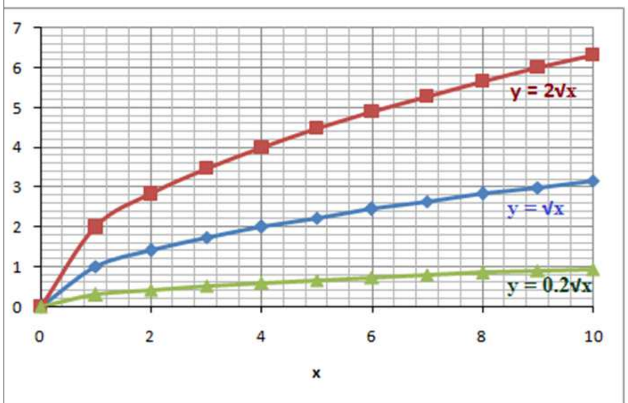
2.1 Radical Functions and Transformations



Focus On ...

- investigating the function y = √x using a table of values and a graph
- graphing radical functions using transformations
- identifying the domain and range of radical functions

Key Terms

radical function square root of a function

radical function

- a function that involves a radical with a variable in the radicand
- $y = \sqrt{3x}$ and $y = 4\sqrt[3]{5 + x}$ are radical functions.

Does a feather fall more slowly than a rock? Galileo Galilei, a mathematician and scientist, pondered this question more than 400 years ago.

In 1971, during the Apollo 15 lunar landing, Commander David Scott performed a similar demonstration on live television. Because the surface of the moon is essentially a vacuum, a hammer and a feather fell at the same rate.

Investigation

page 16 in your textbook

Let's do this as a group...

Investigate a Radical Function

Materials

- grid paper
- graphing technology (optional)

For objects falling near the surface of Earth, the function $d = 5t^2$ approximately models the time, t, in seconds, for an object to fall a distance, d, in metres, if the resistance caused by air can be ignored.

1. a) Identify any restrictions on the domain of this function. Why are these restrictions necessary? What is the range of the function?
   b) Create a table of values and a graph showing the distance fallen as a function of time.
2. Express time in terms of distance for the distance-time function from step 1. Represent the new function graphically and using a table of values.
3. For each representation, how is the equation of the new function from step 2 related to the original function?

Reflect and Respond

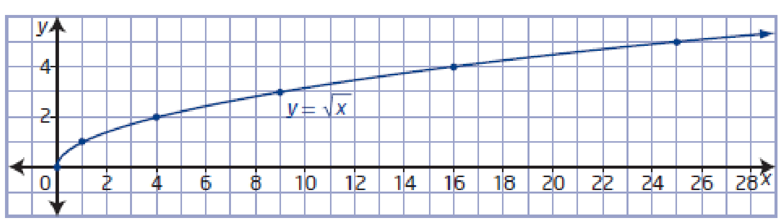
4. a) The original function is a distance-time function. What would you call the new function? Under what circumstances would you use each function?
   b) What is the shape of the graph of the original function? Describe the shape of the graph of the new function.

Radical Functions

Using technology, graph the following function...

$$y = \sqrt{x}$$

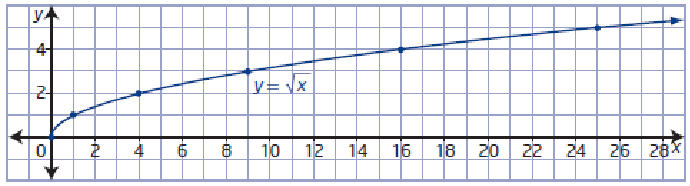
x	y
0	0
1	1
4	2
9	3
16	4
25	5



- domain and range
- shape
- endpoint

$$y = \sqrt{x}$$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

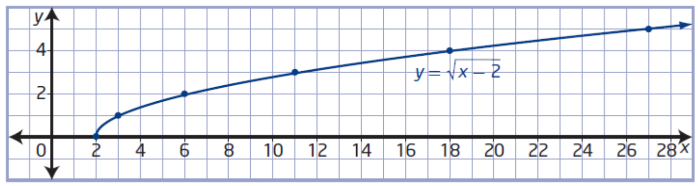


Now let's apply a transformation...

Use your knowledge of transformations to predict!

