Chapter 1 Sequences and Series

Section 1.1 Arithmetic Sequences

Section 1.1 Page 16 Question 1

a) $32 - 16 = 16$, $48 - 32 = 16$, $64 - 48 = 16$, …
Since successive differences are constant, the sequence is arithmetic.
$t_1 = 16$; $d = 16$.
The next three terms are:
$80 + 16 = 96$
$96 + 16 = 112$
$112 + 16 = 128$

b) Not arithmetic, because the differences of consecutive terms are not constant:
$4 - 2 = 2$, $8 - 4 = 4$,…

c) $-7 - (-4) = -3$, $-10 - (-7) = -3$, $-13 - (-10) = -3$, …
Since successive differences are constant, the sequence is arithmetic.
$t_1 = -4$; $d = -3$.
The next three terms are:
$-16 + (-3) = -19$
$-19 + (-3) = -22$
$-22 + (-3) = -25$

d) $0 - 3 = -3$, $-3 - 0 = -3$, $-6 - (-3) = -3$, …
Since successive differences are constant, the sequence is arithmetic.
$t_1 = 3$; $d = -3$.
The next three terms are:
$-9 + (-3) = -12$
$-12 + (-3) = -15$
$-15 + (-3) = -18$

Section 1.1 Page 16 Question 2

a) $t_1 = 5$, $t_2 = 5 + 3$ or 8, $t_3 = 8 + 3$ or 11, $t_4 = 11 + 3 = 14$
The first four terms are 5, 8, 11, 14.

b) $t_1 = -1$, $t_2 = -1 + (-4)$ or -5, $t_3 = -5 + (-4)$ or -9, $t_4 = -9 + (-4)$ or -13
The first four terms are $-1, -5, -9, -13$.

c) $t_1 = 4$, $t_2 = 4 + \frac{1}{5}$ or $4\frac{1}{5}$, $t_3 = 4\frac{1}{5} + \frac{1}{5}$ or $4\frac{2}{5}$, $t_4 = 4\frac{2}{5} + \frac{1}{5}$ or $4\frac{3}{5}$
The first four terms are 4, $4\frac{1}{5}$, $4\frac{2}{5}$, $4\frac{3}{5}$.

d) $t_1 = 1.25$, $t_2 = 1.25 + (-0.25)$ or 1, $t_3 = 1 + (-0.25)$ or 0.75, $t_4 = 0.75 + (-0.25) = 0.5$
The first four terms are 1.25, 1, 0.75, 0.5.

Section 1.1 Page 16 Question 3

a) $t_n = 3n + 8$
$t_1 = 3(1) + 8$
$t_1 = 11$

b) $t_7 = 3(7) + 8$
$t_7 = 29$

c) $t_{14} = 3(14) + 8$
$t_{14} = 50$

Section 1.1 Page 16 Question 4

a) $t_4 = 19$, $t_5 = 23$
$t_5 - t_4 = 23 - 19$ or 4
$d = 4$
$t_n = t_1 + (n - 1)d$
$19 = t_1 + (4 - 1)4$
$19 = t_1 + 12$
$t_1 = 7$
Then, $t_2 = 7 + 4$ or 11, and $t_3 = 11 + 4$ or 15.
The missing terms are 7, 11, 15; $t_1 = 7$ and $d = 4$.

b) $t_3 = 3$, $t_4 = \frac{3}{2}$

$t_4 - t_3 = \frac{3}{2} - 3$ or $-\frac{3}{2}$

$d = -\frac{3}{2}$

$t_n = t_1 + (n - 1)d$

$3 = t_1 + (3 - 1)\left(-\frac{3}{2}\right)$

$3 = t_1 + (-3)$
$t_1 = 6$

Then, $t_2 = 6 + \left(-\frac{3}{2}\right)$ or $4\frac{1}{2}$.

The missing terms are 6, $4\frac{1}{2}$; $t_1 = 6$ and $d = -\frac{3}{2}$.

c) $t_2 = 4$, $t_5 = 10$
$t_5 = t_2 + 3d$
$10 = 4 + 3d$
$6 = 3d$
$d = 2$
The missing terms are 2 and 6, 8; $t_1 = 2$ and $d = 2$.

Section 1.1 Page 16 Question 5

a) From the pattern of the sequence, $t_1 = -4$ and $d = 6$.

$$t_n = t_1 + (n - 1)d$$
$$170 = -4 + (n - 1)6$$
$$170 = -4 + 6n - 6$$
$$170 + 10 = 6n$$
$$180 = 6n$$
$$n = 30$$

So, 170 is the 30th term of the sequence.

b) From the pattern of the sequence, $t_1 = 2\frac{1}{5}$ and $d = -\frac{1}{5}$.

$$t_n = t_1 + (n-1)d$$
$$-14 = 2\frac{1}{5} + (n-1)\left(-\frac{1}{5}\right)$$
$$-14 = 2\frac{1}{5} - \frac{1}{5}n + \frac{1}{5}$$
$$\frac{1}{5}n = 14 + 2\frac{2}{5}$$
$$\frac{1}{5}n = 16\frac{2}{5}$$
$$n = 5\left(16\frac{2}{5}\right)$$
$$n = 82$$

So, −14 is the 82nd term of the sequence.

c) From the pattern of the sequence, $t_1 = -3$ and $d = 4$.

$$t_n = t_1 + (n - 1)d$$
$$97 = -3 + (n - 1)4$$
$$97 = -3 + 4n - 4$$
$$97 + 7 = 4n$$
$$104 = 4n$$
$$n = 26$$

So, 97 is the 26th term of the sequence.

d) From the pattern of the sequence, $t_1 = 14$ and $d = -1.5$.

$$t_n = t_1 + (n - 1)d$$
$$-10 = 14 + (n - 1)(-1.5)$$
$$-10 = 14 - 1.5n + 1.5$$
$$1.5n = 15.5 + 10$$
$$1.5n = 25.5$$

$$n = \frac{25.5}{1.5}$$
$$n = 17$$
So, −10 is the 17th term of the sequence.

Section 1.1 Page 16 Question 6

a) $t_1 = 6$, $t_4 = 33$
Substitute for t_4 in $t_n = t_1 + (n - 1)d$.
$$\begin{aligned} 33 &= 6 + 3d \\ 33 - 6 &= 3d \\ 27 &= 3d \\ d &= 9 \end{aligned}$$
Then, the second term is 6 + 9 or 15, and the third term is 15 + 9 or 24.

b) $t_1 = 8$, $t_4 = 41$
Substitute for t_4 in $t_n = t_1 + (n - 1)d$.
$$\begin{aligned} 41 &= 8 + 3d \\ 41 - 8 &= 3d \\ 33 &= 3d \\ d &= 11 \end{aligned}$$
Then, the second term is 8 + 11 or 19, and the third term is 19 + 11 or 30.

c) $t_1 = 42$, $t_4 = 27$
Substitute for t_4 in $t_n = t_1 + (n - 1)d$.
$$\begin{aligned} 27 &= 42 + 3d \\ 27 - 42 &= 3d \\ -15 &= 3d \\ d &= -5 \end{aligned}$$
Then, the second term is 42 + (−5) or 37, and the third term is 37 + (−5) or 32.

Section 1.1 Page 17 Question 7

a) The y-values of each point are the terms. It appears that the first five terms are:
5, 8, 11, 14, 17.

b) From the pattern of the sequence, $t_1 = 5$ and $d = 3$.
$t_n = 5 + (n - 1)3$
$t_n = 2 + 3n$

c) $t_{50} = 2 + 3(50)$ $\quad$ $t_{200} = 2 + 3(200)$
$t_{50} = 152$ $\quad$ $t_{200} = 602$

d) Answers may vary. Example:
Use the points (1, 5) and (2, 8).

$$\text{slope} = \frac{8-5}{2-1}$$
$$\text{slope} = 3$$

The slope is the same as the common difference. It is the coefficient of the variable term in the general term, $2 + 3n$.

e) If a line were drawn through the points, the y-intercept would be 2. This is the same as the constant value in the general term, $2 + 3n$.

Section 1.1 Page 17 Question 8

Consider **A**:

$t_n = 6 + (n - 1)4$
$34 = 6 + 4n - 4$
$34 - 2 = 4n$
$32 = 4n$
$n = 8$

The sequence defined by $t_n = 6 + (n - 1)4$ has 34 as its 8th term.

Consider **B**:

$t_n = 3n - 1$
$34 = 3n - 1$
$35 = 3n$

This does not give a natural number for n, the term number, so 34 cannot be a term of this sequence.

Consider **C**:

$t_1 = 12$, $d = 5.5$

Then, $t_n = t_1 + (n - 1)d$
$t_n = 12 + (n - 1)5.5$
$34 = 12 + 5.5n - 5.5$
$34 - 6.5 = 5.5n$
$27.5 = 5.5n$
$$n = \frac{27.5}{5.5}$$
$n = 5$

The sequence for which $t_1 = 12$ and $d = 5.5$ has 34 as its fifth term.

Consider **D**:

From the pattern of the sequence, $t_1 = 3$ and $d = 4$.

Then, $t_n = 3 + (n - 1)4$
$34 = 3 + 4n - 4$
$34 + 1 = 4n$

This does not give a natural number for n, the term number, so 34 cannot be a term of this sequence.

Section 1.1 Page 17 Question 9

$t_{16} = 110$, $d = 7$
Substitute for t_{16} in $t_n = t_1 + (n - 1)d$.
$110 = t_1 + 15(7)$
$110 = t_1 + 105$
$t_1 = 5$

The first term of the sequence is 5.

Section 1.1 Page 17 Question 10

$t_1 = 5y$, $d = -3y$
Substitute into $t_n = t_1 + (n - 1)d$
$t_n = 5y + (n - 1)(-3y)$
$t_n = 5y - 3ny + 3y$
$t_n = 8y - 3ny$

For t_{15}, substitute $n = 15$.
$t_{15} = 8y - 3(15)y$
$t_{15} = 8y - 45y$
$t_{15} = -37y$

Section 1.1 Page 17 Question 11

The difference between consecutive terms is the same for terms of an arithmetic sequence.

$$\begin{aligned}7x - 4 - (5x + 2) &= 10x + 6 - (7x - 4)\\ 7x - 4 - 5x - 2 &= 10x + 6 - 7x + 4\\ 2x - 6 &= 3x + 10\\ x &= -16\end{aligned}$$

Substitute to find the three terms:

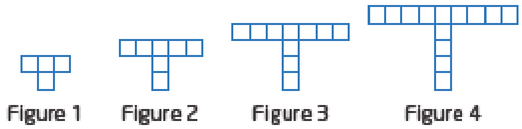
$5x + 2 = 5(-16) + 2 = -78$ $7x - 4 = 7(-16) - 4 = -116$ $10x + 6 = 10(-16) + 6 = -154$

Check that these terms have a common difference:
$-116 - (-78) = -38$
$-154 - (-116) = -38$
So, the value of x is -16 and the three terms are -78, -116, -154.

Section 1.1 Page 17 Question 12

The difference between consecutive terms is the same for terms of an arithmetic sequence.
$z - y = y - x$
$z = 2y - x$

Section 1.1 Page 17 Question 13



a) Perimeters of the four figures shown: 10, 16, 22, 28.
The perimeters are an arithmetic sequence with $t_1 = 10$ and $d = 6$. So, an equation for the perimeter of figure n is $P_n = 10 + (n - 1)6$ or $P_n = 6n + 4$.

b) For the perimeter of Figure 9, substitute $n = 9$.
$P_9 = 10 + (9 - 1)6$
$P_9 = 58$
The perimeter of Figure 9 is 58 units.

c) Determine the value of n when $P_n = 76$.
$76 = 10 + (n - 1)6$
$76 = 10 + 6n - 6$
$72 = 6n$
$n = 12$
Figure 12 has a perimeter of 76 units.

Section 1.1 Page 17 Question 14

a) $t_1 = 0$, $d = 8$
The tee-off times are: 8:00, 8:08, 8:16, 8:24, …
Considering 8:00 to be time 0, the sequence is 0, 8, 16, 24.

b) Extend the sequence to 60 min.
0, 8, 16, 24, 32, 40, 48, 56, …
So within the first hour, 8 groups of four will have teed-off. This means 32 players will be on the course after 1 h.

c) $t_n = 0 + (n - 1)8$
$t_n = 8n - 8$

d) For 132 players, there will need to be 132 ÷ 4 or 33 groups teeing-off.
Substitute $n = 33$ into t_n.
$t_n = 8(33) - 8$
$t_n = 256$

This means that the last group will tee-off 256 min after the first group.
256 min = 4 h 16 min, so the last group will tee-off at 8:00 + 4:16 or 12:16.

e) Answers may vary. Examples: Rain may interrupt the tee-off times or players in a group might not be quite ready at their tee-off time.

Section 1.1 Page 18 Question 15

Area of the wall hanging $= 22 \times 27$
$= 594$

Substitute $t_1 = 48$, $n = 27$, and $t_n = 594$ into $t_n = t_1 + (n - 1)d$.

$594 = 48 + (27 - 1)d$
$594 = 48 + 26d$
$594 - 48 = 26d$
$546 = 26d$
$d = 21$

Lucy completed 21 square inches of the wall hanging on each subsequent day.

Section 1.1 Page 18 Question 16

a) $t_6 = 11$, $t_{15} = 29$

Substitute into $t_n = t_1 + (n - 1)d$.

For $n = 6$: $11 = t_1 + 5d$ ①

For $n = 15$: $29 = t_1 + 14d$ ②

Subtract the first equation from the second.

$18 = 9d$
$d = 2$

Substitute $d = 2$ into ① to determine t_1.

$11 = t_1 + 5(2)$
$t_1 = 1$

The general term that relates the number of sit-ups to the number of days is
$t_n = 1 + (n - 1)2$ or $t_n = 2n - 1$.

b) Substitute $t_n = 100$ into the general term and solve for n.

$100 = 1 + (n - 1)2$
$101 = 2n$
$n = 50.5$

Susan will be able to do 100 sit-ups on the 51st day of her program.

c) Answers may vary. Example: Assume that she is physically able to continue increasing the number of sit-ups by 2 each day.

Section 1.1 Page 18 Question 17

a)

Carbon Atoms	1	2	3	4
Hydrogen Atoms	4	6	8	10

b) From the pattern in the table, $t_1 = 4$ and $d = 2$.

$t_n = t_1 + (n - 1)d$
$t_n = 4 + (n - 1)2$
$t_n = 4 + 2n - 2$
$t_n = 2 + 2n$ or $H = 2 + 2C$

c) Substitute $t_n = 202$ and solve for n.

$202 = 2 + 2n$
$200 = 2n$
$n = 100$

In hectane, 100 carbon atoms support 202 hydrogen atoms.

Section 1.1 Page 18 Question 18

Multiples of	28	7	15
Between	1 and 1000	500 and 600	50 and 500
First Term, t_1	28	504	60
Common Difference, d	28	7	15
nth Term, t_n	980	595	495
General Term	$28n$	$497 + 7n$	$45 + 15n$
Number of Terms	35	14	30

Calculations for the last row:
For the first column of entries:
To determine the number of multiples of 28 less than 1000:

$1000 = 28n$
$n \approx 35.7$

Check $28(35) = 980$
So, there are 35 multiples of 28 between 1 and 1000.

