &= -9 \\
-4x &= -10 \\
x &= \frac{10}{4} \\
x &= \frac{5}{2}
\end{aligned}
$$

The value $\frac{5}{2}$ satisfies the condition $x > \frac{1}{4}$.

The solution is $x = -2$ or $x = \frac{5}{2}$.

The lesser solution to the absolute value equation |1 – 4*x*| = 9 is *x* = **–2**.

Unit 3 Test Page 419 Question 13

Find the non-permissible values of the expression $\frac{1}{x^2 - 4}$.

$$
\begin{aligned}
x^2 - 4 &= 0 \\
(x - 2)(x + 2) &= 0 \\
x - 2 = 0 \quad &\text{or } x + 2 = 0 \\
x = 2 \quad &\quad\ \ x = -2
\end{aligned}
$$

The graph of the reciprocal function $f(x) = \frac{1}{x^2 - 4}$ has vertical asymptotes with equations *x* = **–2** and *x* = **2**.

Unit 3 Test Page 419 Question 14

$3\sqrt{7} = \sqrt{9(7)} = \sqrt{63}$ $\quad 4\sqrt{5} = \sqrt{16(5)} = \sqrt{80}$ $\quad \sqrt{18}$ $\quad 6\sqrt{2} = \sqrt{36(2)} = \sqrt{72}$ $\quad 5 = \sqrt{25}$

The numbers from least to greatest are 5, $3\sqrt{7}, 6\sqrt{2}$ and $4\sqrt{5}$.

Unit 3 Test Page 419 Question 15

a) The first step to solve the radical equation $\sqrt{3x+4} = \sqrt{2x-5}$ is to square both sides.

b) For $\sqrt{3x+4}$, $x \geq -\frac{4}{3}$. For $\sqrt{2x-5}$, $x \geq 2.5$. So, the restriction on the values for the variable *x* in the equation $\sqrt{3x+4} = \sqrt{2x-5}$ is $x \geq 2.5$.

c) $\sqrt{3x+4} = \sqrt{2x-5}$

$$\left(\sqrt{3x+4}\right)^2 = \left(\sqrt{2x-5}\right)^2$$
$$3x+4 = 2x-5$$
$$x = -9$$

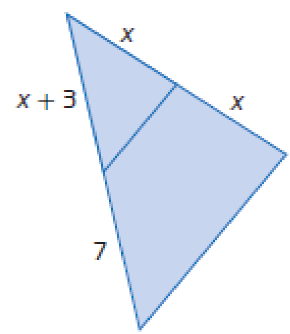
There is no solution.

d) There are no solutions to verify.

Unit 3 Test Page 419 Question 16

$$\frac{4x^2+4x-8}{x^2-5x+4} \div \frac{2x^2+3x-2}{4x^2+8x-5} = \frac{4\cancel{(x+2)}\cancel{(x-1)}}{\cancel{(x-1)}(x-4)}\left(\frac{\cancel{(2x-1)}(2x+5)}{\cancel{(2x-1)}\cancel{(x+2)}}\right)$$
$$= \frac{4(2x+5)}{x-4}, x \neq -2.5, -2, 0.5, 1, 4$$

Unit 3 Test Page 419 Question 17



a) Example: A proportion that relates the side of the similar triangles is $\frac{x}{2x} = \frac{x+3}{x+10}$.

b) The non-permissible values are $x \neq 0, -10$.

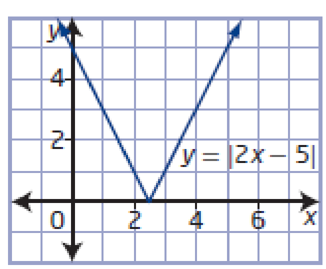
c)
$$\frac{x}{2x} = \frac{x+3}{x+10}$$
$$(2x)(x+10)\left(\frac{x}{2x}\right) = \left(\frac{x+3}{x+10}\right)(2x)(x+10)$$
$$x^2+10x = 2x^2+6x$$
$$0 = x^2-4x$$
$$0 = x(x-4)$$

$x = 0$ or $x - 4 = 0$

$x = 4$

The value of x that makes the triangles similar is 4.

Unit 3 Test Page 419 Question 18



a) Use a table of values to sketch the graph.

b) x-intercept: 2.5, y-intercept: 5

c) domain: $\{x \mid x \in \mathrm{R}\}$, range: $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d) When $x \geq 2.5$, the graph of $y = |2x - 5|$ is the graph of $y = 2x - 5$. When $x < 2.5$, the graph of $y = |2x - 5|$ is the graph of $y = -(2x - 5)$ or $y = -2x + 5$. The absolute value function $y = |2x - 5|$ expressed as a piecewise function is $y = \begin{cases} 2x - 5, \text{ if } x \geq 2.5 \\ -2x + 5, \text{ if } x < 2.5 \end{cases}$.

Unit 3 Test Page 419 Question 19

Examine the two cases.

Case 1

The expression $|x^2 - 3x|$ equals $x^2 - 3x$ when $x \leq 0$ or $x \geq 3$.

$x^2 - 3x = 2$

$x^2 - 3x - 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{17}}{2}$$

$x = \frac{3 + \sqrt{17}}{2}$ or $x = \frac{3 - \sqrt{17}}{2}$

$x = 3.5615\ldots$ $x = -0.5615\ldots$

Both values satisfy the conditions.