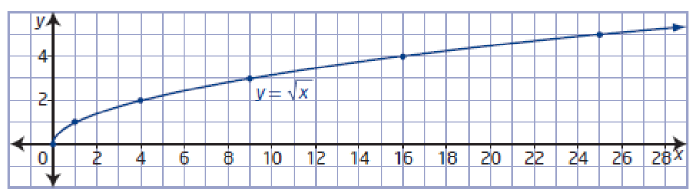
$$y = \sqrt{x-2}$$

x	y
2	0
3	1
6	2
11	3
18	4
27	5



$$y = \sqrt{x}$$

x	y
0	0
1	1
4	2
9	3
16	4
25	5



$$y = \sqrt{x-2}$$

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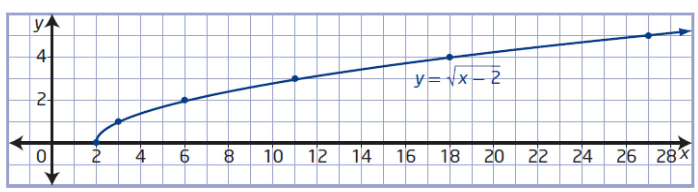
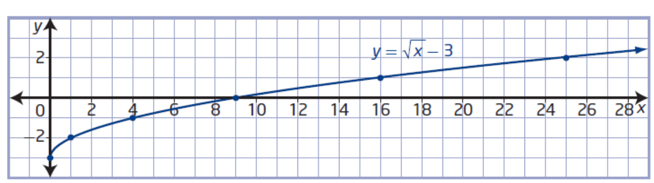


$$y = \sqrt{x} - 3$$

x	y
0	−3
1	−2
4	−1
9	0
16	1
25	2



How are they the same? How are they different?
Domain? Range?

Your turn...

Sketch the graph of $y = \sqrt{x+5}$ by creating a table of values and the sketching the graph. What is the domain? Range?

Previously, we have explored transforming functions with a, b, h, and k

$$y = af(b(x - h)) + k$$

Now, if $y = \sqrt{x}$ then we can write...

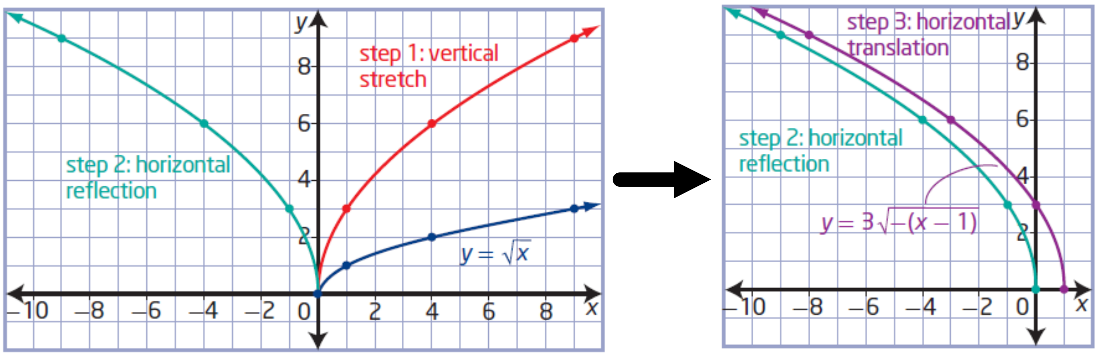
$$y = a\sqrt{b(x-h)} + k$$

How will a, b, h, and k affect the graph?

Example

Sketch the graph of the function $y = 3\sqrt{-(x-1)}$
Compare the domain and range

Method 1 Transform the Graph



method 2 next page

Method 2 Mapping Points

$$y = 3\sqrt{-(x-1)}$$

Transformation of $y = \sqrt{x}$	Mapping
Vertical stretch by a factor of 3	$(0, 0) \to (0, 0)$ $(1, 1) \to (1, 3)$ $(4, 2) \to (4, 6)$ $(9, 3) \to (9, 9)$
Horizontal reflection in the y-axis	$(0, 0) \to (0, 0)$ $(1, 3) \to (-1, 3)$ $(4, 6) \to (-4, 6)$ $(9, 9) \to (-9, 9)$
Horizontal translation of 1 unit to the right	$(0, 0) \to (1, 0)$ $(-1, 3) \to (0, 3)$ $(-4, 6) \to (-3, 6)$ $(-9, 9) \to (-8, 9)$

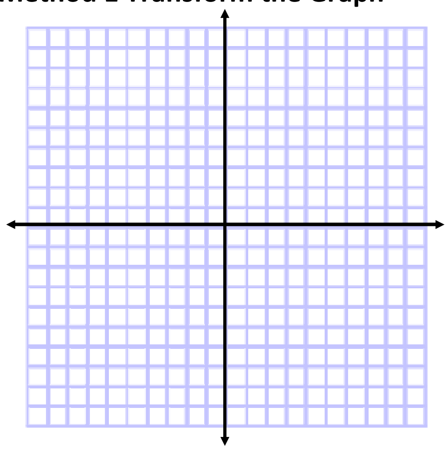
Example

Sketch the graph of the function $y=-2\sqrt{x+3}-1$ by transforming the graph of $y=\sqrt{x}$

