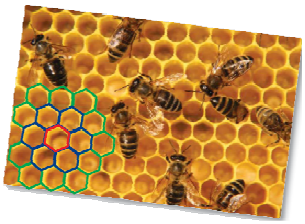
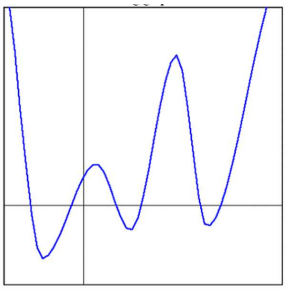
3.1 Characteristics of Polynomial Functions

Focus On ...

- identifying polynomial functions
- analysing polynomial functions





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Investigate Graphs of Polynomial Functions

Materials
- graphing calculator or computer with graphing software

1. Graph each set of functions on a different set of coordinate axes using graphing technology. Sketch the results.

Type of Function	Set A	Set B	Set C	Set D
linear	$y = x$	$y = -3x$	$y = x + 1$	
quadratic	$y = x^2$	$y = -2x^2$	$y = x^2 - 3$	$y = x^2 - x - 2$
cubic	$y = x^3$	$y = -4x^3$	$y = x^3 - 4$	$y = x^3 + 4x^2 + x - 6$
quartic	$y = x^4$	$y = -2x^4$	$y = x^4 + 2$	$y = x^4 + 2x^3 - 7x^2 - 8x + 12$
quintic	$y = x^5$	$y = -x^5$	$y = x^5 - 1$	$y = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$

Work through the investigation in groups of 2.
Do all questions # 1 - 8

Polynomial Functions

A real polynomial function is any function of the form

$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2 x^2 + a_1 x^1 + a_o$

where $a_o \ldots a_n$ are real numbers and n is a whole number

Examples

$f(x) = 2x - 1$ **$f(x) = x^2 + x - 6$**

$f(x) = x^3 + 2x^2 - 5x - 6$

Terminology

The **degree** is the exponent of the greatest power of the variable.

$$y = 2x^3 + x^2 - x + 4$$

The **leading coefficient** is the coefficient of the greatest power.

The **constant term** is the value that is not affected by the variable.

Practice

Example	Degree	Leading Coefficient	Constant
$3x^2 - 5x + 2$			
$-2x^4 - 2$			
$0x^2 + 2x - 3$			
$3x - 3$			
$x^{1/2} - 4$			

end behaviour

- the behaviour of the *y*-values of a function as $|x|$ becomes very large

Example 1

Identify Polynomial Functions

Which functions are polynomials? Justify your answer. State the degree, the leading coefficient, and the constant term of each polynomial function.

a) $g(x) = \sqrt{x} + 5$

b) $f(x) = 3x^4$

c) $y = |x|$

d) $y = 2x^3 + 3x^2 - 4x - 1$

Solution

Your Turn

Identify whether each function is a polynomial function. Justify your answer. State the degree, the leading coefficient, and the constant term of each polynomial function.

a) $h(x) = \frac{1}{x}$

b) $y = 3x^2 - 2x^5 + 4$

c) $y = -4x^4 - 4x + 3$

d) $y = x^{\frac{1}{2}} - 7$

Example 3

Application of a Polynomial Function

A bank vault is built in the shape of a rectangular prism. Its volume, V, is related to the width, w, in metres, of the vault doorway by the function $V(w) = w^3 + 13w^2 + 54w + 72$.

a) What is the volume, in cubic metres, of the vault if the door is 1 m wide?

b) What is the least volume of the vault? What is the width of the door for this volume? Why is this situation not realistic?

Your Turn

A toaster oven is built in the shape of a rectangular prism. Its volume, V, in cubic inches, is related to the height, h, in inches, of the oven door by the function $V(h) = h^3 + 10h^2 + 31h + 30$.

a) What is the volume, in cubic inches, of the toaster oven if the oven door height is 8 in.?

b) What is the height of the oven door for the least toaster oven volume? Explain.

30.10	2	3	4
Outcome 3b: I can demonstrate understanding of polynomial functions of degree higher than 2 by graphing	Match a polynomial function with its graph based on degree, end behavior, number of x intercepts Given a graph determine the least possible degree, sign of leading coefficient, x intercepts, intervals where functions is positive and negative Analyze factored equations to sketch polynomial functions	Analyze Equations to sketch Polynomial functions	Solve problems Explain relationships between zeros and roots.

Assignment

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Level 4

Do 1 - 10 all