Case 2

The expression $|x^2 - 3x|$ equals $-(x^2 - 3x)$ when $0 < x < 3$.

$-(x^2 - 3x) = 2$

$x^2 - 3x = -2$

$x^2 - 3x + 2 = 0$

$(x - 2)(x - 1) = 0$

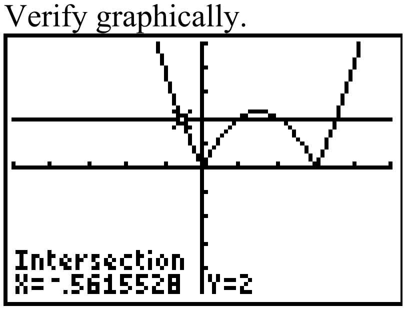
$x - 2 = 0$ or $x - 1 = 0$

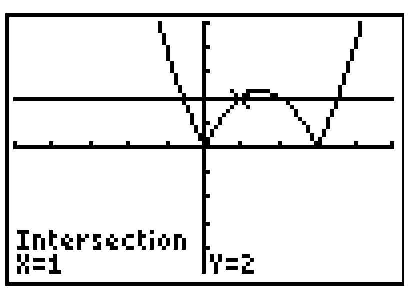
$x = 2$ $x = 1$

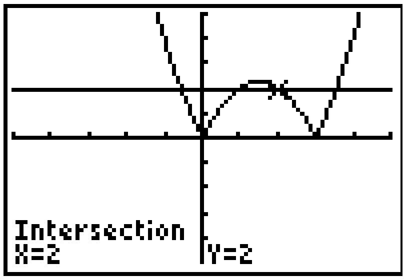
Both values satisfy the condition.

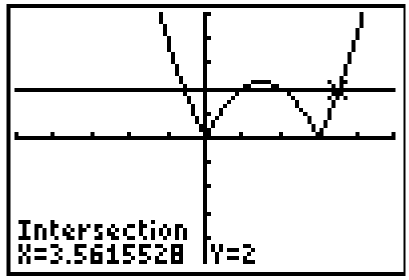
The solutions are $x = \frac{3 - \sqrt{17}}{2}$, $x = 1$, $x = 2$, and $x = \frac{3 + \sqrt{17}}{2}$.

Verify graphically.



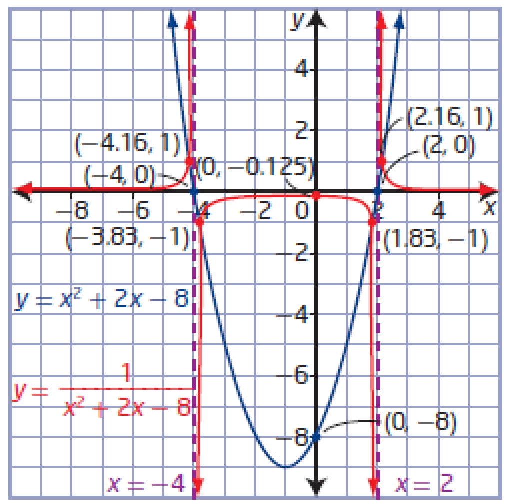






Unit 3 Test Page 419 Question 20

Use the location of the vertex and intercepts to plot the graph of $f(x)$.



• Determine the coordinates of the vertex for $f(x) = x^2 + 2x - 8$ using $x = -\frac{b}{2a}$ and its corresponding value of y. The vertex is located at $(-1, -9)$.

• Determine the zeros of the function, or x-intercepts of the graph, by solving $f(x) = 0$.

$x^2 + 2x - 8 = 0$

$(x + 4)(x - 2) = 0$

$x + 4 = 0$ or $x - 2 = 0$

$x = -4$ $\quad$ $x = 2$

• The y-intercept is -8.

To sketch the graph of the reciprocal function, consider the following characteristics:

• The reciprocal function has vertical asymptotes $x = -4$ and $x = 2$.

• Determine the invariant points by solving $f(x) = \pm 1$.

$x^2 + 2x - 8 = 1$ $\qquad$ $x^2 + 2x - 8 = -1$

$x^2 + 2x - 9 = 0$ $\qquad$ $x^2 + 2x - 7 = 0$

In each case, use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-9)}}{2(1)} \qquad x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{40}}{2} \qquad x = \frac{-2 \pm \sqrt{32}}{2}$$

$x \approx 2.16$ or $x \approx -4.16$ $\qquad$ $x \approx 1.83$ or $x \approx -3.83$

Invariant points are $(2.16, 1)$, $(-4.16, 1)$, $(1.83, -1)$, and $(-3.83, -1)$.

The y-intercept of $y = \frac{1}{x^2 + 2x - 8}$ is $-\frac{1}{8}$ or -0.125.

Unit 3 Test　　　　Page 419　　Question 21

a) Substitute $d = 72$ into $d = st$ and isolate s.

$d = st$
$72 = st$
$s = \frac{72}{t}$

b) Substitute $t = 14.5$ into $s = \frac{72}{t}$.

$s = \frac{72}{t}$
$s = \frac{72}{14.5}$
$s \approx 4.97$

The weight of the stone is approximately 4.97 ft/s.

c) Substitute $s = 6.3$ into $s = \frac{72}{t}$.

$s = \frac{72}{t}$
$6.3 = \frac{72}{t}$
$t \approx 11.43$

The time required to travel between hog lines is approximately 11.43 s.