For the second column of entries:
To determine the number of multiples of 7 greater than 500 but less than 600:

$600 = 497 + 7n$
$103 = 7n$
$n \approx 14.7$

Check: $504 + 7(14 - 1) = 595$

For the third column of entries:
To determine numbers multiples of 15 greater than 50 but less than 500:

$500 = 45 + 15n$
$455 = 15n$
$n \approx 30.3$

Check: $60 + 15(29) = 495$

Section 1.1 Page 19 Question 19

a)

Depth (ft)	0	30	60	90
Water Pressure (psi)	14.7	29.4	44.1	58.8

The first four terms of the sequence of water pressure with depth are 14.7, 29.4, 44.1, 58.8. The general term of this sequence is $t_n = 14.7n$, where n is the number of 30-ft descents.

b) First determine n for 1000 ft.

$$\frac{1000}{30} = 33\frac{1}{3}$$

$$\begin{aligned}\text{Water pressure} &= 14.7 + \left(33\frac{1}{3} - 1\right)14.7 \\ &= 490\end{aligned}$$

The pressure at a depth of 1000 ft is 490 psi.

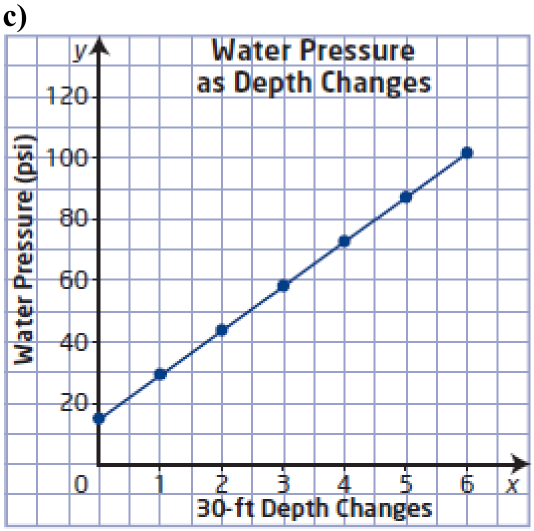
Determine n for 2000 ft.

$$\frac{2000}{30} = 66\frac{2}{3}$$

$$\begin{aligned}\text{Water pressure} &= 14.7 + \left(66\frac{2}{3} - 1\right)14.7 \\ &= 980\end{aligned}$$

The pressure at a depth of 1000 ft is 980 psi.

c)



d) The y-intercept is 14.7.

e) The slope is 14.7.

f) The y-intercept is t_1 and the slope is the common difference.

Section 1.1 Page 19 Question 20

Let *a* represent the length of the shortest side and *d* the common difference. Then, the four sides are: t_1, $t_1 + d$, $t_1 + 2d$, and $t_1 + 3d$.
Use the perimeter:

$$t_1 + t_1 + d + t_1 + 2d + t_1 + 3d = 60$$
$$4t_1 + 6d = 60$$
$$2t_1 + 3d = 30 \quad ①$$

Given that the longest side is 24 cm,

$$t_1 + 3d = 24 \quad ②$$

Subtract ② from ①.

$$t_1 = 6$$

Substitute $t_1 = 6$ in (2) to find *d*.

$$6 + 3d = 24$$
$$3d = 18$$
$$d = 6$$

The other three sides lengths are 6 cm, 12 cm and 18 cm.
Check: 6 + 12 + 18 + 24 = 60.

Section 1.1 Page 19 Question 21

a)

Term Number	1	2	3	4	5
Number of minutes	4	8	12	16	20
Number of degrees	1	2	3	4	5

The sequence of number of minutes is 4, 8, 12, 16, 20, …

b) The number of minutes, t_n, is given by $t_n = 4n$ where *n* is the number of degrees of turn.

c) The time for a rotation of 80º is 4(80) or 320 min. This is 5 h 20 min.

Section 1.1 Page 19 Question 22

Year	1986	1987	…	2006	2007
Term Number	1				22
Number of Beekeepers	1657				1048

$t_1 = 1657$, $t_n = 1048$, $n = 22$

$$t_n = t_1 + (n - 1)d$$
$$1048 = 1657 + (21)d$$
$$-609 = 21d$$
$$d = -29$$

From 1986 to 2007, the number of beekeepers decreased by about 29 each year.

Section 1.1 Page 19 Question 23

Year	2003	2004	…	2022	2023
Term Number	1				20
Millions of Carats	3.8				113.2

$t_1 = 3.8$, $t_n = 113.2$, $n = 20$

$t_n = t_1 + (n - 1)d$

$113.2 = 3.8 + 19d$

$109.4 = 19d$

$d \approx 5.76$

The common difference is approximately 5.8 million carats. This is the increase in the number of diamond carats extracted each year between 2003 and 2023.

Section 1.1 Page 20 Question 24

The radius for the circle traversed by wheel 12 will be 50 + 11(20) or 270 m.
Use $C = \pi d$.

$C = \pi(270)(2)$

$C \approx 1696.460\ldots$

Wheel 12 traverses a circle with circumference of about 1696.5 m.

Section 1.1 Page 21 Question 25

a) Consider each time to be number of minutes after 13:00.
Then, the first five terms are 54, 59, 64, 69, 74; $t_1 = 54$, $d = 5$

b) $t_n = t_1 + (n - 1)d$

$t_n = 54 + (n - 1)5$

$t_n = 5n + 49$

c) The terms are numbers of minutes after 13:00.

d) Determine the time when $n = 24$.

$t_{24} = 5(24) + 49$

$t_{24} = 169$

So, 169 min after 13:00 the sun was completely eclipsed. This is 2 h 49 min after 13:00, so the complete eclipse occurred at 15:49.

Section 1.1 Page 21 Question 26

a) An arithmetic sequence is an increasing sequence if and only the difference between one term and the next is a positive number. An arithmetic sequence is an increasing sequence if and only if $\boldsymbol{d > 0}$.

b) An arithmetic sequence is an decreasing sequence if and only the difference between one term and the next is a negative number. An arithmetic sequence is a decreasing sequence if and only if $\boldsymbol{d < 0}$.

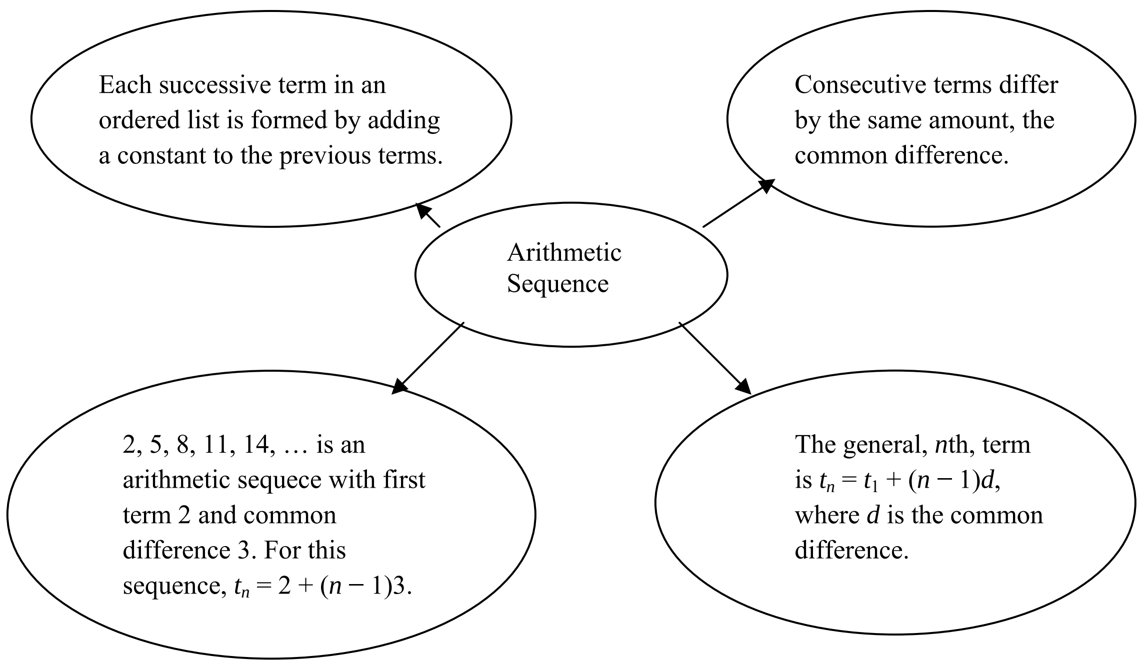
c) An arithmetic sequence is constant if and only all the terms are the same, or in other words there is no difference between successive terms. An arithmetic sequence is constant if and only if $\boldsymbol{d = 0}$.

d) The first term of a sequence is $\boldsymbol{t_1}$.

e) The symbol for the general term of a sequence is $\boldsymbol{t_n}$.

Section 1.1 Page 21 Question 27

Answers may vary. Example:



Section 1.1 Page 21 Question 28

Answers may vary. Example:
Step 1: The graph that models an arithmetic sequence is always a straight line. Successive terms are obtained by adding a constant, so successive points form steps of a line. No other shape is possible.

Step 2: a) As the value of the first term increases, the points are raised vertically. As the value of the first term decreases, each point is lowered by that amount.
b) The graph is still a line with the same slope.

Step 3: a) As the value of the common difference increases, successive points are raised vertically more and more. As the value of the common difference decreases, successive points are lowered vertically more and more.
b) As the value of the common difference increases, the slope of the line is steeper. As the value of the common difference decreases, the line is less steep.

Step 4: The slope would be the same as the common difference.

Step 5: The slope is the same as the value of d in the general term for the sequence. This is the coefficient of the variable n in t_n.

Section 1.2 Arithmetic Series

Section 1.2 Page 27 Question 1

a) $t_1 = 5$, $d = 3$, $t_n = 53$
Step 1: Determine n.
Substitute into $t_n = t_1 + (n - 1)d$.

$$53 = 5 + (n - 1)3$$
$$53 = 5 + 3n - 3$$
$$51 = 3n$$
$$n = 17$$

Step 2: Determine the sum of the series.

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$S_{17} = \frac{17}{2}(5 + 53)$$
$$S_{17} = 493$$

b) $t_1 = 7$, $d = 7$, $t_n = 98$
Step 1: Determine n.
Substitute into $t_n = t_1 + (n - 1)d$.

$$98 = 7 + (n - 1)7$$
$$98 = 7 + 7n - 7$$
$$98 = 7n$$
$$n = 14$$

Step 2: Determine the sum of the series.

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$S_{14} = \frac{14}{2}(7 + 98)$$
$$S_{14} = 735$$

c) $t_1 = 8$, $d = -5$, $t_n = -102$
Step 1: Determine n.
Substitute into $t_n = t_1 + (n - 1)d$.

$$-102 = 8 + (n - 1)(-5)$$
$$-102 = 8 - 5n + 5$$
$$5n = 115$$
$$n = 23$$

Step 2: Determine the sum of the series.

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$S_{23} = \frac{23}{2}(8 + (-102))$$
$$S_{23} = -1081$$

d) $t_1 = \frac{2}{3}$, $d = 1$, $t_n = \frac{41}{3}$
Step 1: Determine n.
Substitute into $t_n = t_1 + (n - 1)d$.

$$\frac{41}{3} = \frac{2}{3} + (n-1)1$$
$$13 + 1 = n$$
$$n = 14$$

Step 2: Determine the sum of the series.

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$S_{14} = \frac{14}{2}\left(\frac{2}{3} + \frac{41}{3}\right)$$
$$S_{14} = \frac{301}{3} \text{ or } 100\frac{1}{3}$$

Section 1.2 Page 27 Question 2

a) $t_1 = 1$, $d = 2$, $n = 8$

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_8 = \frac{8}{2}[2(1) + (8-1)2]$$
$$S_8 = 4[2 + 14]$$
$$S_8 = 64$$

b) $t_1 = 40$, $d = -5$, $n = 11$

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_{11} = \frac{11}{2}[2(40) + (11-1)(-5)]$$
$$S_{11} = \frac{11}{2}[80 - 50]$$
$$S_{11} = 165$$

c) $t_1 = \frac{1}{2}$, $d = 1$, $n = 7$

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_7 = \frac{7}{2}\left[2\left(\frac{1}{2}\right) + (7-1)1\right]$$
$$S_8 = \frac{7}{2}[1 + 6]$$
$$S_8 = \frac{49}{2} \text{ or } 24.5$$

d) $t_1 = -3.5$, $d = 2.25$, $n = 6$

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_6 = \frac{6}{2}[2(-3.5) + (6-1)2.25]$$
$$S_6 = 3[-7 + 11.25]$$
$$S_6 = 12.75$$

Section 1.2 Page 27 Question 3

a) $t_1 = 7$, $t_n = 79$, $n = 8$

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$S_8 = \frac{8}{2}(7 + 79)$$
$$S_8 = 4(86)$$
$$S_8 = 344$$

b) $t_1 = 58$, $t_n = -7$, $n = 26$

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$S_{26} = \frac{26}{2}(58 + (-7))$$
$$S_{26} = 13(51)$$
$$S_{26} = 663$$

c) $t_1 = -12$, $t_n = 51$, $n = 10$

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$S_{10} = \frac{10}{2}(-12 + 51)$$
$$S_{10} = 5(39)$$
$$S_{10} = 195$$

d) $t_1 = 12$, $d = 8$, $n = 9$

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_9 = \frac{9}{2}[2(12) + (9-1)(8)]$$
$$S_9 = \frac{9}{2}[24 + 64]$$
$$S_9 = 396$$

e) $t_1 = 42$, $d = -5$, $n = 14$

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_{14} = \frac{14}{2}[2(42) + (14-1)(-5)]$$
$$S_{14} = 7[84 - 65]$$
$$S_{14} = 133$$

Section 1.2 Page 27 Question 4

a) $d = 6$, $S_n = 574$, $n = 14$

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$574 = \frac{14}{2}[2t_1 + (14-1)6]$$
$$574 = 7[2t_1 + 78]$$
$$574 = 14t_1 + 546$$
$$28 = 14t_1$$
$$t_1 = 2$$

b) $d = -6$, $S_n = 32$, $n = 13$

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$32 = \frac{13}{2}[2t_1 + (13-1)(-6)]$$
$$32 = \frac{13}{2}[2t_1 - 72]$$
$$32 = 13t_1 - 468$$
$$500 = 13t_1$$
$$t_1 = \frac{500}{13} \text{ or } 38\frac{6}{13}$$

c) $d = 0.5$, $S_n = 218.5$, $n = 23$

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$218.5 = \frac{23}{2}[2t_1 + (23-1)0.5]$$
$$218.5 = \frac{23}{2}[2t_1 + 11]$$
$$218.5 = 23t_1 + 126.5$$
$$92 = 23t_1$$
$$t_1 = 4$$

d) $d = -3$, $S_n = 279$, $n = 18$

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$279 = \frac{18}{2}[2t_1 + (18-1)(-3)]$$
$$279 = 9[2t_1 - 51]$$
$$279 = 18t_1 - 459$$
$$738 = 18t_1$$
$$t_1 = 41$$

Section 1.2 Page 27 Question 5

a) $t_1 = 8$, $t_n = 68$, $S_n = 608$

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$608 = \frac{n}{2}(8 + 68)$$
$$608 = 38n$$
$$n = 16$$

b) $t_1 = -6$, $t_n = 21$, $S_n = 75$

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$75 = \frac{n}{2}(-6 + 21)$$
$$75 = 7.5n$$
$$n = 10$$

Section 1.2 Page 27 Question 6

a) $t_1 = 5$, $d = 5$, $n = 10$

For t_{10}, substitute into $t_n = t_1 + (n - 1)d$.
$t_{10} = 5 + (10 - 1)5$
$t_{10} = 45$

For S_{10}, substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_{10} = \frac{10}{2}[2(5) + (10-1)5]$$
$$S_{10} = 5[10 + 45]$$
$$S_{10} = 275$$

b) $t_1 = 10$, $d = -3$, $n = 10$

For t_{10}, substitute into $t_n = t_1 + (n - 1)d$.
$t_{10} = 10 + (10 - 1)(-3)$
$t_{10} = -17$

For S_{10}, substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_{10} = \frac{10}{2}[2(10) + (10 - 1)(-3)]$$
$$S_{10} = 5[20 - 27]$$
$$S_{10} = -35$$

c) $t_1 = -10$, $d = -4$, $n = 10$

For t_{10}, substitute into $t_n = t_1 + (n - 1)d$.
$t_{10} = -10 + (10 - 1)(-4)$
$t_{10} = -10 - 36$
$t_{10} = -46$

For S_{10}, substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_{10} = \frac{10}{2}[2(-10) + (10 - 1)(-4)]$$
$$S_{10} = 5[-20 - 36]$$
$$S_{10} = -280$$

d) $t_1 = 2.5$, $d = 0.5$, $n = 10$

For t_{10}, substitute into $t_n = t_1 + (n - 1)d$.
$t_{10} = 2.5 + (10 - 1)(0.5)$
$t_{10} = 7$

For S_{10}, substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_{10} = \frac{10}{2}[2(2.5) + (10 - 1)(0.5)]$$
$$S_{10} = 5[5 + 4.5]$$
$$S_{10} = 47.5$$

Section 1.2 Page 27 Question 7

a) $t_1 = 4$, $t_n = 996$, $d = 4$
Step 1: Determine n.
Substitute into $t_n = t_1 + (n - 1)d$.
$996 = 4 + (n - 1)4$
$996 = 4 + 4n - 4$
$996 = 4n$
$n = 249$

Step 2: Determine the sum of the series.