Identify the domain and the range of $y=\sqrt{x}$ and describe how they are affected by the transformations

Answer

Method 1 Transform the Graph



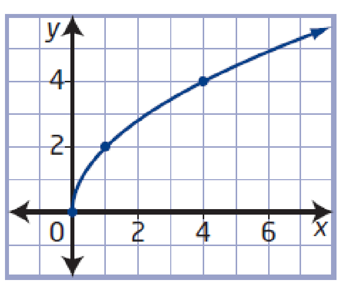
$y=\sqrt{x}$

$y=-2\sqrt{x+3}-1$

Method 2 Mapping Points

Example

Determine a radical from a graph

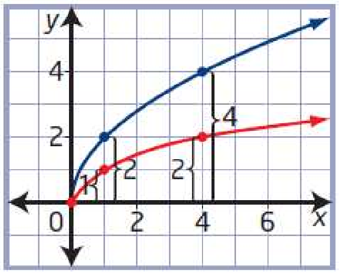


solution next two pages

Graphically

View as a Vertical Stretch ($y = a\sqrt{x}$)

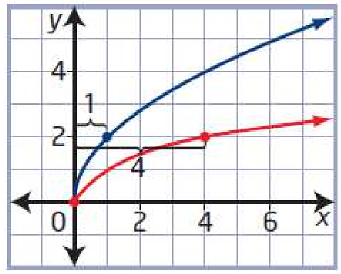
Each vertical distance is 2 times the corresponding distance for $y = \sqrt{x}$.



This represents a vertical stretch by a factor of 2, which means $a = 2$. The equation $y = 2\sqrt{x}$ represents the function.

View as a Horizontal Stretch ($y = \sqrt{bx}$)

Each horizontal distance is $\frac{1}{4}$ the corresponding distance for $y = \sqrt{x}$.



This represents a horizontal stretch by a factor of $\frac{1}{4}$, which means $b = 4$. The equation $y = \sqrt{4x}$ represents the function.

Algebraically

View as a Vertical Stretch

Substitute 1 for x and 2 for y in the equation $y = a\sqrt{x}$. Then, solve for a.

$y = a\sqrt{x}$

$2 = a\sqrt{1}$

$2 = a(1)$

$2 = a$

The equation of the function is $y = 2\sqrt{x}$.

View as a Horizontal Stretch

Substitute the coordinates (1, 2) in the equation $y = \sqrt{bx}$ and solve for b.

$y = \sqrt{bx}$

$2 = \sqrt{b(1)}$

$2 = \sqrt{b}$

$2^2 = (\sqrt{b})^2$

$4 = b$

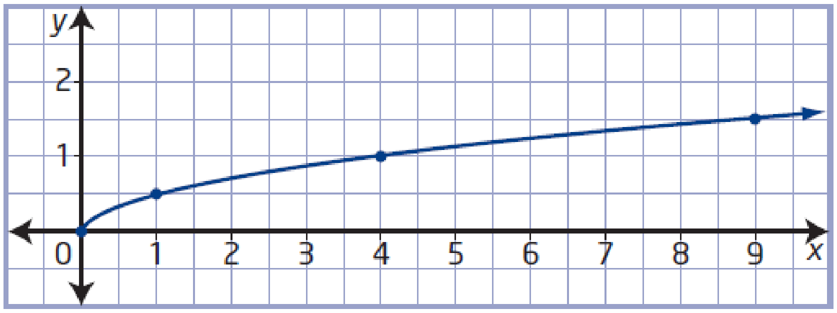
The equation can also be expressed as $y = \sqrt{4x}$.

Your Turn

a) Determine two forms of the equation for the function shown. The function is a transformation of the function $y = \sqrt{x}$.

b) Show algebraically that the two equations are equivalent.

c) What is the equation of the curve reflected in each quadrant?



30.11	**2**	**3**	**4**
Outcome 2a: I can demonstrate understanding of radical functions.	I can demonstrate the process of: • sketch the graph of $y = \sqrt{(x)}$ using a table of values • identify the role of a, b, h, k given an equation	I can use transformations to graph $y - k = a\sqrt{b(x-h)}$ I can explain the role of a, b, h, and k given an equation. Sketch the graph of $y = \sqrt{f(x)}$ given the graph of $y = f(x)$ I can compare the domains and ranges of $y = \sqrt{f(x)}$ and $y = f(x)$ Graphically solve Radical Equations	I can determine a radical function from its graph Explain level 2 and 3 concepts Express level 2 and 3 answers in simplest form with no errors

ASSIGNMENT

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 # Level 2

 # Level 3

 # Level 4

1 - 8, 10 - 12, 14, 16

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