

### 3.4 Graphing Polynomials

**Did You Know?**  
Polynomiography is a fusion of art, mathematics, and computer science. It creates a visualization of the approximation of zeros of polynomial functions.

### Focus On ...

- describing the relationship between zeros, roots, and x-intercepts of polynomial functions and equations
- sketching the graph of a polynomial function without technology
- modelling and solving problems involving polynomial functions

### Investigate Sketching the Graph of a Polynomial Function

**Materials**  
graphing calculator or computer with graphing software

*optional*

**A: The Relationship Among the Roots, x-Intercepts, and Zeros of a Function**

- Graph the function  $f(x) = x^4 + x^3 - 10x^2 - 4x + 24$  using graphing technology.
  - Determine the x-intercepts from the graph.
  - Factor  $f(x)$ . Then, use the factors to determine the zeros of  $f(x)$ . What are the possible integral factors of this polynomial?
- Set the polynomial function  $f(x) = x^4 + x^3 - 10x^2 - 4x + 24$  equal to 0. Solve the equation  $x^4 + x^3 - 10x^2 - 4x + 24 = 0$  to determine the roots.
  - What do you notice about the roots of the equation and the x-intercepts of the graph of the function?

**Reflect and Respond**

- What is the relationship between the zeros of a function, the x-intercepts of the corresponding graph, and the roots of the polynomial equation?

### Multiplicity of a Zero

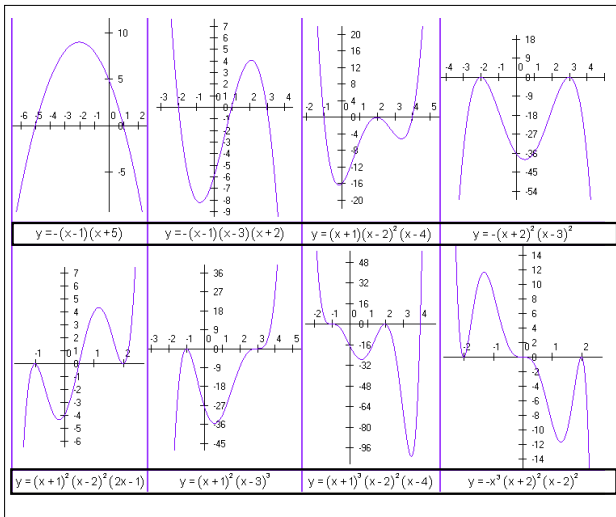
the number of times a zero of a polynomial function occurs

$$f(x) = (x - 1)^2(x + 2)$$

$$f(x) = x^3(x + 4)^2(x + 1)(x - 7)^2$$

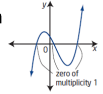
### Multiplicity of a Zero

the shape of the graph of a function close to a zero depends on its multiplicity

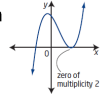


**Summary**

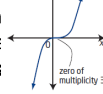
1. If a real number zero of a polynomial function has a multiplicity of one, then at that zero the graph of the polynomial will: **Cross the x-axis**



2. If a real number zero of a polynomial function has a multiplicity of two, then at that zero the graph of the polynomial will: **Tangent to the x-axis**



3. If a real number zero of a polynomial function has a multiplicity of three, then at that zero the graph of the polynomial will: **Tangent to and crosses the x-axis**



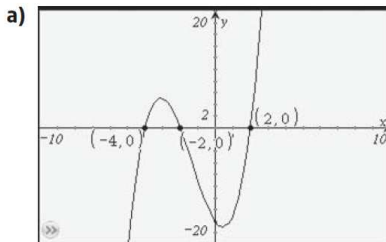
**Example 1**

**Analyse Graphs of Polynomial Functions**

*Solution*

For each graph of a polynomial function, determine

- the least possible degree
- the sign of the leading coefficient
- the x-intercepts and the factors of the function with least possible degree
- the intervals where the function is positive and the intervals where it is negative



**Example 2**

**Analyse Equations to Sketch Graphs of Polynomial Functions**

*Solution*

Sketch the graph of each polynomial function.

a)  $y = (x - 1)(x + 2)(x + 3)$

**Example 2**

**Analyse Equations to Sketch Graphs of Polynomial Functions**

Sketch the graph of each polynomial function.

b)  $f(x) = -(x + 2)^3(x - 4)$

**Example 2**

**Analyse Equations to Sketch Graphs of Polynomial Functions**

Sketch the graph of each polynomial function.

a)  $y = (x - 1)(x + 2)(x + 3)$

b)  $f(x) = -(x + 2)^3(x - 4)$

c)  $y = -2x^3 + 6x - 4$

**Your Turn**

Sketch a graph of each polynomial function by hand. State the characteristics of the polynomial functions that you used to sketch the graphs.

- a)  $g(x) = (x - 2)^3(x + 1)$
- b)  $f(x) = -x^3 + 13x + 12$

30.10	2	3	4
Outcome 3b: I can demonstrate understanding of polynomial functions of degree higher than 2 by graphing	Match a polynomial function with its graph based on degree, end behavior, number of x intercepts  Given a graph determine the least possible degree, sign of leading coefficient, x intercepts, intervals where functions is positive and negative  Analyze factored equations to sketch polynomial functions	Analyze Equations to sketch Polynomial functions  Determine an equation given specific characteristics of the polynomial function	Solve situational questions  Explain relationships between zeros and roots.

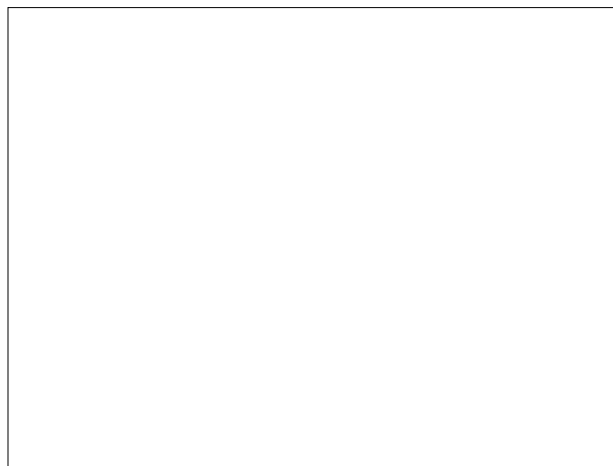
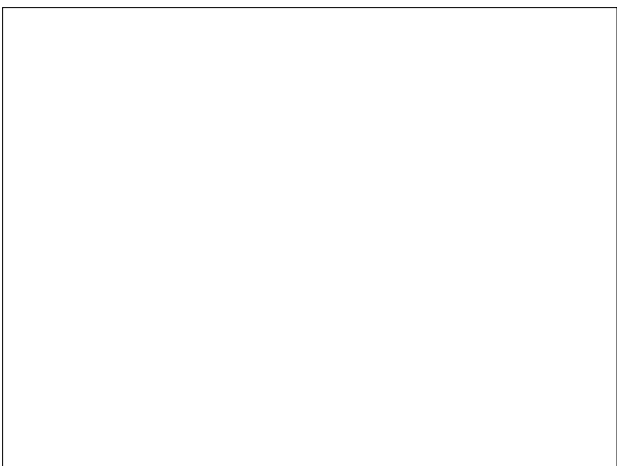
# Assignment

Page 147

# 1a, 2c, 3, 4, 5, 9e, 10      Level 2

# 9abcd, 14      Level 3

# 12, 16, 18      Level 4



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