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$S_{249} = \frac{249}{2}(4 + 996)$$
$$S_{249} = 124\ 500$$

The sum of all the multiples of 4 between 1 and 999 is 124 500.

b) $t_1 = 12$, $t_n = 996$, $d = 6$
Step 1: Determine n.
Substitute into $t_n = t_1 + (n - 1)d$.
$996 = 12 + (n - 1)6$
$990 = 6n$
$n = 165$

Step 2: Determine the sum of the series.

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$

$$S_{165} = \frac{165}{2}(6 + 996)$$
$$S_{165} = 82\ 665$$

The sum of all the multiples of 6 between 6 and 999 is 82 665.

Section 1.2 Page 28 Question 8

The number of chimes in a 24-h period will be double the sum of the series
$1 + 2 + 3 + \ldots + 12$.

Substitute $t_1 = 1$, $t_n = 12$, $n = 12$ into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$S_{12} = \frac{12}{2}(1 + 12)$$
$$S_{12} = 78$$

In a 24-period the clock will chime 78(2) or 156 times.

Section 1.2 Page 28 Question 9

a) $t_5 = 14$, $d = 3$, $n = 5$
Substitute into $t_n = t_1 + (n - 1)d$.
$14 = t_1 + (5 - 1)3$
$14 = t_1 + 12$
$t_1 = 2$
The pilot flew 2 circuits on the first day.

b) Total by the end of the fifth day = 2 + 5 + 8 + 11 + 14
= 40
The pilot flew a total of 40 circuits by the end of the fifth day.

c) $S_n = \frac{n}{2}[2t_1 + (n-1)d]$

$S_n = \frac{n}{2}[2(2) + (n-1)3]$

$S_n = \frac{n}{2}[4 + 3n - 3]$

$S_n = \frac{n}{2}[1 + 3n]$

The total number of circuits by the end of the nth day is given by $\frac{n}{2}(1+3n)$.

Section 1.2 Page 28 Question 10

$t_2 = 40$, $t_5 = 121$
Step 1: Determine the values of t_1 and d.
Substitute into $t_n = t_1 + (n - 1)d$.

$$\begin{array}{ll} \text{For } t_2: & 40 = t_1 + d \\ \text{For } t_5: & \underline{121 = t_1 + 4d} \\ \text{Subtracting} & -81 = -3d \\ & 27 = d \end{array}$$

Then, $t_1 = 40 - 27$
$t_1 = 13$

Step 2: Determine S_{25}.
Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$S_{25} = \frac{25}{2}[2(13) + (25-1)27]$

$S_{25} = \frac{25}{2}[26 + 648]$

$S_{25} = 8425$

The sum of the first 25 terms of the series is 8425.

Section 1.2 Page 28 Question 11

$S_5 = 85$, $S_6 = 123$
Then, $t_6 = 123 - 85$ or 38.
For t_6: $38 = t_1 + 5d$ ①
For S_5: $85 = \frac{5}{2}[2t_1 + 4d]$ ②
Multiply ① by 2 and simplify ②:
$76 = 2t_1 + 10d$ ③
$\underline{85 = 5t_1 + 10d}$ ④
$-9 = -3t_1$ ③ − ④
$t_1 = 3$
Substitute $t_1 = 3$ into ① to obtain $d = 7$.

Then, the first four terms of the series are 3 + 10 + 17 + 24.

Section 1.2 Page 28 Question 12

a) Substitute $t_1 = 5$, $d = 10$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_n = \frac{n}{2}[2(5) + (n-1)10]$$

$$S_n = \frac{n}{2}[10 + 10n - 10]$$

$$S_n = \frac{n}{2}[10n]$$

$$S_n = 5n^2$$

b) $S_{100} = \frac{100}{2}[2(5) + (100-1)10]$ and $d(n) = 5n^2$

$S_{100} = 50[10 + 1000 - 10]$ $d(100) = 5(100)^2$

$S_{100} = 50[1000]$ $d(100) = 5(10\ 000)$

$S_{100} = 50\ 000$ $d(100) = 50\ 000$

They are equivalent.

Section 1.2 Page 28 Question 13

Consider the top row to be t_1 and the bottom row to be t_{18}. Each row has one more can than the row above.
$t_1 = 1$, $d = 1$, $t_{18} = 18$

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$S_{18} = \frac{18}{2}(1 + 18)$
$S_{18} = 9(19)$
$S_{18} = 171$
There are 171 cans in the display.

Section 1.2 Page 29 Question 14

a) The series represents the number of handshakes among 6 people.

b) 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9

c) For 30 people, the number of handshakes will be 1 + 2 + 3 + … + 29.
So, $t_1 = 1$, $d = 1$, and $t_{29} = 29$.
Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$S_{29} = \frac{29}{2}(1 + 29)$
$S_{29} = 435$

d) Answers will vary. Example: Finding the number of wire connections between terminals, when all must be joined to each other.

Section 1.2 Page 29 Question 15

a) Since the terms form an arithmetic sequence, the difference of successive terms is constant.

$$\begin{aligned} 8.6 - (2x - 5) &= 2x - 5 - x \\ 8.6 - 2x + 5 &= x - 5 \\ 18.6 &= 3x \\ x &= 6.2 \end{aligned}$$

Then, $d = 2x - 5 - x$
$d = 2(6.2) - 5 - 6.2$
$d = 1.2$
The first term is 6.2 and the common difference is 1.2.

b) Substitute in $t_n = t_1 + (n - 1)d$.
$t_{20} = 6.2 + 19(1.2)$
$t_{20} = 29$
The 20th term is 29.

c) Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$S_{20} = \frac{20}{2}(6.2 + 29)$
$S_{20} = 10(35.2)$
$S_{20} = 352$

Section 1.2 Page 29 Question 16

There are 18 rings of diameter 20 cm down to diameter 3 cm. There are 17 overlaps of 2 cm each. So, find the sum of the 18 diameters and subtract the overlap.

Substitute $t_1 = 20$, $t_n = 3$, and $n = 18$ into $S_n = \frac{n}{2}(t_1 + t_n)$.

$S_{18} = \frac{18}{2}(20 + 3)$
$S_{18} = 207$
Then, the distance from the top of the top ring to the bottom of the lowest ring is $207 - 2(17)$ or 173 cm.

Section 1.2 Page 29 Question 17

a) True. If each term is doubled, then $S_n = \frac{n}{2}(t_1 + t_n)$ will become $S_n = \frac{n}{2}(2t_1 + 2t_n)$ which is the same as twice the original sum.

b) False. The first sum is $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ and double this would be
$2S_n = n[2t_1 + (n - 1)d]$
$2S_n = 2nt_1 + n^2d - nd$
However, the second sum will be
$S_{2n} = \frac{2n}{2}[2t_1 + (2n-1)d]$
$S_{2n} = 2nt_1 + 2n^2d - nd$
Comparing the second sum is n^2d greater than the first.

c) True. An arithmetic sequence has the form, t_1, $t_1 + d$, $t_1 + 2d$, … Multiplying each term by a constant, k, gives kt_1, $kt_1 + kd$, $kt_1 + 2kd$, …which has the same pattern just with first term kt_1 and common difference kd.
For example, 2, 5, 8, 11, … has $t_1 = 2$ and $d = 3$.
When $k = 5$, the sequence becomes 10, 25, 40, 55, …. Here $t_1 = 10$ and $d = 15$.

Section 1.2 Page 30 Question 18

a) Substitute $n = 1$ into $S_n = 2n^2 + 5n$ to determine $S_1 = 7 = t_1$
Substitute $n = 2$, $S_2 = 18$. Then, $t_2 = 18 - 7 = 11$.
So $d = 4$ and $t_3 = 15$.
The first three terms of the series are 7 + 11 + 15.

b) Substitute in $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_{10} = \frac{10}{2}[2(7) + (10-1)4]$$
$$S_{10} = 5[14+36]$$
$$S_{10} = 250$$

c) Substitute in $S_n = 2n^2 + 5n$.
$S_{10} = 2(10)^2 + 5(10)$
$S_{10} = 2(100) + 50$
$S_{10} = 250$

d)
$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$
$$S_n = \frac{n}{2}[2(7) + (n-1)4]$$
$$S_n = \frac{n}{2}[14 + 4n - 4]$$
$$S_n = \frac{n}{2}[10 + 4n]$$
$$S_n = 5n + 2n^2$$

Section 1.2 Page 30 Question 19

a) Amount by the end of the 7th hour = 240 + 250 + 260 + 270 + 280 + 290 + 300

b) Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_n = \frac{n}{2}[2(240) + (n-1)10]$$
$$S_n = \frac{n}{2}[480 + 10n - 10]$$
$$S_n = \frac{n}{2}[470 + 10n]$$
$$S_n = 235n + 5n^2$$

c) $S_7 = 235(7) + 5(7)^2$
$S_7 = 1645 + 245$
$S_7 = 1890$
By the end of the 7th hour Nathan has harvested 1890 bushels.

d) No assumptions if conditions were as stated. He would be working non-stop from 11:00 a.m. until 6:00 p.m. and there would be no delays for rain or machinery break-down.

Section 1.2 Page 30 Question 20

Substitute $S_{15} = 120$, $t_{15} = 43$, and $n = 15$ into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$\begin{aligned} 120 &= \frac{15}{2}(t_1 + 43) \\ 240 &= 15t_1 + 645 \\ -405 &= 15t_1 \\ t_1 &= -27 \end{aligned}$$

Substitute $t_{15} = 43$, $t_1 = -27$, and $n = 15$ into $t_n = t_1 + (n - 1)d$.
$43 = -27 + (15 - 1)d$
$43 = -27 + 14d$
$70 = 14d$
$d = 5$
The first three terms of the series are $-27 + (-22) + (-17)$.

Section 1.2 Page 30 Question 21

The formula that Pierre used is, in effect, the same as the one that Jeannette used. In the first formula, substitute $t_n = t_1 + (n - 1)d$.

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$S_n = \frac{n}{2}[t_1 + t_1 + (n-1)d]$$

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$

Section 1.2 Page 30 Question 22

a) Substitute $t_1 = 1$, $d = 2$, and $n = 10$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_{10} = \frac{10}{2}[2(1) + (10-1)2]$$
$$S_{10} = 5[2+18]$$
$$S_{10} = 100$$

b) Blue triangles = 1 + 2 + 3 + … + 9

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$
$$S_9 = \frac{9}{2}[2(1) + (9-1)1]$$
$$S_9 = \frac{9}{2}[2+8]$$
$$S_9 = 45$$

Green triangles = 1 + 2 + 3 + … + 10

$$S_n = \frac{n}{2}[2t_1 + (n-1)d]$$
$$S_{10} = \frac{10}{2}[2(1) + (10-1)1]$$
$$S_9 = 5[2+9]$$
$$S_9 = 55$$

Sum of blue triangles + sum of green triangles = 45 + 55 = 100. This is the same as the sum in part a).

Section 1.2 Page 31 Question 23

a) The tenth triangular number is given by 1 + 2 + 3…+ 9 + 10.

Substitute $t_1 = 1$, $t_n = 10$, and $n = 10$ into $S_n = \frac{n}{2}(t_1 + t_n)$.

$$S_{10} = \frac{10}{2}(1+10)$$
$$S_{10} = 5(11)$$
$$S_{10} = 55$$

The tenth triangular number is 55.

b) Substitute $t_1 = 1$ and $d = 1$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_n = \frac{n}{2}[2(1) + (n-1)1]$$

$$S_n = \frac{n}{2}[2 + n - 1]$$

$$S_n = \frac{n}{2}(n+1)$$

Section 1.3 Geometric Sequences

Section 1.3 Page 39 Question 1

a) geometric with $t_1 = 1$ and $r = 2$
$t_n = 1(2)^{n-1}$
$t_n = 2^{n-1}$

b) not geometric

c) geometric with $t_1 = 3$ and $r = -3$
$t_n = 3(-3)^{n-1}$

d) not geometric

e) geometric with $t_1 = 10$ and $r = 1.5$
$t_n = 10(1.5)^{n-1}$

f) geometric with $t_1 = -1$ and $r = 5$
$t_n = -1(5)^{n-1}$

Section 1.3 Page 39 Question 2

	Geometric Sequence	Common Ratio	6th Term	10th Term
a)	6, 18, 54	3	$6(3)^{6-1} = 6(3)^5$ $= 1458$	$6(3)^{10-1} = 6(3)^9$ $= 118\ 098$
b)	1.28, 0.64, 0.32	0.5	$1.28(0.5)^{6-1}$ $= 1.28(0.5)^5$ $= 0.04$	$1.28(0.5)^{10-1}$ $= 1.28(0.5)^9$ $= 0.0025$
c)	$\frac{1}{5}, \frac{3}{5}, \frac{9}{5}$	3	$\left(\frac{1}{5}\right)3^{6-1} = \left(\frac{1}{5}\right)3^5$ $= \frac{243}{5}$ or 48.6	$\left(\frac{1}{5}\right)3^{10-1} = \left(\frac{1}{5}\right)3^9$ $= \frac{19\ 683}{5}$ or 3936.6

Section 1.3 Page 39 Question 3

a) Substitute $t_1 = 2$ and $r = 3$ into $t_n = t_1r^{n-1}$.

$t_2 = 2(3)^1$ $t_3 = 2(3)^2$ $t_4 = 2(3)^3$

$t_2 = 6$ $t_3 = 18$ $t_4 = 54$

The first four terms of the sequence are 2, 6, 18, 54.

b) Substitute $t_1 = -3$ and $r = -4$ into $t_n = t_1r^{n-1}$.

$t_2 = -3(-4)^1$ $t_3 = -3(-4)^2$ $t_4 = -3(-4)^3$

$t_2 = 12$ $t_3 = -48$ $t_4 = 192$

The first four terms of the sequence are −3, 12, −48, 192.

c) Substitute $t_1 = 4$ and $r = -3$ into $t_n = t_1r^{n-1}$.

$t_2 = 4(-3)^1$ $t_3 = 4(-3)^2$ $t_4 = 4(-3)^3$

$t_2 = -12$ $t_3 = 36$ $t_4 = -108$

The first four terms of the sequence are 4, −12, 36, −108.

d) Substitute $t_1 = 2$ and $r = 0.5$ into $t_n = t_1r^{n-1}$.

$t_2 = 2(0.5)^1$ $t_3 = 2(0.5)^2$ $t_4 = 2(0.5)^3$

$t_2 = 1$ $t_3 = 0.5$ $t_4 = 0.25$

The first four terms of the sequence are 2, 1, 0.5, 0.25.

