3.2 & 3.3 Remainder and Factor Theorem

Focus On ...

- describing the relationship between polynomial long division and synthetic division
- dividing polynomials by binomials of the form $x - a$ using long division or synthetic division
- explaining the relationship between the remainder when a polynomial is divided by a binomial of the form $x - a$ and the value of the polynomial at $x = a$

Terminology

Quotient

Divisor

$$14\overline{)307}\quad 21$$

Dividend

Result

Remainder

$$\boxed{307 = 14(21) + \frac{13}{14}}$$

14 is not a factor of 307 b/c the remiander was not 0

Long Division

Let's review the process of long division...

$$12\overline{)327}$$

Solution

Long Division

The same process can be used to divide higher order polynomials...

$$x+3\overline{)x^2+7x+17}$$

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$$

Solution $\frac{x^2+7x+17}{x+3} = (x+4) + \frac{5}{x+3}$

The Quotient

$$\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$$

P(x) is a polynomial

x-a is the divisor

Q(x) is the quotient

R is the remainder

Note: by rearranging the equation, we can also write....

$$P(x) = Q(x)(x-a) + R$$

Rewrite the division statement

$$\frac{x^2+7x+17}{x+3} = (x+4) + \frac{5}{x+3}$$

Restrictions

You cannot divide by zero, then...

$$x - a \neq 0$$
$$x \neq a$$

State the restrictions on the divisor

$$\frac{x^2+7x+17}{x+3} = (x+4) + \frac{5}{x+3}$$

Example

a) Divide the polynomial $P(x) = x^4 - 2x^3 + x^2 - 3x + 4$ by $x - 1$. Express the result in the form $\frac{P(x)}{x-a} = Q(x) + \frac{R}{x-a}$.

b) Identify any restrictions on the variable.

c) Verify your answer.

Your turn...

$$\frac{2x^4 - 5x^2 - 2x}{x-2}$$

- Divide the polynomial
- State the restrictions
- Check your answer

Synthetic Division

$$x - 5\overline{)x^3 - 4x^2 - 7x + 10}$$

The 'set-up'

Note: this number always has the opposite sign

5	1	-4	-7	10
x				

Example

Divide using synthetic division

$$\frac{x^3 + 7x^2 - 3x + 4}{x-2}$$

The Remainder Theorem provides us with a shortcut to find out what the remainder will be

REMAINDER THEOREM

If a polynomial p(x) is divided by a binomial (x - a), then the remainder can be found by determining p(a)

Note: 'a' is backwards

$$\frac{x^3 + 7x^2 - 3x + 4}{x-2}$$

What is the remainder when $x^3 - 4x^2 - 7x + 10$ is divided by x - 2?

Long Division **Synthetic Division** **Remainder Theorem**

Your Turn

What is the remainder when $11x - 4x^4 - 7$ is divided by $x - 3$? Verify your answer using either long or synthetic division.

Key Ideas

- Use long division to divide a polynomial by a binomial.
- Synthetic division is an alternate form of long division.
- The result of the division of a polynomial in x, $P(x)$, by a binomial of the form $x - a$ can be written as $\frac{P(x)}{x - a} = Q(x) + \frac{R}{x - a}$ or $P(x) = (x - a)Q(x) + R$, where $Q(x)$ is the quotient and R is the remainder.
- To check the result of a division, multiply the quotient, $Q(x)$, by the divisor, $x - a$, and add the remainder, R, to the product. The result should be the dividend, $P(x)$.
- The remainder theorem states that when a polynomial in x, $P(x)$, is divided by a binomial, $x - a$, the remainder is $P(a)$. A non-zero remainder means that the binomial is not a factor of $P(x)$.

ASSIGNMENT

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# 8 - 10	Level 3
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Your Turn

The volume of a rectangular prism is given by $V(x) = x^3 + 3x^2 - 36x + 32$. Determine possible measures for w and h in terms of x if the length, l, is $x - 4$.