Section 1.3 Page 39 Question 4

Substitute $t_1 = 8.1$, $n = 5$, and $t_n = 240.1$ into $t_n = t_1r^{n-1}$.

$$240.1 = 8.1r^4$$
$$r^4 = \frac{240.1}{8.1}$$
$$r = \sqrt[4]{\frac{240.1}{8.1}}$$
$$r = 2.333...$$

$t_2 = 8.1(2.333\ldots)^1$ $t_3 = 8.1(2.333\ldots)^2$ $t_4 = 8.1(2.333\ldots)^3$

$t_2 = 18.9$ $t_3 = 44.1$ $t_4 = 102.9$

The missing terms, t_2, t_3, and t_4, are 18.9, 44.1, and 102.9.

Section 1.3 Page 39 Question 5

a) Substitute $r = 2$ and $t_1 = 3$ into $t_n = t_1r^{n-1}$.

$t_n = 3(2)^{n-1}$

b) From the pattern of the sequence, $t_1 = 192$ and r = $-\frac{1}{4}$. Substitute into $t_n = t_1r^{n-1}$.

$$t_n = 192\left(-\frac{1}{4}\right)^{n-1}$$

c) Substitute into $t_n = t_1r^{n-1}$.

For t_3:

$5 = t_1r^2$

$t_1 = \frac{5}{r^2}$

For t_6:

$135 = t_1r^5$

$135 = \left(\frac{5}{r^2}\right)r^5$

$135 = 5r^3$

$r = \sqrt[3]{27}$

$r = 3$

Then, $t_1 = \frac{5}{r^2} = \frac{5}{9}$.

So, $t_n = \frac{5}{9}(3)^{n-1}$.

d) Substitute $t_1 = 4$, $n = 13$, and $t_n = 16\ 384$ into $t_n = t_1r^{n-1}$.

$16\ 384 = 4r^{12}$

$r^{12} = 4096$

$r = \sqrt[12]{4096}$

$r = 2$

So, $t_n = 4(2)^{n-1}$.

Section 1.3 Page 39 Question 6

a) Substitute $t_1 = 5$, $r = 3$, and $t_n = 135$ into $t_n = t_1r^{n-1}$.

$135 = 5(3)^{n-1}$

$3^{n-1} = 27$

$3^{n-1} = 3^3$

So, $n - 1 = 3$

$n = 4$

The number of terms, n, is 4.

b) Substitute $t_1 = -2$, $r = -3$, and $t_n = -1458$ into $t_n = t_1r^{n-1}$.

$-1458 = -2(-3)^{n-1}$

$729 = (-3)^{n-1}$

$(-3)^6 = (-3)^{n-1}$

So, $6 = n - 1$

$n = 7$

The number of terms, n, is 7.

c) Substitute $t_1 = \frac{1}{3}$, $r = \frac{1}{2}$, and $t_n = \frac{1}{48}$ into $t_n = t_1r^{n-1}$.

$$\frac{1}{48} = \frac{1}{3}\left(\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^{4} = \left(\frac{1}{2}\right)^{n-1}$$

So, $4 = n - 1$

$n = 5$

The number of terms, n, is 5.

d) Substitute $t_1 = 4$, $r = 4$, and $t_n = 4096$ into $t_n = t_1r^{n-1}$.

$4096 = 4(4)^{n-1}$

$1024 = (4)^{n-1}$

$(4)^5 = (4)^{n-1}$

So, $5 = n - 1$

$n = 6$

The number of terms, n, is 6.

e) Substitute $t_1 = -\frac{1}{6}$, $r = 2$, and $t_n = -\frac{128}{3}$ into $t_n = t_1r^{n-1}$.

$$-\frac{128}{3} = -\frac{1}{6}(2)^{n-1}$$

$$-6\left(-\frac{128}{3}\right) = -6\left(-\frac{1}{6}(2)^{n-1}\right)$$

$$256 = 2^{n-1}$$

$$2^8 = 2^{n-1}$$

So, comparing exponents, $n = 9$.

The number of terms, n, is 9.

f) Substitute $t_1 = \frac{p^2}{2}$, $r = \frac{p}{2}$, and $t_n = \frac{p^9}{256}$ into $t_n = t_1r^{n-1}$.

$$\frac{p^9}{256} = \frac{p^2}{2}\left(\frac{p}{2}\right)^{n-1}$$

$$\frac{p^7}{128} = \left(\frac{p}{2}\right)^{n-1}$$

$$\left(\frac{p}{2}\right)^{7} = \left(\frac{p}{2}\right)^{n-1}$$

So, $7 = n - 1$

$n = 8$

The number of terms, n, is 8.

Section 1.3 Page 40 Question 7

For a geometric sequence, successive terms have the same ratio.

So, $\frac{5y+7}{48}=\frac{12}{3}$

$$\begin{aligned}5y+7&=48(4)\\5y+7&=192\\5y&=192-7\\5y&=185\\y&=37\end{aligned}$$

The value of y is 37.

Section 1.3 Page 40 Question 8

The first three terms are $t_1 = 16$, $t_2 = 12$, and $t_3 = 9$.
For a geometric sequence, successive terms have the same ratio.

$r=\frac{12}{16}$

$r=\frac{3}{4}$ or 0.75

Then, substitute in $t_n = t_1r^{n-1}$.

$t_n=16\left(\frac{3}{4}\right)^{n-1}$

Section 1.3 Page 40 Question 9

a) $t_1 = 3.0$, $r = 0.75$

b) Substitute $t_1 = 3.0$, $r = 0.75$ in $t_n = t_1r^{n-1}$.
$t_n = 3.0(0.75)^{n-1}$

c) After the 6th bounce the ball will reach its 7th height. Substitute $n = 7$ in $t_n = t_1r^{n-1}$.
$t_7 = 3.0(0.75)^6$
$t_7 = 0.5339……$
After the sixth bounce the ball reaches a height of about 0.53 m.

d) Find n when $t_n = 40$ cm.

$$\begin{aligned}0.40&=3.0(0.75)^{n-1}\\\frac{0.40}{3.0}&=(0.75)^{n-1}\\0.133\ 333...&=(0.75)^{n-1}\end{aligned}$$

Try values for n:

$n = 6$ $(0.75)^5 = 0.237…$ Too big.
$n = 8$ $(0.75)^7 = 0.133\ 48…$Close, but a little too big.
$n = 9$ $(0.75)^8 = 0.100…$ Too small.
After 7 bounces the ball will reach a height of approximately 40 cm.

Section 1.3 Page 40 Question 10

a) If 5% of the colour is lost, 95% remains after one washing.

b) $t_1 = 100$, $t_2 = 95$, $t_3 = 95(0.95)$ or 90.25, $t_4 = 90.25(0.95)$ or 85.7375
The first four terms of the sequence are 100, 95, 90.25, and 85.7375.

c) $r = 0.95$

d) For this situation, $t_n = 100(0.95)^{n-1}$. For 10 washings, $n = 11$.
So, $t_{11} = 100(0.95)^{10}$
$t_{11} = 59.8736…$
After 10 washings, approximately 60% of the colour remains.

e) Find the value of n, for which $t_n = 25$.

$$25 = 100(0.95)^{n-1}$$
$$0.25 = (0.95)^{n-1}$$

Try values for n:
$n = 17$ $(0.95)^{16} = 0.440…$ Too big.
$n = 26$ $(0.95)^{25} = 0.277…$ Close, but still too big.
$n = 28$ $(0.95)^{27} = 0.250…$ Ok.
After 27 washings only 25% of the original colour remains.
An assumption is that washing conditions remain the same and the jeans don't get faded another way such as being out in the sun a lot.

Section 1.3 Page 40 Question 11

Consider 2004 to be t_1, so $t_1 = 326$.
Then, 2010 is t_7, and $t_7 = 10\ 000$.
Substitute into $t_n = t_1r^{n-1}$.

$$10\ 000 = 326r^{7-1}$$
$$\frac{10\ 000}{326} = r^6$$
$$r = \sqrt[6]{\frac{10\ 000}{326}}$$
$$r = 1.769...$$

The value of the annual rate of growth from 2004 to 2010 was approximately 1.77.

Section 1.3 Page 41 Question 12

a) The sequence of terms for the first five days is 1, 2, 4, 8, 16.

b) $t_n = 2^{n-1}$

c) $t_{30} = 2^{29}$
$t_{30} = 536\ 870\ 912$
On day 30, Rani would receive 536 870 912 grains of rice.

Section 1.3 Page 41 Question 13

a) $t_1 = 191.41$, $t_2 = 197.34$, $t_3 = 203.46$

$r = \dfrac{197.34}{191.41}$ or $r = \dfrac{203.46}{197.34}$

$r = 1.0309...$ $\quad r = 1.0310...$

Each jump Georges improved his performance by a ratio of 1.031, to three decimal places.

b) Find t_5.
Substitute in $t_n = t_1r^{n-1}$.
$t_5 = 191.41(1.031)^4$
$t_5 = 216.271\ldots$
Georges' winning jump was 216.3 cm, to the nearest tenth.

c) Find n when $t_n = 10\ 200$

$1020 = 197.41(1.031)^{n-1}$

$\dfrac{1020}{197.41} = (1.031)^{n-1}$

$5.166\ 911... = (1.031)^{n-1}$

Try values for n:
$n = 51$ $\quad (1.031)^{50} = 4.601\ldots$ Too small.
$n = 61$ $\quad (1.031)^{60} = 6.244\ldots$ Too big.
$n = 56$ $\quad (1.031)^{55} = 5.360\ldots$ Still a bit too big.
$n = 55$ $\quad (1.031)^{54} = 5.199$ Too small.
If Georges continued to increase his jumps in the same geometric sequence, he would beat Santjie's record on the 56th jump.

Section 1.3 Page 41 Question 14

a) The cell growth of yeast follows the sequence 1, 2, 4, 8, 16, 32.

b) $t_n = 2^{n-1}$

c) Substitute $n = 26$, for 25 doublings.
$t_{26} = 2^{25}$

$t_{26} = 33\ 554\ 432$
After 25 doublings, there would be 33 554 432 cells.

d) The assumption is that all cells continue living.

Section 1.3 Page 42 Question 15

$t_1 = 700$, $t_{38} = 2000$

$$t_n = t_1 r^{n-1}$$
$$2000 = 700r^{37}$$
$$r = \sqrt[37]{\frac{2000}{700}}$$
$$r = 1.0287...$$

The growth rate was 2.9%, to the nearest tenth of a percent.

Section 1.3 Page 42 Question 16

$t_1 = 2$, $t_2 = 4$, $t_3 = 8$
The given terms form a geometric sequence with $t_1 = 2$ and $r = 2$.
So, $t_n = 2(2)^{n-1}$.
Determine n when $t_n = 142$.
$142 = 2(2)^{n-1}$
$71 = (2)^{n-1}$
Test values of n:

Try $n = 8$	$2^7 = 128$	Too big.
Try $n = 7$	$2^6 = 64$	Too small.

The required number, 71, is between 7 and 8. So, it took Jason 8 weeks to reach his competition number of 142 sledges.

Section 1.3 Page 42 Question 17

$r = 0.96$, $t_{20} = 30$
Solve for t_1 in $t_n = t_1(0.96)^{n-1}$, when $n = 20$.
$30 = t_1(0.96)^{19}$

$$t_1 = \frac{30}{(0.96)^{19}}$$
$$t_1 = 65.158...$$

The arc length for the first swing is 65.2 m, to the nearest tenth of a metre.

Section 1.3 Page 43 Question 18

$t_1 = 60$, $t_{50} = 1$
Substitute and solve for r in $t_n = t_1r^{n-1}$.

$$1 = 60(r^{50-1})$$
$$60 = r^{49}$$
$$r = \sqrt[49]{\frac{1}{60}}$$
$$r = 0.9198...$$

The common ratio for the decrease in doll size is 0.920, to three decimal places.

Section 1.3 Page 43 Question 19

a) $t_1 = 250$, $r = 100\% - 18\%$ or 0.82, $n = \frac{12}{2}$ or 6
Since the amount is decreasing every 2 h, we need to use $n = 7$. Substitute and solve for t_7 in $t_n = t_1r^{n-1}$.

$$t_7 = 250(0.82)^6$$
$$t_7 = 76.001\ldots$$

After 12 h approximately 76.0 mL of the medicine remains.

b) Substitute and solve for n, when $t_n = 20$.

$$20 = 250(0.82)^{n-1}$$
$$\frac{20}{250} = (0.82)^{n-1}$$
$$0.08 = (0.82)^{n-1}$$

Test values for n.

Try $n = 20$	$(0.82)^{19} = 0.023\ldots$	Too low.
Try $n = 10$	$(0.82)^9 = 0.167\ldots$	Too high.
Try $n = 13$	$(0.82)^{12} = 0.0924\ldots$	Close, but a bit too high.
Try $n = 14$	$(0.82)^{13} = 0.0757\ldots$	Is now less than the target.

The reduction takes place every 2 h, so the amount will be less than 20 mL after approximately 26 h.

Section 1.3 Page 43 Question 20

a)

Time, *d* (days)	**Charge Level, *C* (%)**
0	100
1	$100(0.98) = 98$
2	$100(0.98)^2 = 96.04$
3	$100(0.98)^3 = 94.1192$

b) $t_n = 100(0.98)^{n-1}$

c) In the formula in part b), n represents the number of times the 2% reduction has occurred plus one…i.e. $n = 2$ for the first 2% reduction. We could say "$n – 1$" represents the number of reductions. In the formula $C = 100(0.98)^d$, d is the day number. So, $d = n − 1$ or $n = d + 1$.

d) $t_{11} = 100(0.98)^{10}$
$t_{11} = 81.7\,072…$
After 10 days, approximately 81.7% of the batteries charge remains.

Section 1.3 Page 43 Question 21

a) $t_1 = 6$, $r = 1.22$
Substitute in $t_n = t_1r^{n-1}$ to find t_8.
$t_8 = 6(1.22)^7$
$t_8 = 24.136…$
The radius of the 8th coil is approximately 24.14 mm.

b) First find the radius of the 18th coil.
Substitute in $t_n = t_1r^{n-1}$ to find t_{18}.
$t_{18} = 6(1.22)^{17}$
$t_{18} = 176.306\,519…$
Then, use the formula for the circumference $C = 2\pi r$.
$C = 2\pi(176.306\,519…)$
$C = 1107.766…$
The circumference of the top of the basket is approximately 1107.77 mm.

Section 1.3 Page 44 Question 22

If $6^a, 6^b, 6^c,…$ forms a geometric sequence, then the ratio of successive terms is constant.

So, $$\frac{6^b}{6^a} = \frac{6^c}{6^b}$$
$$6^{b-a} = 6^{c-b}$$
$$b - a = c - b$$

In words, this means that there is a constant difference between the exponents which means they form an arithmetic sequence.

Section 1.3 Page 44 Question 23

If $x + 2, 2x + 1, 4x − 3,…$ forms a geometric sequence, then the ratio of successive terms is constant.

$$\frac{2x+1}{x+2} = \frac{4x-3}{2x+1}$$
$$(2x + 1)(2x + 1) = (4x − 3)(x + 2)$$
$$4x^2 + 4x + 1 = 4x^2 + 5x − 6$$
$$x = 7$$

So the terms are 9, 15, and 25 and the common ratio is $\frac{5}{3}$.

Section 1.3 Page 44 Question 24

Some answers may vary due to rounding of the value of r.

a) $t_1 = 38$, $t_2 = 35.87$, $t_3 = 33.86$

First determine r:

$\frac{33.86}{35.87} = 0.943\ 96...$ $\frac{35.87}{38} = 0.943\ 94...$

So, an approximation for r is 0.9439.

Now substitute in $t_n = t_1r^{n-1}$ to find t_9.

$t_9 = 38(0.9439)^8$

$t_9 = 23.943\ldots$

The 8th fret is approximately 23.94 cm from the bridge.

b) Substitute in $t_n = t_1r^{n-1}$ to find t_{13}.

$t_{13} = 38(0.9439)^{12}$

$t_{13} = 19.006\ldots$

The 12th fret is approximately 19.01 cm from the bridge.

c) $38 - 35.87 = 2.13$

The distance from the nut to the first fret is 2.13 cm.

d) $35.87 - 33.86 = 2.01$

The distance from the first fret to the second fret is 2.01 cm.

e) First, find the distance from the bridge to the third and fourth frets.

Substitute in $t_n = t_1r^{n-1}$ to find t_4.

$t_4 = 38(0.9439)^3$

$t_4 = 31.9566\ldots$

The third fret is approximately 31.96 cm from the bridge.

Substitute in $t_n = t_1r^{n-1}$ to find t_5.

$t_5 = 38(0.9439)^4$

$t_4 = 30.1639\ldots$

The fourth fret is approximately 30.16 cm from the bridge.

Distance between 2nd and 3rd fret is $33.86 - 31.96$, or 1.9 cm.
Distance between 3rd and 4th fret is $31.96 - 30.16$, or 1.8 cm.

The sequence of distances between frets is 2.01, 1.9, 1.8.
Their common difference is not constant, so the sequence is not arithmetic.

Check the ratio of successive terms:

$\frac{1.8}{1.9} = 0.947\ 368...$ $\frac{1.9}{2.01} = 0.945\ 273...$

The ratio of successive terms is similar – the differences may be due to rounding errors.

Section 1.3 Page 44 Question 25

Mala's solution is correct. Alex's method is fine, but he did not determine the correct value for r. If the aquarium loses 8% each day, then the next term is 92% of the previous term. This means $r = 0.92$. Paul's method is incorrect because the water is evaporating at a constant rate, not a constant amount, each day. The sequence is geometric.

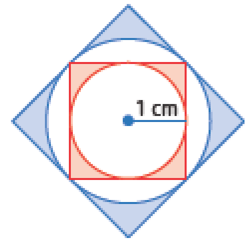
Section 1.3 Page 45 Question 26

	$\frac{1}{500}$			$\frac{50}{3}$				
	$\frac{1}{100}$	$\frac{1}{10}$	1	10				
	$\frac{1}{20}$		2	6	18	54		
$\frac{1}{16}$	$\frac{1}{4}$	1	4			9		
	$\frac{5}{4}$		8			$\frac{3}{2}$		
	$\frac{25}{4}$		16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
	$\frac{125}{4}$		32					
	$\frac{625}{4}$	100	64					

Section 1.3 Page 45 Question 27

a) Area of red shaded portion = area of square − area of circle
Area $= 2 \times 2 - \pi(1)^2$
Area $= 4 - \pi$
Area $= 0.8584\ldots$
The area of the red shaded portion is 0.86 cm^2, to the nearest hundredth of a square centimetre.

b) First find the side length of the blue square.
Consider one right triangle that has the side of the red square as its hypotenuse. Let each of its equal legs be x.
Use the Pythagorean Theorem.



$$x^2 + x^2 = 2^2$$
$$2x^2 = 4$$
$$x = \sqrt{2}$$

Then, the side length of the blue square is double x, or $2\sqrt{2}$ cm.

Area of blue shaded portion = area of larger square − area of larger circle
Area = $2\sqrt{2} \times 2\sqrt{2} - \pi(\sqrt{2})^2$
Area = $8 - 2\pi$
Area = 1.7168…
The area of the blue shaded portion is 1.72 cm^2, to the nearest hundredth of a square centimetre.

c) As in part b), find the side length of the orange square by considering one of the right triangles on its corners.
Let y represent each equal leg length.

$$y^2 + y^2 = (2\sqrt{2})^2$$
$$2y^2 = 8$$
$$y^2 = 4$$
$$y = 2$$

Then, area of orange shaded region = area of orange square − area of largest circle
Area = $4 \times 4 - \pi(2)^2$
Area = 3.4336…

The area of the orange shaded region is 3.43 cm^2, to the nearest hundredth of a square centimetre.

d) Explore whether the area is an arithmetic or geometric sequence.
$t_1 = 0.86$, $t_2 = 1.72$, $t_3 = 3.43$
This is not an arithmetic sequence, as the difference between successive terms is not constant.
Consider the ratio of successive terms:

$$\frac{1.72}{0.86} = 2 \qquad \frac{3.43}{1.72} = 1.994\ 186...$$

Allowing for rounding errors, it appears that each area is 2 times the previous area.
In $t_n = t_1r^{n-1}$, substitute $r = 2$, $t_1 = 4 - \pi$ and $n = 8$.
$t_8 = (4 - \pi)(2)^7$
$t_8 = 109.876…$
The area of the newly shaded part of the 8th square would be 109.88 cm^2, to the nearest hundredth of a square centimetre.

Section 1.4 Geometric Series

Section 1.4 Page 53 Question 1

a) The series 4 + 24 + 144 + 864 + … is geometric because each term is 6 times the previous term.

b) The series −40 + 20 − 10 + 5 − … is geometric because each term is −0.5 times the previous term.

c) The series 3 + 9 + 18 + 54 + … is not geometric.

d) The series 10 + 11 + 12.1 + 13.31 + … is geometric because each term is 1.1 times the previous term.

Section 1.4 Page 53 Question 2

a) $t_1 = 6$, $r = \frac{3}{2}$, $n = 10$

Substitute into $S_n = \frac{t_1(r^n - 1)}{r - 1}$ to find S_{10}.

$$S_{10} = \frac{6\left[\left(\frac{3}{2}\right)^{10} - 1\right]}{\frac{3}{2} - 1}$$

$$S_{10} = \frac{6\left[\frac{59\ 049}{1024} - 1\right]}{\frac{1}{2}}$$

$$S_{10} = 6\left[\frac{58\ 025}{1024}\right](2)$$

$$S_{10} = \frac{174\ 075}{256}$$

$S_{10} = 679.98$, to the nearest hundredth.

b) $t_1 = 18$, $r = -\frac{1}{2}$, $n = 12$

Substitute into $S_n = \frac{t_1(r^n - 1)}{r - 1}$ to find S_{12}.

$$S_{12} = \frac{18\left[\left(-\frac{1}{2}\right)^{12} - 1\right]}{\left(-\frac{1}{2}\right) - 1}$$

$$S_{12} = \frac{18\left[\frac{1}{4096} - 1\right]}{\frac{-3}{2}}$$

$$S_{12} = \cancel{18}^{\cancel{-6}^{-3}}\left[-\frac{4095}{\cancel{4096}_{\cancel{2048}_{1024}}}\right]\left(\frac{\cancel{2}}{\cancel{-3}}\right)$$

$$S_{12} = \frac{12\ 285}{1024}$$

S_{12} = 12.00, to the nearest hundredth.

c) $t_1 = 2.1$, $r = 2$, $n = 9$

Substitute into $S_n = \frac{t_1(r^n - 1)}{r - 1}$ to find S_9.

$$S_9 = \frac{2.1(2^9 - 1)}{2 - 1}$$

$$S_9 = 1073.1$$

$$S_9 = \frac{10\ 731}{10}$$

d) $t_1 = 0.3$, $r = 0.01$, $n = 12$

Substitute into $S_n = \frac{t_1(r^n - 1)}{r - 1}$ to find S_{12}.

$$S_{12} = \frac{0.3[(0.01)^{12} - 1]}{0.01 - 1}$$

$$S_{12} = \frac{-0.3}{-0.99}$$

$$S_{12} = \frac{10}{33}$$

S_{12} = 0.30, to nearest hundredth.

Section 1.4 Page 53 Question 3

a) Substitute $t_1 = 12$, $r = 2$, $n = 10$ into $S_n = \dfrac{t_1(r^n - 1)}{r - 1}$.

$$S_{10} = \frac{12(2^{10} - 1)}{2 - 1}$$
$$S_{10} = 12(1023)$$
$$S_{10} = 12\ 276$$

b) Substitute $t_1 = 27$, $r = \dfrac{1}{3}$, $n = 8$ into $S_n = \dfrac{t_1(r^n - 1)}{r - 1}$.

$$S_8 = \frac{27\left[\left(\frac{1}{3}\right)^8 - 1\right]}{\frac{1}{3} - 1}$$
$$S_8 = 27\left[\frac{1}{6561} - 1\right]\left(-\frac{3}{2}\right)$$
$$S_8 = \cancel{27}\left(\frac{\overset{3280}{\cancel{-6560}}}{\underset{\underset{81}{\cancel{2187}}}{\cancel{6561}}}\right)\left(\frac{\cancel{3}}{\cancel{-2}}\right)$$
$$S_8 = \frac{3280}{81}$$

c) Substitute $t_1 = \dfrac{1}{256}$, $r = -4$, $n = 10$ into $S_n = \dfrac{t_1(r^n - 1)}{r - 1}$.

$$S_{10} = \frac{\left(\frac{1}{256}\right)[(-4)^{10} - 1]}{-4 - 1}$$
$$S_{10} = \frac{\left(\frac{1}{256}\right)1\ 048\ 575}{-5}$$
$$S_{10} = \left(\frac{1\ 048\ 575}{256}\right)\left(\frac{-1}{5}\right)$$
$$S_{10} = -\frac{209\ 715}{256}$$

d) Substitute $t_1 = 72$, $r = \frac{1}{2}$, $n = 12$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$S_{12} = \frac{72\left[\left(\frac{1}{2}\right)^{12} - 1\right]}{\frac{1}{2} - 1}$$

$$S_{12} = \frac{\left(72\left[\frac{1}{4096} - 1\right]\right)}{-\frac{1}{2}}$$

$$S_{12} = \overset{9}{\cancel{72}}\left(\frac{-4095}{\underset{\underset{-256}{\cancel{512}}}{\cancel{4096}}}\right)(\cancel{-2})$$

$$S_{12} = \frac{36\ 855}{256}$$

Section 1.4 Page 53 Question 4

a) Substitute $t_1 = 27$, $t_n = \frac{1}{243}$, $r = \frac{1}{3}$ into $S_n = \frac{rt_n - t_1}{r - 1}$.

$$S_n = \frac{\left(\frac{1}{3}\right)\left(\frac{1}{243}\right) - 27}{\frac{1}{3} - 1}$$

$$S_n = \left[\left(\frac{1}{729}\right) - 27\right]\left(-\frac{3}{2}\right)$$

$$S_n = \left(-\frac{19\ 682}{729}\right)\left(-\frac{3}{2}\right)$$

$$S_n = \frac{9841}{243} \text{ or } S_n \approx 40.50$$

b) Substitute $t_1 = \frac{1}{3}$, $t_n = \frac{128}{6561}$, $r = \frac{2}{3}$ into $S_n = \frac{rt_n - t_1}{r-1}$.

$$S_n = \frac{\left(\frac{2}{3}\right)\left(\frac{128}{6561}\right) - \frac{1}{3}}{\frac{2}{3} - 1}$$

$$S_n = \left(\frac{-6305}{19\ 683}\right)(-3)$$

$$S_n = \frac{6305}{6561} \text{ or } S_n \approx 0.96$$

c) Substitute $t_1 = 5$, $t_n = 81\ 920$, $r = 4$ into $S_n = \frac{rt_n - t_1}{r-1}$.

$$S_n = \frac{4(81\ 920) - 5}{4 - 1}$$

$$S_n = 109\ 225$$

d) Substitute $t_1 = 3$, $t_n = 46\ 875$, $r = -5$ into $S_n = \frac{rt_n - t_1}{r-1}$.

$$S_n = \frac{-5(46\ 875) - 3}{-5 - 1}$$

$$S_n = 39\ 063$$

Section 1.4 Page 54 Question 5

a) Substitute $S_n = 33$, $t_n = 48$, $r = -2$ into $S_n = \frac{rt_n - t_1}{r-1}$.

$$33 = \frac{-2(48) - t_1}{-2 - 1}$$

$$3(33) = 96 + t_1$$

$$99 - 96 = t_1$$

$$t_1 = 3$$

b) Substitute $S_n = 443$, $n = 6$, $r = \frac{1}{3}$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$443 = \frac{t_1\left[\left(\frac{1}{3}\right)^6 - 1\right]}{\frac{1}{3} - 1}$$

$$443 = t_1\left(\frac{1}{729} - 1\right)\left(-\frac{3}{2}\right)$$

$$443 = t_1\left(\frac{-728}{729}\right)\left(-\frac{3}{2}\right)$$

$$443 = t_1\left(\frac{364}{243}\right)$$

$$t_1 = 443\left(\frac{243}{364}\right)$$

$$t_1 \approx 295.7$$

Section 1.4 Page 54 Question 6

Substitute $S_n = 4372$, $t_1 = 4$, $r = 3$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$4372 = \frac{4(3^n - 1)}{3 - 1}$$

$$4372 = 2(3^n - 1)$$

$$2186 = 3^n - 1$$

$$2187 = 3^n$$

$$3^7 = 3^n$$

$$n = 7$$

There are seven terms in the series.

Section 1.4 Page 54 Question 7

a) Substitute $S_n = 121$, $r = \frac{1}{3}$, and $n = 5$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$121 = \frac{t_1\left[\left(\frac{1}{3}\right)^5 - 1\right]}{\frac{1}{3} - 1}$$

$$121 = t_1\left(\frac{1}{243} - 1\right)\left(-\frac{3}{2}\right)$$

$$121 = t_1\left(\frac{-242}{243}\right)\left(-\frac{3}{2}\right)$$

$$121 = t_1\left(\frac{121}{81}\right)$$

$$t_1 = 81$$

b) The first five terms of the series are 81 + 27 + 9 + 3 + 1.

Section 1.4 Page 54 Question 8

Substitute $t_3 = \frac{9}{4}$ and $t_6 = -\frac{16}{81}$ into $t_6 = t_3(r^3)$.

$$-\frac{16}{81} = \left(\frac{9}{4}\right)r^3$$

$$r^3 = -\frac{16}{81}\left(\frac{4}{9}\right)$$

$$r = -\frac{4}{9}$$

Then, in $t_n = t_1 r^{n-1}$ substitute $n = 3$, $t_3 = \frac{9}{4}$, and $r = -\frac{4}{9}$.

$$\frac{9}{4} = t_1\left(-\frac{4}{9}\right)^2$$

$$\left(\frac{9}{4}\right)\left(\frac{9^2}{4^2}\right) = t_1$$

$$t_1 = \frac{9^3}{4^3} \text{ or } \frac{729}{64}$$

Then, $t_2 = \left(\frac{9^3}{4^3}\right)\left(-\frac{4}{9}\right)$ or $-\frac{81}{16}$.

Substitute to find S_6 using $S_n = \frac{rt_n - t_1}{r - 1}$.

$$S_6 = \frac{\left(-\frac{4}{9}\right)\left(-\frac{16}{81}\right) - \left(\frac{729}{64}\right)}{\left(-\frac{4}{9}\right) - 1}$$

$$S_6 = \left(\frac{64}{729} - \frac{729}{64}\right)\left(-\frac{9}{13}\right)$$

$$S_6 = 7.825...$$

The sum of the first six terms is approximately 7.8.

Section 1.4 Page 54 Question 9

a) Including the person in charge, the series is 1 + 4 + 16 + 64 + …

b) Substitute $t_1 = 1$, $r = 4$, $n = 10$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

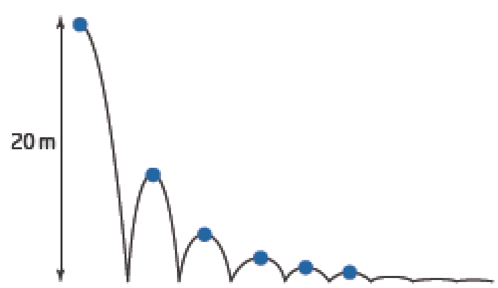
$$S_{10} = \frac{1(4^{10} - 1)}{4 - 1}$$

$$S_{10} = \frac{1\ 048\ 576}{3}$$

$$S_{10} \approx 349\ 525$$

After 10 levels, 349 525 people are notified.

Section 1.4 Page 54 Question 10



$t_1 = 20$, $t_2 = 2(0.4)20$ or 16, $t_3 = 2(0.4)^2(20)$ or 6.4

Determine the sum of the series of up and down bounces, 16 + 6.4 +…. Then, add the first drop of 20 m to the sum.

Substitute $t_1 = 16$, $r = 0.4$, $n = 5$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$S_5 = \frac{16[(0.4)^5 - 1]}{0.4 - 1}$$

$$S_5 = 26.3936$$

The total distance travelled when the ball hits the ground for the sixth time is 20 m + 26.39 m or 46.4 m, to the nearest tenth of a metre.

Section 1.4 Page 54 Question 11

Substitute $t_1 = 25$, $r = 1.1$, $n = 15$ into $S_n = \dfrac{t_1(r^n - 1)}{r - 1}$.

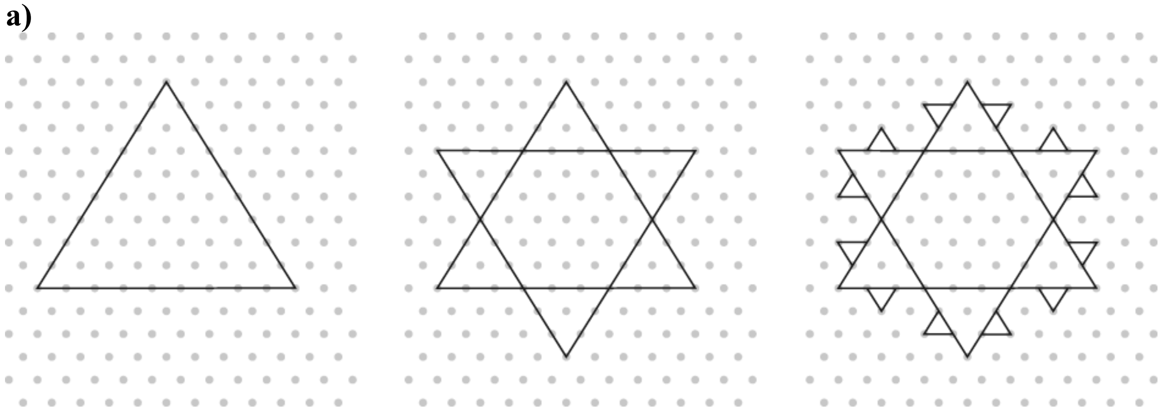
$$S_{15} = \frac{25[(1.1)^{15} - 1]}{1.1 - 1}$$

$$S_{15} = 794.312...$$

By the end of the 15th week, Celia will have run 794.3 km, to the nearest tenth of a kilometre.

Section 1.4 Page 54 Question 12

a)



Stage 1 Stage 2 Stage 3

b)

Stage Number	Length of Each Line Segment	Number of Line Segments	Perimeter of Snowflake
1	1	3	3
2	$\frac{1}{3}$	12	4
3	$\frac{1}{9}$	48	$\frac{16}{3}$
4	$\frac{1}{27}$	192	$\frac{64}{9}$
5	$\frac{1}{81}$	768	$\frac{256}{27}$

c) In each of the following general terms, n is the stage number.

Length of each line segment: $t_n = \left(\frac{1}{3}\right)^{n-1}$

Number of line segments: $t_n = 3(4)^{n-1}$

Perimeter of snowflake $= 3\left(\frac{4}{3}\right)^{n-1}$

d) When $n = 6$, perimeter $= 3\left(\frac{4}{3}\right)^{5}$ or $\frac{1024}{81}$.

The total perimeter at stage 6 is $\frac{1024}{81}$, or approximately 12.64.

Section 1.4 Page 55 Question 13

Substitute $t_1 = 1000(1.4)$, $r = 1.4$, and $n = \frac{100}{10}$ or 10 into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$S_{10} = \frac{1000(1.4)[(1.4)^{10} - 1]}{1.4 - 1}$$
$$S_{10} = 97\ 739.129\ldots$$

After 100 days, 98 739 people will be aware of the product.

Section 1.4 Page 55 Question 14

Substitute $t_1 = 24$, $r = \frac{3}{4}$ or 0.75, and $n = 10$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$S_{10} = \frac{24[(0.75)^{10} - 1]}{0.75 - 1}$$
$$S_{10} = 90.593\ldots$$

The total length of the line of 10 beads was 91 mm, to the nearest millimetre.

Section 1.4 Page 56 Question 15

a) $t_1 = 200$, $t_2 = 200 + 200(0.12)$
$t_3 = 200 + [200 + 200(0.12)](0.12)$
$t_3 = 200 + 200(0.12) + 200(0.12)^2$
$t_3 = 226.88$
The amount of ampicillin in the body after taking the third tablet is 226.9 mg, to the nearest tenth.

b) Substitute $t_1 = 200$, $r = 0.12$, $n = 6$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$S_6 = \frac{200[(0.12)^6 - 1]}{0.12 - 1}$

$S_6 = 227.272...$

The amount of ampicillin in the body after taking the sixth tablet is 227.3 mg, to the nearest tenth.

Section 1.4 Page 56 Question 16

Substitute $t_1 = 3$, $r = 3$, and $S_n = 9840$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$\begin{aligned}
9840 &= \frac{3(3^n - 1)}{3 - 1} \\
\frac{2(9840)}{3} &= 3^n - 1 \\
6560 + 1 &= 3^n \\
6561 &= 3^n \\
3^8 &= 3^n \\
n &= 8
\end{aligned}$$

The series has eight terms.

Section 1.4 Page 56 Question 17

$t_3 = t_1(r^2) = 24$ and $t_4 = t_1(r^3) = 36$

So, $r = \frac{36}{24}$ or $\frac{3}{2}$.

Then, substituting in t_3:

$$\begin{aligned}
t_1\left(\frac{3}{2}\right)^2 &= 24 \\
t_1 &= \frac{24(4)}{9} \\
t_1 &= \frac{32}{3}
\end{aligned}$$

Substitute $t_1 = \frac{32}{3}$, $r = \frac{3}{2}$, and $n = 10$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$S_{10}=\frac{\frac{32}{3}\left[\left(\frac{3}{2}\right)^{10}-1\right]}{\frac{3}{2}-1}$$

$$S_{10}=\frac{32}{3}\left[\frac{59\ 049}{1024}-1\right](2)$$

$$S_{10}=\frac{\cancel{32}}{3}\left(\frac{58\ 025}{\underset{\cancel{32}\ 16}{\cancel{1024}}}\right)(\cancel{2})$$

$$S_{10}=\frac{58\ 025}{48}$$

The sum of the first 10 terms is $\frac{58\ 025}{48}$.

Section 1.4 Page 56 Question 18

$t_1 = a$, $t_2 = ar$, $t_3 = ar^2$

So, $a + ar + ar^2 = 35$ and $a(ar)(ar^2) = 1000$

$a(1 + r + r^2) = 35$ ① $a^3r^3 = 1000$ ②

From equation ②:

$$a=\sqrt[3]{\frac{1000}{r^3}}$$

$$a=\frac{10}{r}$$

Substitute into equation ①.

$$\frac{10}{r}(1+r+r^2)=35$$

$$10+10r+10r^2=35r$$

$$10r^2-25r+10=0$$

$$2r^2-5r+2=0$$

$$(2r-1)(r-2)=0$$

$$r=\frac{1}{2}\text{ or }r=2$$

Substitute $r=\frac{1}{2}$ back in $a=\frac{10}{r}$, gives $a = 20$.

Then $b = 10$ and $c = 5$.

Substitute $r = 2$ gives $a = 5$, then $b = 10$ and $c = 20$.

So the values are $a = 5$, $b = 10$ and $c = 20$ or $a = 20$, $b = 10$ and $c = 5$.

Section 1.4 Page 56 Question 19

Since the sum of the first 8 terms is 104 and the sum of the first 7 terms is 89, the 8th term must be 104 − 89 or 15.

Section 1.4 Page 56 Question 20

Circle #	1	2	3	4	5
Radius (cm)	8	4	2	1	$\frac{1}{2}$
Area (cm^2)	64π	16π	4π	π	$\frac{1}{4}\pi$

Area of the five circles = $64\pi + 16\pi + 4\pi + \pi + \frac{1}{4}\pi$

Area of the five circles is $\frac{341}{4}\pi$, or $85\frac{1}{4}\pi$ cm^2.

Section 1.4 Page 56 Question 21

Answers will vary.

<table>
<tr><td colspan="4">Sequences
• Some kind of ordered list of items or numbers
• May have an arithmetic or a geometric pattern, or some other type of pattern</td></tr>
<tr><td colspan="2">Arithmetic
• There is a common difference between successive terms.</td><td colspan="2">Geometric
• Successive pairs of terms have the same ratio, or other words each term is the same multiple (r) of the previous term</td></tr>
<tr><td>General Term
$t_n = t_1 + (n - 1)d$</td><td>Example
3, 5, 7, 9, …</td><td>General Term
$t_n = t_1 r^{n-1}$</td><td>Example
3, 6, 12, 24, …</td></tr>
</table>

<table>
<tr><td colspan="4">Series
• The sum of the terms of a sequence of numbers.</td></tr>
<tr><td colspan="2">Arithmetic
• The sum of the terms of an arithmetic sequence.</td><td colspan="2">Geometric
• The sum of the terms of a geometric sequence.</td></tr>
<tr><td>General Sum
$S_n = \frac{n}{2}[2t_1 + (n-1)d]$
$S_n = \frac{n}{2}(t_1 + t_n)$</td><td>Example
3 + 5 +7 + 9 + …
$S_4 = \frac{4}{2}[2(3) + (4-1)2]$
$S_4 = 2[6 + 3(2)]$
$S_4 = 24$</td><td>General Sum
$S_n = \frac{t_1(r^n - 1)}{r - 1}, r \neq 1$
$S_n = \frac{rt_n - t_1}{r - 1}, r \neq 1$</td><td>Example
3 + 6 + 12 + 24 + …
$S_4 = \frac{3(2^4 - 1)}{2 - 1}$
$S_4 = 3(16 - 1)$
$S_4 = 45$</td></tr>
</table>

Section 1.4 Page 57 Question 22

Answers may vary. Examples:

a) Tom is assuming that all 400 eggs do in fact produce a butterfly.

b) Tom's assumption is very optimistic. Some eggs would not survive due to weather, predators, or other unfavourable circumstances. Also he has calculated the total number in the first to fifth generations, not the fifth generation only.

c) Tom's estimate is probably way too much. See the reasons given in part b).

d) I would research the life span of the butterflies and the normal number of successful hatchings expected from 400 eggs.

Section 1.5 Infinite Geometric Series

Section 1.5 Page 63 Question 1

a) Since $r > 1$, the series is divergent.

b) Since $-1 < r < 1$, the series is convergent.

c) $r = \frac{1}{5}$; since $-1 < r < 1$, the series is convergent.

d) $r = 2$; since $r > 1$, the series is divergent.

e) $r = \left(-\frac{9}{5}\right) \div \left(\frac{27}{25}\right)$

$$r = \left(-\frac{9}{5}\right)\left(\frac{25}{27}\right)$$

$$r = -\frac{5}{3}$$

Since $r < -1$, the series is divergent.

Section 1.5 Page 63 Question 2

a) Substitute $t_1 = 8$ and $r = -\frac{1}{4}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{t_1}{1-r}$$

$$S_\infty = \frac{8}{1-\left(-\frac{1}{4}\right)}$$

$$S_\infty = 8\left(\frac{4}{5}\right)$$

$$S_\infty = \frac{32}{5} \text{ or } 6\frac{2}{5}$$

b) Since $r = \frac{4}{3}$, $r > 1$ and the series is divergent and has no infinite sum.

c) If $r = 1$, the series is not geometric. The series would just be 5 + 5 + 5 + 5 + ….
It has no infinite sum.

d) $r = \frac{1}{2}$. Substitute $t_1 = 1$ and $r = \frac{1}{2}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{1}{1-\frac{1}{2}}$$

$$S_\infty = 2$$

e) $r = -\frac{12}{5} \div 4$

$$r = -\frac{12}{5}\left(\frac{1}{4}\right)$$

$$r = -\frac{3}{5}$$

Substitute $t_1 = 4$ and $r = -\frac{3}{5}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{4}{1-\left(-\frac{3}{5}\right)}$$

$$S_\infty = 4\left(\frac{5}{8}\right)$$

$$S_8 = \frac{5}{2} \text{ or } 2\frac{1}{2}$$

Section 1.5 Page 63 Question 3

a) $0.\overline{87} = 0.878\ 787\ 87...$

$\quad = 0.87 + 0.0087 + 0.000\ 087 + ...$

Substitute $t_1 = \frac{87}{100}$ and $r = \frac{1}{100}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{\frac{87}{100}}{1-\frac{1}{100}}$$

$$S_\infty = \left(\frac{87}{100}\right)\left(\frac{100}{99}\right)$$

$$S_\infty = \frac{87}{99} \text{ or } \frac{29}{33}$$

b) $0.\overline{437} = 0.437\ 437\ 437...$

$\quad = 0.437 + 0.000\ 437 + 0.000\ 000\ 437 + ...$

Substitute $t_1 = \frac{437}{1000}$ and $r = \frac{1}{1000}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{\frac{437}{1000}}{1-\frac{1}{1000}}$$

$$S_\infty = \left(\frac{437}{1000}\right)\left(\frac{1000}{999}\right)$$

$$S_\infty = \frac{437}{999}$$

Section 1.5 Page 63 Question 4

0.999… = 0.9 + 0.09 + 0.009 + …

Substitute $t_1 = 0.9$ and $r = 0.1$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{0.9}{1-0.1}$$
$$S_\infty = \frac{0.9}{0.9}$$
$$S_\infty = 1$$

Yes, 0.999…= 1.

Section 1.5 Page 63 Question 5

a) Substitute $t_1 = 5$ and $r = \frac{2}{3}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{5}{1-\frac{2}{3}}$$
$$S_\infty = 15$$

b) Substitute $t_1 = 1$ and $r = -\frac{1}{4}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{1}{1-\left(-\frac{1}{4}\right)}$$
$$S_\infty = \frac{4}{5}$$

c) Substitute $t_1 = 7$ and $r = \frac{1}{2}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{7}{1-\frac{1}{2}}$$
$$S_\infty = 14$$

Section 1.5 Page 63 Question 6

Substitute $S_\infty = 81$ and $r = \frac{2}{3}$ into $S_\infty = \frac{t_1}{1-r}$.

$$81 = \frac{t_1}{1-\frac{2}{3}}$$
$$81 = 3t_1$$
$$t_1 = 27$$

Then, $t_2 = 27\left(\frac{2}{3}\right)$ or 18 and $t_3 = 27\left(\frac{2}{3}\right)^2$ or 12.

The first three terms of the series are 27 + 18 + 12.

Section 1.5 Page 63 Question 7

Substitute $t_1 = -8$ and $S_\infty = -\frac{40}{3}$ into $S_\infty = \frac{t_1}{1-r}$.

$$-\frac{40}{3} = \frac{-8}{1-r}$$
$$-40(1-r) = -24$$
$$40r = 40 - 24$$
$$r = \frac{16}{40}$$
$$r = \frac{2}{5}$$

$t_2 = -8\left(\frac{2}{5}\right)$ or $-\frac{16}{5}$, $t_3 = -8\left(\frac{2}{5}\right)^2$ or $-\frac{32}{25}$, $t_4 = -8\left(\frac{2}{5}\right)^3$ or $-\frac{64}{125}$

The first four terms of the series are $-8 - \frac{16}{5} - \frac{32}{25} - \frac{64}{125}$.

Section 1.5 Page 63 Question 8

a) Substitute $t_1 = 24\ 000$ and $r = 0.94$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{24\ 000}{1-0.94}$$
$$S_\infty = 400\ 000$$

If the trend continues, the lifetime production would be 400 000 barrels of crude.

b) The assumption is that the trend does continue and that the well is kept operational until it runs dry. This is not reasonable: once production is low, the well would not be profitable to operate and would probably be closed.

Section 1.5 Page 64 Question 9

Substitute $t_1 = 1$, $r = 3x$, and $S_\infty = 4$ into $S_\infty = \frac{t_1}{1-r}$.

$$4 = \frac{1}{1-3x}$$
$$4(1-3x) = 1, \qquad x \neq \frac{1}{3}$$
$$3 = 12x$$
$$x = \frac{1}{4}$$

The value of x is $\frac{1}{4}$. The first four terms of the series are $1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64}$.

Section 1.5 Page 64 Question 10

Let $t_1 = x$, then $S_\infty = 2x$. Substitute into $S_\infty = \frac{t_1}{1-r}$.

$$2x = \frac{x}{1-r}$$
$$1-r = \frac{x}{2x}$$
$$1-r = \frac{1}{2}$$
$$r = \frac{1}{2}$$

Section 1.5 Page 64 Question 11

A series is convergent if $-1 < r < 1$.

a) $r = x$, so the series is convergent if $-1 < x < 1$.

b) $r = \frac{x}{3}$, so the series is convergent if $-1 < \frac{x}{3} < 1$ or $-3 < x < 3$.

c) $r = 2x$, so the series is convergent if $-1 < 2x < 1$ or $-\frac{1}{2} < x < \frac{1}{2}$.

Section 1.5 Page 64 Question 12

The perimeter of the largest triangle is 3 cm. Each smaller triangle is half this.

Substitute $t_1 = 3$ and $r = \frac{1}{2}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{3}{1-\frac{1}{2}}$$
$$S_\infty = 6$$

The sum of the perimeters in this infinite series is 6 cm.

Section 1.5 Page 64 Question 13

Substitute $t_1 = 50$ and $r = 0.8$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{50}{1-0.8}$$
$$S_\infty = 250$$

The pendulum will swing a total of 250 cm.

Section 1.5 Page 64 Question 14

Andrew's answer is not reasonable. The series is not convergent as $r = 1.1$, which is greater than 1. The series has no sum. Just looking at the terms, each term is greater than 1 so the sum of the first five or six terms is already greater than 10.

Section 1.5 Page 64 Question 15

The pattern of vertical heights is: 16, $2(16)\left(\frac{1}{2}\right)$, $2(16)\left(\frac{1}{2}\right)^2$, $2(16)\left(\frac{1}{2}\right)^3$, $2(16)\left(\frac{1}{2}\right)^4$, …

One way to find the sum of all vertical heights is to find the sum of the downward distances and the sum of the upward distances. The downward sum has one extra term, 16.

For the downward sum, substitute $t_1 = 16$ and $r = \frac{1}{2}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{16}{1-\frac{1}{2}}$$
$$s_\infty = 32$$

For the upward sum, substitute $t_1 = 8$ and $r = \frac{1}{2}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{8}{1-\frac{1}{2}}$$

$$s_\infty = 16$$

The total vertical distance that the ball travels is 32 + 16, or 48 m.

Section 1.5 Page 64 Question 16

a) If the sequence is geometric, then $r = \frac{27}{30}$ or $\frac{9}{10}$.

Substitute into $S_n = \frac{t_1(r^n - 1)}{r-1}$ to find S_8.

$$S_8 = \frac{30\left[\left(\frac{9}{10}\right)^8 - 1\right]}{\frac{9}{10} - 1}$$

$$S_8 = 170.859...$$

After 8 times, the post is pounded 170.86 cm into the ground, to the nearest hundredth of a centimetre.

b) Substitute $t_1 = 30$ and $r = \frac{9}{10}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{30}{1-\frac{9}{10}}$$

$$S_\infty = 300$$

If the post is pounded indefinitely, it will be pounded 300 cm into the ground.

Section 1.5 Page 64 Question 17

a) Rita is correct.

b) $r = -\frac{4}{3}$ or $-1\frac{1}{3}$. Since $r < -1$ the series is divergent and has no sum.

Section 1.5 Page 64 Question 18

Substitute $t_1 = 25$ and $r = 0.8$ into $S_\infty = \frac{t_1}{1-r}$.

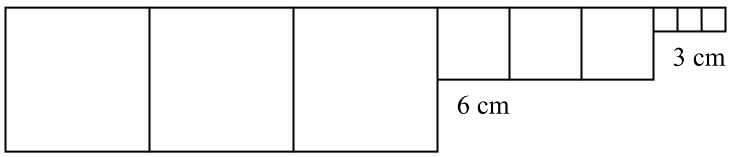
$$S_\infty = \frac{25}{1-0.8}$$

$$S_\infty = 125$$

The balloon's maximum altitude would be 125 m.

Section 1.5 Page 64 Question 19

The first three steps in the process are shown.



The length of the arrangement is $3\left[12+12\left(\frac{1}{2}\right)+12\left(\frac{1}{2}\right)^2+\ldots\right]$.

Substitute $t_1 = 12$ and $r = 0.5$ into $S_\infty = \frac{t_1}{1-r}$ and multiply the sum by 3.

$$S_\infty = \frac{12}{1-0.5}$$
$$S_\infty = 24$$

The length of the arrangement is 3(24) or 72 cm.

Section 1.5 Page 65 Question 20

Answers will vary. Example:

a) Let $z = 3$, then the pair of series are

$$\frac{1}{4}+\left(\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)^3+\ldots+\left(\frac{1}{4}\right)^n \quad \text{and} \quad \frac{3}{4}+\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3+\ldots+\left(\frac{3}{4}\right)^n$$

b) Substitute $t_1 = \frac{1}{4}$ and $r = \frac{1}{4}$ into $S_\infty = \frac{t_1}{1-r}$, and substitute $t_1 = \frac{3}{4}$ and $r = \frac{3}{4}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{\frac{1}{4}}{1-\frac{1}{4}} \qquad\qquad S_\infty = \frac{\frac{3}{4}}{1-\frac{3}{4}}$$

$$S_\infty = \left(\frac{1}{4}\right)\left(\frac{4}{3}\right) \qquad\qquad S_\infty = \left(\frac{3}{4}\right)\left(\frac{4}{1}\right)$$

$$S_\infty = \frac{1}{3} \qquad\qquad S_\infty = 3$$

Section 1.5 Page 65 Question 21

An infinite geometric series converges if the common ratio is between −1 and 1.

Section 1.5 Page 65 Question 22

$t_1 = 1$, $t_2 = \frac{1}{4}$

a) If the series is arithmetic, then $d = t_2 - t_1 = -\frac{3}{4}$.

Substitute into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$$S_n = \frac{n}{2}\left[2(1) + (n-1)\left(-\frac{3}{4}\right)\right]$$

$$S_n = \frac{n}{2}\left[2 - \frac{3}{4}n + \frac{3}{4}\right]$$

$$S_n = \frac{n}{2}\left[\frac{11}{4} - \frac{3}{4}n\right]$$

$$S_n = \frac{11}{8}n - \frac{3}{8}n^2$$

b) If the series is geometric, then $r = \frac{1}{4}$.

Substitute into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$S_n = \frac{1\left[\left(\frac{1}{4}\right)^n - 1\right]}{\frac{1}{4} - 1}$$

$$S_n = \left[\left(\frac{1}{4}\right)^n - 1\right]\left(-\frac{4}{3}\right)$$

$$S_n = \frac{4}{3}\left[1 - \left(\frac{1}{4}\right)^n\right]$$

c) For the sum of an infinite geometric series substitute into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{1}{1-\frac{1}{4}}$$

$$S_\infty = \frac{4}{3}$$

Section 1.5 Page 65 Question 23

Step 3

$\boldsymbol{n}$	1	2	3	4
Fraction of Paper	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$	$\frac{1}{256}$

Step 4

Area received by each student is $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}$.

The areas form an infinite geometric series, with $t_1 = \frac{1}{4}$ and $r = \frac{1}{4}$.

$$S_\infty = \frac{\frac{1}{4}}{1-\frac{1}{4}}$$

$$S_\infty = \left(\frac{1}{4}\right)\left(\frac{4}{3}\right)$$

$$S_\infty = \frac{1}{3}$$

Chapter 1 Review

Chapter 1 Review Page 66 Question 1

a) arithmetic, $d = 4$

b) arithmetic, $d = -5$

c) not arithmetic

d) not arithmetic

Chapter 1 Review Page 66 Question 2

a) From the pattern of the sequence, $t_1 = 18$ and $d = 12$.
Substitute into $t_n = t_1 + (n - 1)d$.
$t_n = 18 + (n - 1)12$
$t_n = 12n + 6$
This matches **C**.

b) From the pattern of the sequence, $t_1 = 7$ and $d = 5$.
Substitute into $t_n = t_1 + (n - 1)d$.
$t_n = 7 + (n - 1)5$
$t_n = 5n + 2$
This matches **D**.

c) From the pattern of the sequence, $t_1 = 2$ and $d = 2$.
Substitute into $t_n = t_1 + (n - 1)d$.
$t_n = 2 + (n - 1)2$
$t_n = 2n$
This matches **E**.

d) From the pattern of the sequence, $t_1 = -8$ and $d = -4$.
Substitute into $t_n = t_1 + (n - 1)d$.
$t_n = -8 + (n - 1)(-4)$
$t_n = -4n - 4$
$t_n = -4(n + 1)$
This matches **B**.

e) From the pattern of the sequence, $t_1 = 4$ and $d = 3$.
Substitute into $t_n = t_1 + (n - 1)d$.
$t_n = 4 + (n - 1)3$
$t_n = 3n + 1$
This matches **A**.

Chapter 1 Review Page 66 Question 3

$t_1 = 7$, $d = 7$

a) All of the terms are multiples of 7 and 98 is a multiple of 7.
Substitute into $t_n = t_1 + (n - 1)d$ and solve for n.
$98 = 7 + (n - 1)7$
$98 = 7n$
$n = \frac{98}{7}$
$n = 14$

b) Since 110 is not a multiple of 7, it is not a term of this sequence.

c) 378 is a multiple of 7.

$n = \dfrac{378}{7}$

$n = 54$

d) Since 575 is not a multiple of 7, it is not a term of this sequence.

Chapter 1 Review Page 66 Question 4

a) Both sequences are arithmetic.
For sequence 1: $t_1 = 2$ and $d = 7$.
Substitute $n = 17$ into $t_n = t_1 + (n - 1)d$ to determine t_{17}.

$t_{17} = 2 + (17 - 1)7$

$t_{17} = 114$

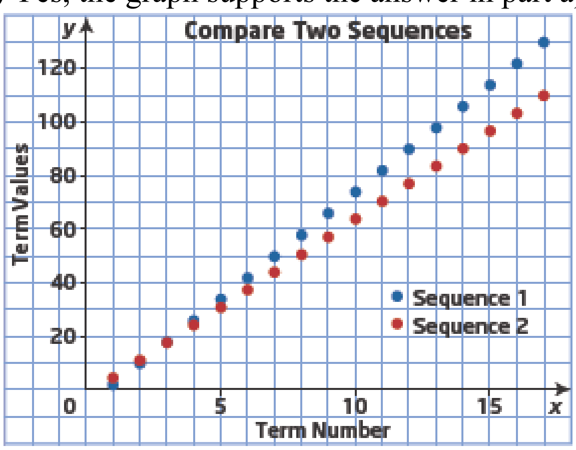
For sequence 2: $t_1 = 4$ and $d = 6$.
Substitute $n = 17$ into $t_n = t_1 + (n - 1)d$ to determine t_{17}.

$t_{17} = 4 + (17 - 1)6$

$t_{17} = 100$

Statement **A** is correct, t_{17} is greater in sequence 1.

b) Yes, the graph supports the answer in part a).



The graph shows that sequence 1 is growing at a faster rate than sequence 2. The two sequences have the same third term, 16, then further terms are greater in sequence 1.

Chapter 1 Review Page 66 Question 5

$t_1 = 5$, $t_4 = 17$
Substitute for t_4 in $t_n = t_1 + (n - 1)d$ and solve for d.

$17 = 5 + (4 - 1)d$

$12 = 3d$

$d = 4$

Then, determine t_{10}.

$t_{10} = 5 + (10 - 1)4$
$t_{10} = 41$
The tenth term of the sequence is 41.

Chapter 1 Review Page 66 Question 6

Substitute $t_1 = 200$, $d = 2$ and $n = 2020 - 1967 + 1 = 54$ into $t_n = t_1 + (n - 1)d$.
$t_{54} = 200 + (54 - 1)2$
$t_{54} = 306$
By 2020, the dam will have moved 306 cm downstream.

Chapter 1 Review Page 66 Question 7

a) Substitute $t_1 = 6$, $d = 3$, and $n = 10$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$S_{10} = \frac{10}{2}[2(6) + (10-1)3]$
$S_{10} = 5[12 + 9(3)]$
$S_{10} = 195$

b) Substitute $t_1 = 4.5$, $d = 3.5$, and $n = 12$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$S_{12} = \frac{12}{2}[2(4.5) + (12-1)3.5]$
$S_{12} = 6[9 + 11(3.5)]$
$S_{12} = 285$

c) Substitute $t_1 = 6$, $d = -3$, and $n = 10$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$S_{10} = \frac{10}{2}[2(6) + (10-1)(-3)]$
$S_{10} = 5[12 + 9(-3)]$
$S_{10} = -75$

d) Substitute $t_1 = 60$, $d = 10$, and $n = 20$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$S_{20} = \frac{20}{2}[2(60) + (20-1)10]$
$S_{20} = 10[120 + 19(10)]$
$S_{20} = 3100$

Chapter 1 Review Page 66 Question 8

Substitute $S_n = 186$ and $n = 12$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ to obtain an equation.

$186 = \frac{12}{2}[2t_1 + (12-1)d]$
$186 = 6[2t_1 + 11d]$
$31 = 2t_1 + 11d$ ①

Substitute $t_n = 83$ and $n = 20$ into $t_n = t_1 + (n - 1)d$ to obtain a second equation.
$83 = t_1 + 19d$ ②
Multiply ② by 2.
$166 = 2t_1 + 38d$ ③
$\underline{\ \ 31 = 2t_1 + 11d\ \ }$ ①
$135 = 27d$
$d = 5$
Substitute $d = 5$ in ① to find t_1.
$31 = 2t_1 + 11(5)$
$2t_1 = 31 - 55$
$t_1 = -12$
Now, substitute $t_1 = -12$, $d = 5$, and $n = 40$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$S_{40} = \frac{40}{2}[2(-12) + (40-1)5]$
$S_{40} = 20[-24 + 39(5)]$
$S_{40} = 3420$
The sum of the first 40 terms is 3420.

Chapter 1 Review Page 66 Question 9

a) Substitute $t_1 = 1$, $d = 2$, and $n = 15$ into $t_n = t_1 + (n - 1)d$.
$t_{15} = 1 + 14(2)$
$t_{15} = 29$
On the 15th day, you would contact 29 people.

b) Substitute $t_1 = 1$, $d = 2$, and $n = 15$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$S_{15} = \frac{15}{2}[2(1) + (15-1)2]$

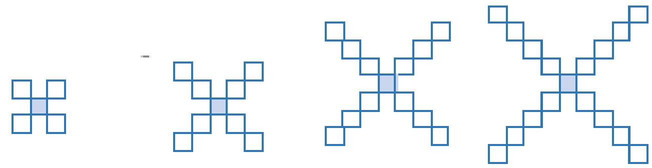
$S_{15} = \frac{15}{2}[2 + 14(2)]$

$S_{15} = 225$
By the end of the 15th day, you would have contacted 225 people.

c) Substitute $S_n = 625$, $t_1 = 1$, and $d = 2$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$ and solve for n.

$625 = \frac{n}{2}[2(1) + (n-1)2]$
$625 = n^2$
$n = 25$
You would need 25 days to contact all 625 people in the neighbourhood.

Chapter 1 Review Page 67 Question 10



a) The sequence of number of squares is 5, 5 + 4, 5 + 2(4), 5 + 3(4), …
The total number of squares in the 15th step will be:
5 + 14(4) = 61
There are 61 squares in the 15th step of the design.

b) $t_1 = 5$, $t_n = 61$, $n = 15$
Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$S_{15} = \frac{15}{2}(5 + 61)$
$S_{15} = 495$
To build all 15 steps, 495 squares are needed.

Chapter 1 Review Page 67 Question 11

Substitute $t_1 = 10$, $d = 2$ and $n = 30$ into $S_n = \frac{n}{2}[2t_1 + (n-1)d]$.

$S_{30} = \frac{30}{2}[2(10) + (30-1)2]$
$S_{30} = 15[20 + 29(2)]$
$S_{30} = 1170$
The entire concert hall has 1170 seats.

Chapter 1 Review Page 67 Question 12

a) Not geometric, because there is no common ratio between successive terms.

b) geometric: $t_1 = 1$, $r = -2$, $t_n = (-2)^{n-1}$

c) geometric: $t_1 = 1$, $r = \frac{1}{2}$, $t_n = \left(\frac{1}{2}\right)^{n-1}$

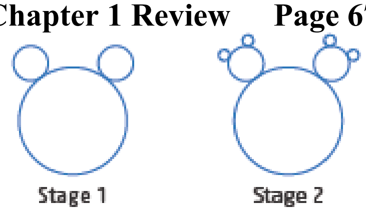
d) Not geometric, because there is no common ratio between successive terms.

Chapter 1 Review Page 67 Question 13

a) $t_1 = 5000$, $r = 100\% + 8\%$ or 1.08
At the end of the 5th hour, the 6th term will occur.
Substitute for t_6 in $t_n = t_1r^{n-1}$.
$t_6 = 5000(1.08)^5$
$t_6 = 7346.64\ldots$
The number of bacteria present at the end of 5 h is 7346.

b) $t_n = t_1r^{n-1}$
$t_n = 5000(1.08)^{n-1}$
A formula for the number of bacteria present at the start of the nth hour is $5000(1.08)^{n-1}$.
So after the nth hour, the number is $5000(1.08)^{n-1}(1.08)$ or $5000(1.08)^n$.

Chapter 1 Review Page 67 Question 14



Stage 1: radius $81\left(\frac{1}{3}\right)$ or 27 cm

Stage 2: radius $81\left(\frac{1}{3}\right)^2$ or 9 cm

Stage 3: radius $81\left(\frac{1}{3}\right)^3$ or 3 cm

Stage 4: radius $81\left(\frac{1}{3}\right)^4$ or 1 cm

So, $C = 2\pi(1) = 2\pi$ or approximately 6.28.

Chapter 1 Review Page 67 Question 15

Answers will vary.

Arithmetic Sequence
↓
Definition: A pattern of numbers in which there is a constant difference between successive terms
↓
Formula: $t_n = t_1 + (n - 1)d$
↓
Example: 4, 7, 10, 13, … $t_n = 4 + (n - 1)3$ $t_n = 1 + 3n$

Geometric Sequence
↓
Definition: A pattern of numbers in which there is a constant ratio between successive terms
↓
Formula: $t_n = t_1r^{n-1}$
↓
Example: 4, 8, 16, 32, … $t_n = 4(2)^{n-1}$

Chapter 1 Review Page 67 Question 16

a) arithmetic series

b) geometric series

c) geometric series

d) arithmetic series

e) arithmetic series with $d = \frac{1}{4}$

f) geometric series with $r = \frac{2}{3}$

Chapter 1 Review Page 68 Question 17

a) Substitute $t_1 = 6$, $r = 1.5$, and $n = 10$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$S_{10} = \frac{6(1.5^{10} - 1)}{1.5 - 1}$$
$$S_{10} = 679.980...$$

S_{10} is approximately 679.98.

b) Substitute $t_1 = 18$, $r = 0.5$, and $n = 12$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$S_{12} = \frac{18(0.5^{12} - 1)}{0.5 - 1}$$
$$S_{12} = 35.991...$$

S_{12} is approximately 35.99.

c) Substitute $t_1 = 6000$, $r = 0.1$, and $n = 20$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

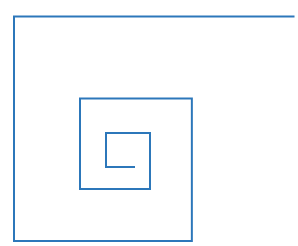
$$S_{20} = \frac{6000(0.1^{20} - 1)}{0.1 - 1}$$
$$S_{20} = 6666.666...$$
S_{20} is approximately 6666.67.

d) Substitute $t_1 = 80$, $r = \frac{1}{4}$, and $n = 9$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$S_9 = \frac{80\left[\left(\frac{1}{4}\right)^9 - 1\right]}{\frac{1}{4} - 1}$$
$$S_9 = 80\left(\frac{-262\,143}{\cancel{4^9}_{\,4^8}}\right)\left(\frac{\cancel{4}}{-3}\right)$$
$$S_9 = 5\left(\frac{87\,381}{4^6}\right)$$
$$S_9 = \frac{436\,905}{4096}$$
$$S_9 = 106.666...$$
S_9 is approximately 106.67.

Chapter 1 Review Page 68 Question 18

a) Substitute $t_1 = 4$, $r = 1.25$, and $n = 8$ into $t_n = t_1r^{n-1}$.
$t_8 = 4(1.25)^7$
$t_8 = 19.073\ldots$



The eighth line segment will be 19.1 mm long, to the nearest tenth of a millimetre.

b) Substitute $t_1 = 4$, $r = 1.25$, and $n = 20$ into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$S_{20} = \frac{4(1.25^{20} - 1)}{1.25 - 1}$$
$$S_{20} = 1371.778...$$

The total length of the spiral shape, when 20 line segments have been drawn is about 1372 mm or 1.37 m, to the nearest hundredth of a metre.

Chapter 1 Review Page 68 Question 19

a) Substitute $t_1 = 5$ and $r = \frac{2}{3}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{5}{1-\frac{2}{3}}$$

$$S_\infty = 15$$

b) Substitute $t_1 = 1$ and $r = -\frac{1}{3}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{1}{1-\left(-\frac{1}{3}\right)}$$

$$S_\infty = \frac{3}{4}$$

Chapter 1 Review Page 68 Question 20

a) Convergent; substitute $t_1 = 8$ and $r = \frac{1}{2}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{8}{1-\frac{1}{2}}$$

$$S_\infty = 16$$

b) divergent

c) Convergent; substitute $t_1 = -42$ and $r = -\frac{1}{2}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{-42}{1-\left(-\frac{1}{2}\right)}$$

$$S_\infty = -28$$

d) Convergent; substitute $t_1 = \frac{3}{4}$ and $r = \frac{1}{2}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{\frac{3}{4}}{1-\frac{1}{2}}$$

$$S_\infty = \left(\frac{3}{4}\right)\left(\frac{2}{1}\right)$$

$$S_\infty = \frac{3}{2}$$

Chapter 1 Review Page 68 Question 21

a) $r = \frac{-2.8}{7} = -\frac{2}{5}$ or -0.4

b) $S_1 = 7$
$S_2 = 7 - 2.8 = 4.2$
$S_3 = 7 - 2.8 + 1.12 = 5.32$
$S_4 = 7 - 2.8 + 1.12 - 0.448 = 4.872$
$S_5 = 7 - 2.8 + 1.12 - 0.448 + (0.448)(0.4) = 5.0512$

c) The sums seem to be approaching 5.

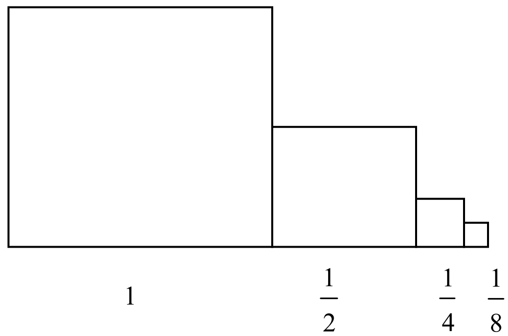
d) Substitute $t_1 = 7$ and $r = -0.4$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{7}{1-(-0.4)}$$

$$S_\infty = 5$$

The sum of the series is 5.

Chapter 1 Review Page 68 Question 22



a) Areas of the sequence of squares are: 1, $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$, $\left(\frac{1}{8}\right)^2 = \frac{1}{64}$

The areas form a geometric sequence with $t_1 = 1$ and $r = \frac{1}{4}$.

b) Sum of the areas of the four squares is $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} = \frac{64 + 16 + 4 + 1}{64}$ or $\frac{85}{64}$ square units.

c) Substitute $t_1 = 1$ and $r = \frac{1}{4}$ into $S_\infty = \frac{t_1}{1 - r}$.

$$S_\infty = \frac{1}{1 - \frac{1}{4}}$$

$$S_\infty = \frac{4}{3}$$

The infinite sum of the areas of the squares is $\frac{4}{3}$ or $1\frac{1}{3}$ square units.

Chapter 1 Review Page 68 Question 23

a)
- A series is geometric if there is a common ratio r such that r is any real number, $\boldsymbol{r \neq 1}$.
- An infinite geometric series converges if $\boldsymbol{-1 < r < 1}$.
- An infinite geometric series diverges if $\boldsymbol{r < -1}$ **or** $\boldsymbol{r > 1}$.

b) Answers will vary. Examples:
Positive common ratio: 3 + 1.5 + 0.75 +….

Substitute $t_1 = 3$ and $r = \frac{1}{2}$ into $S_\infty = \frac{t_1}{1 - r}$.

$$S_\infty = \frac{3}{1-\frac{1}{2}}$$

$$S_\infty = 6$$

Negative common ratio: $10 - 2 + \frac{2}{5} - \frac{2}{25} + \ldots$

Substitute $t_1 = 10$, $r = -\frac{1}{5}$ into $S_\infty = \frac{t_1}{1-r}$.

$$S_\infty = \frac{10}{1-\left(-\frac{1}{5}\right)}$$

$$S_\infty = \frac{25}{3} \text{ or } 8\frac{1}{3}$$

Chapter 1 Practice Test Page 69 Question 1

The difference between the terms 3 and 9 is 6. Use $d = 6$ to determine the other terms.
$3 - 6 = -3$, $9 + 6 = 15$, $15 + 6 = 21$
The best answer is **D**.

Chapter 1 Practice Test Page 69 Question 2

The pattern is an arithmetic sequence with $t_1 = 1$ and $d = 3$.
Substitute into $t_n = t_1 + (n - 1)d$.
$t_n = 1 + (n - 1)3$
$t_n = 3n - 2$
The best answer is **B**.

Chapter 1 Practice Test Page 69 Question 3

First, determine r: $r = \frac{343}{-2401} = -\frac{1}{7}$.

Substitute $t_1 = 16\ 807$, $r = -\frac{1}{7}$, and $n = 5$ into $S_n = \frac{t_1(r^n - 1)}{r-1}$.

$$S_5 = \frac{16\ 807\left[\left(-\frac{1}{7}\right)^5 - 1\right]}{-\frac{1}{7}-1}$$

$$S_5 = 14\ 707$$

The best answer is **B**.

Chapter 1 Practice Test Page 69 Question 4

Consecutive terms of an arithmetic sequence have the same difference.
So, here $d = b - a$. Then, $c = b + b - a$, or $c = 2b - a$.
The best answer is **B**.

Chapter 1 Practice Test Page 69 Question 5

$t_{20} = 524\ 288$ and $t_{14} = 8192$

$t_1r^{19} = 524\ 288$ $t_1r^{13} = 8192$

Dividing: $r^6 = \dfrac{524\ 288}{8192}$

$r = \sqrt[6]{64}$

$r = \pm 2$

Substituting into t_{14} give $t_1 = 1$.
Then, $t_3 = r^2 = \pm 4$
The best answer is **C**.

Chapter 1 Practice Test Page 69 Question 6

Substitute $t_1 = 30$, $r = 0.9$, and $n = 10$ into $t_n = t_1r^{n-1}$.
$t_{10} = 30(0.9)^9$
$t_{10} = 11.6226\ldots$
The radius of the tenth bowl is approximately 11.62 cm.

Chapter 1 Practice Test Page 69 Question 7

Answers may vary.
In an arithmetic sequence each successive term is a constant difference from the previous term, so on a graph the terms form a linear pattern. The terms of a geometric sequence have a common ratio between successive terms, so on a graph the terms form a curve pattern.

Chapter 1 Practice Test Page 70 Question 8

If 3, A, and 27 form an arithmetic sequence, then

$$27 - A = A - 3$$
$$30 = 2A$$
$$A = 15$$

If 3, B, and 27 form a geometric sequence, then

$$\frac{27}{B} = \frac{B}{3}$$
$$81 = B^2$$
$$B = 9, \quad B > 0$$

Chapter 1 Practice Test Page 70 Question 9

Substitute $S_n = 17\ 000$, $r = 1.02$, and $n = 6(52)$ or 312 into $S_n = \frac{t_1(r^n - 1)}{r - 1}$.

$$17\ 000 = \frac{t_1[(1.02)^{312} - 1]}{1.02 - 1}$$
$$t_1 = 0.7065...$$

In the first week, she walked approximately 0.7 km.

Chapter 1 Practice Test Page 70 Question 10

a) Substitute $t_1 = 5$, $t_n = 160$, $n = 6$ into $t_n = t_1 + (n - 1)d$.

$160 = 5 + 5d$
$d = 31$

Then, the missing terms in the sequence are 36, 67, 98, and 129.

b) $t_n = 5 + (n - 1)31$
$t_n = 31n - 26$

c) If the sequence is geometric, $t_6 = t_1(r^5)$.

$160 = 5r^5$
$32 = r^5$
$r = 2$

Then, the missing terms in the sequence are 10, 20, 40, and 80.

d) $t_n = 5(2)^{n-1}$

Chapter 1 Practice Test Page 70 Question 11

a) Find the first five terms by adding 17 successively.
17, 34, 51, 68, 85

b) $t_n = 17 + (n - 1)17$
$t_n = 17n$

c) Convert 6000 km to millimetres to match the units in the formula for t_n.
Substitute $t_n = 6000(1000)(100)(10)$ and solve for n.
$6000(1000)(100)(10) = 17n$
$n = 352\ 941\ 176.5…$
It has taken approximately 352 941 176 years for Europe and North America to separate 6000 km.

d) Assume that the rate of 17 mm per year was constant.

Chapter 1 Practice Test   Page 70   Question 12

a) This is an arithmetic sequence with $t_1 = 30$ and $d = 30$. To keep numbers smaller, work in minutes. The first five terms, in minutes, are 0.5, 1, 1.5, 2, and 2.5.

b) The sequence is arithmetic.

c) Substitute $t_n = 30$ into $t_n = 0.5n$ and solve for n.

$30 = 0.5n$

$n = 60$

In 60 days, the goal of 30 min exposure will be reached.

d) $t_1 = 0.5$, $t_{60} = 30$

Substitute into $S_n = \frac{n}{2}(t_1 + t_n)$.

$S_{60} = 30(0.5 + 30)$

$S_{60} = 915$

The total number of minutes of sun that the patient is exposed to is 915 min.